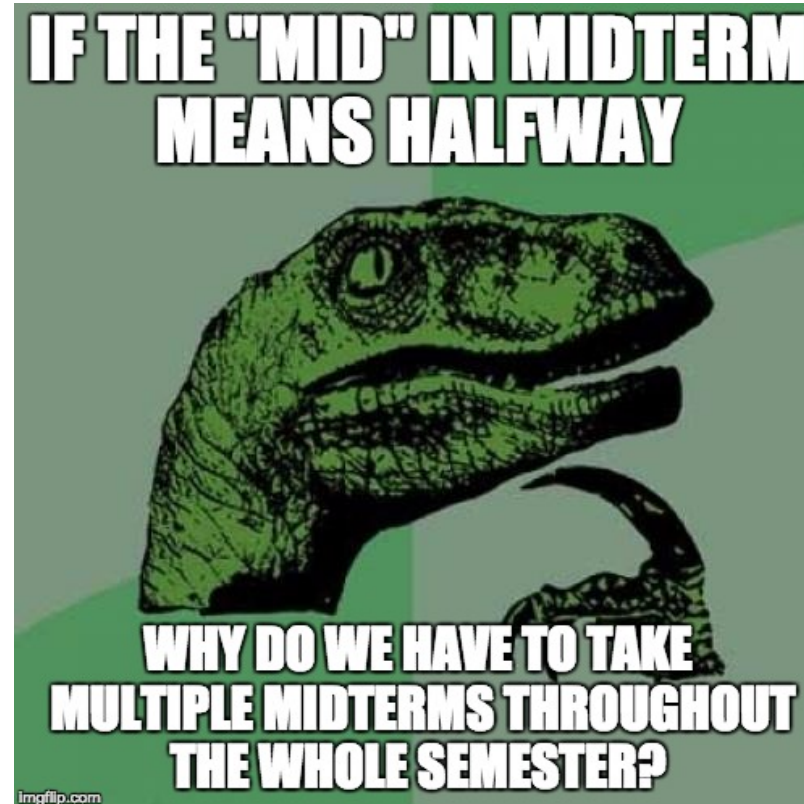


<http://tiny.cc/ift3355>

IFT 3355: INFOGRAPHIE UNE REVUE



Mikhail Bessmeltsev

LES COMPOSITIONS SIMPLES

$$\text{Tr}(x_1, y_1, z_1) \cdot \text{Tr}(x_2, y_2, z_2) = ?$$

$$\text{Tr}(x_2, y_2, z_2) \cdot \text{Tr}(x_1, y_1, z_1) \quad ? \quad \text{Tr}(x_2, y_2, z_2) \cdot \text{Tr}(x_1, y_1, z_1)$$

$$\text{Scale}(a, b, c) \cdot \text{Scale}(d, e, f) = ?$$

$$\text{Scale}(a, b, c) \cdot \text{Scale}(d, e, f) \quad ? \quad \text{Scale}(d, e, f) \cdot \text{Scale}(a, b, c)$$

$$\text{Rot}(\alpha, 0, 0, 1) \cdot \text{Rot}(\beta, 0, 0, 1) = ?$$

$$\text{Rot}(\alpha, 0, 0, 1) \cdot \text{Rot}(\beta, 0, 0, 1) \quad ? \quad \text{Rot}(\beta, 0, 0, 1) \cdot \text{Rot}(\alpha, 0, 0, 1)$$

LES COMPOSITIONS SIMPLES

$$\text{Tr}(x_1, y_1, z_1) \cdot \text{Tr}(x_2, y_2, z_2) = \text{Tr}(x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

$$\text{Tr}(x_2, y_2, z_2) \cdot \text{Tr}(x_1, y_1, z_1) = \text{Tr}(x_2, y_2, z_2) \cdot \text{Tr}(x_1, y_1, z_1)$$

$$\text{Scale}(a, b, c) \cdot \text{Scale}(d, e, f) = \text{Scale}(ad, be, cf)$$

$$\text{Scale}(a, b, c) \cdot \text{Scale}(d, e, f) = \text{Scale}(d, e, f) \cdot \text{Scale}(a, b, c)$$

$$\text{Rot}(\alpha, 0, 0, 1) \cdot \text{Rot}(\beta, 0, 0, 1) = \text{Rot}(\alpha + \beta, 0, 0, 1)$$

$$\text{Rot}(\alpha, 0, 0, 1) \cdot \text{Rot}(\beta, 0, 0, 1) = \text{Rot}(\beta, 0, 0, 1) \cdot \text{Rot}(\alpha, 0, 0, 1)$$

LES COMPOSITIONS PLUS COMPLEXES

$$\textit{Tr}(x, y, z) \cdot \textit{Scale}(a, b, c) ? \textit{Scale}(a, b, c) \cdot \textit{Tr}(x, y, z)$$

LES COMPOSITIONS PLUS COMPLEXES

$$\begin{aligned} & Tr(x, y, z) \cdot Scale(a, b, c) \neq Scale(a, b, c) \cdot Tr(x, y, z) \\ & Tr(x, y, z) \cdot Scale(a, b, c) = Scale(a, b, c) \cdot Tr\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right) \end{aligned}$$

LES COMPOSITIONS PLUS COMPLEXES

$$\begin{aligned} & Tr(x, y, z) \cdot Scale(a, b, c) \neq Scale(a, b, c) \cdot Tr(x, y, z) \\ & Tr(x, y, z) \cdot Scale(a, b, c) = Scale(a, b, c) \cdot Tr\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right) \end{aligned}$$

$$\begin{aligned} & Tr(x, y, z) \cdot Rot(\alpha, 0, 0, 1) \neq Rot(\alpha, 0, 0, 1) \cdot Tr(x, y, z) \\ & Rot(\alpha, 0, 0, 1) \cdot Rot(\beta, 0, 1, 0) ? Rot(\beta, 0, 1, 0) \cdot Rot(\alpha, 0, 0, 1) \end{aligned}$$

LES COMPOSITIONS PLUS COMPLEXES

$$\begin{aligned} Tr(x, y, z) \cdot Scale(a, b, c) &\neq Scale(a, b, c) \cdot Tr(x, y, z) \\ Tr(x, y, z) \cdot Scale(a, b, c) &= Scale(a, b, c) \cdot Tr\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right) \end{aligned}$$

$$\begin{aligned} Tr(x, y, z) \cdot Rot(\alpha, 0, 0, 1) &\neq Rot(\alpha, 0, 0, 1) \cdot Tr(x, y, z) \\ Rot(\alpha, 0, 0, 1) \cdot Rot(\beta, 0, 1, 0) &\neq Rot(\beta, 0, 1, 0) \cdot Rot(\alpha, 0, 0, 1) \end{aligned}$$

LES COMPOSITIONS PLUS COMPLEXES

$$\begin{aligned} Tr(x, y, z) \cdot Scale(a, b, c) &\neq Scale(a, b, c) \cdot Tr(x, y, z) \\ Tr(x, y, z) \cdot Scale(a, b, c) &= Scale(a, b, c) \cdot Tr\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right) \end{aligned}$$

$$\begin{aligned} Tr(x, y, z) \cdot Rot(\alpha, 0, 0, 1) &\neq Rot(\alpha, 0, 0, 1) \cdot Tr(x, y, z) \\ Rot(\alpha, 0, 0, 1) \cdot Rot(\beta, 0, 1, 0) &\neq Rot(\beta, 0, 1, 0) \cdot Rot(\alpha, 0, 0, 1) \\ Scale(a, a, a) \cdot Rot(\beta, 0, 0, 1) &? Rot(\beta, 0, 0, 1) \cdot Scale(a, a, a) \end{aligned}$$

LES COMPOSITIONS PLUS COMPLEXES

$$\begin{aligned} Tr(x, y, z) \cdot Scale(a, b, c) &\neq Scale(a, b, c) \cdot Tr(x, y, z) \\ Tr(x, y, z) \cdot Scale(a, b, c) &= Scale(a, b, c) \cdot Tr\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right) \end{aligned}$$

$$\begin{aligned} Tr(x, y, z) \cdot Rot(\alpha, 0, 0, 1) &\neq Rot(\alpha, 0, 0, 1) \cdot Tr(x, y, z) \\ Rot(\alpha, 0, 0, 1) \cdot Rot(\beta, 0, 1, 0) &\neq Rot(\beta, 0, 1, 0) \cdot Rot(\alpha, 0, 0, 1) \\ Scale(a, a, a) \cdot Rot(\beta, 0, 0, 1) &= Rot(\beta, 0, 0, 1) \cdot Scale(a, a, a) \end{aligned}$$

LES COMPOSITIONS PLUS COMPLEXES

$$\begin{aligned} Tr(x, y, z) \cdot Scale(a, b, c) &\neq Scale(a, b, c) \cdot Tr(x, y, z) \\ Tr(x, y, z) \cdot Scale(a, b, c) &= Scale(a, b, c) \cdot Tr\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right) \end{aligned}$$

$$\begin{aligned} Tr(x, y, z) \cdot Rot(\alpha, 0, 0, 1) &\neq Rot(\alpha, 0, 0, 1) \cdot Tr(x, y, z) \\ Rot(\alpha, 0, 0, 1) \cdot Rot(\beta, 0, 1, 0) &\neq Rot(\beta, 0, 1, 0) \cdot Rot(\alpha, 0, 0, 1) \\ Scale(a, a, a) \cdot Rot(\beta, 0, 0, 1) &= Rot(\beta, 0, 0, 1) \cdot Scale(a, a, a) \end{aligned}$$

$$Scale(a, b, c) \cdot Rot(\beta, 0, 0, 1) \neq Rot(\beta, 0, 0, 1) \cdot Scale(a, b, c)$$

LE CHANGEMENT D'ÉCHELLE NON UNIFORME **APRÈS** LA ROTATION

- OK!

- $M = Rot(\beta, 0, 0, 1) \cdot Scale(a, b, c) =$

$$\begin{pmatrix} \cos(\beta) & -\sin(\beta) & 0 & 0 \\ \sin(\beta) & \cos(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{pmatrix} =$$

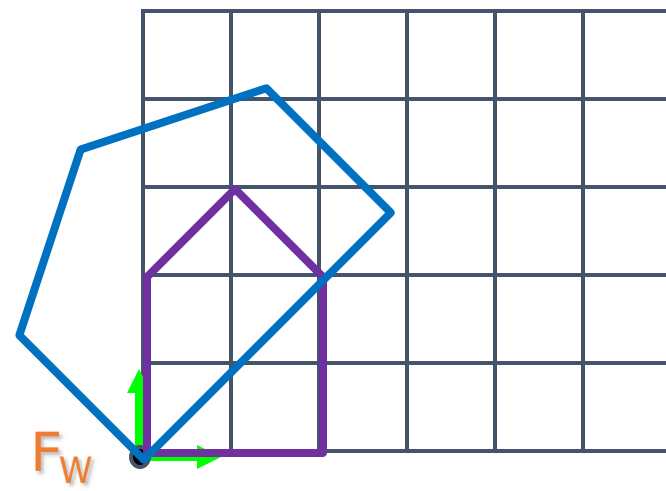
$$\begin{pmatrix} a \cdot \cos(\beta) & -b \cdot \sin(\beta) & 0 & 0 \\ a \cdot \sin(\beta) & b \cdot \cos(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LE CHANGEMENT D'ÉCHELLE NON UNIFORME **APRÈS** LA ROTATION

- OK!
- $M = Rot(\beta, 0, 0, 1) \cdot Scale(a, b, c) =$

$$\begin{pmatrix} a \cdot \cos(\beta) & -b \cdot \sin(\beta) & 0 & 0 \\ a \cdot \sin(\beta) & b \cdot \cos(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Les nouveaux vecteurs de la base sont orthogonaux!
- La rotation + le changement d'échelle, comme on s'y attendrait



LE CHANGEMENT D'ÉCHELLE NON UNIFORME **AVANT** LA ROTATION

- Pas quelque chose qu'on s'attend!
- $M = Scale(a, b, c) \cdot Rot(\beta, 0, 0, 1) =$

$$\begin{pmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\beta) & -\sin(\beta) & 0 & 0 \\ \sin(\beta) & \cos(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$
$$\begin{pmatrix} a \cdot \cos(\beta) & -a \cdot \sin(\beta) & 0 & 0 \\ b \cdot \sin(\beta) & b \cdot \cos(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LE CHANGEMENT D'ÉCHELLE NON UNIFORME **AVANT** LA ROTATION

- Pas quelque chose qu'on s'attend!
- $M = Scale(a, b, c) \cdot Rot(\beta, 0, 0, 1) =$

$$\begin{pmatrix} a \cdot \cos(\beta) & -a \cdot \sin(\beta) & 0 & 0 \\ b \cdot \sin(\beta) & b \cdot \cos(\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Les vecteurs de la base ne sont pas orthogonaux!
- (démonstration)

LES TRANSFORMATIONS INVERSES

$$Tr(x, y, z)^{-1} = ?$$

$$Rot(\alpha, 0, 0, 1)^{-1} = ?$$

$$Scale(a, b, c)^{-1} = ?$$

LES TRANSFORMATIONS INVERSES

$$\text{Tr}(x, y, z)^{-1} = \text{Tr}(-x, -y, -z)$$

$$\text{Rot}(\alpha, 0, 0, 1)^{-1} = \text{Rot}(-\alpha, 0, 0, 1) = \text{Rot}(\alpha, 0, 0, 1)^T \text{ (orthogonal!)}$$

$$\text{Scale}(a, b, c)^{-1} = \text{Scale}\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$$

LES TRANSFORMATIONS INVERSES

$$M = Tr(1,2,3) \cdot Rot(45,0,0,1) \cdot Scale(2,2,2)$$

$$M^{-1} = ?$$

LES TRANSFORMATIONS INVERSES

$$M = Tr(1,2,3) \cdot Rot(45,0,0,1) \cdot Scale(2,2,2)$$

$$M^{-1} = (Scale(2,2,2))^{-1} \cdot (Rot(45,0,0,1))^{-1} \cdot (Tr(1,2,3))^{-1}$$

$$M^{-1} = Scale(0.5,0.5,0.5) \cdot Rot(-45,0,0,1) \cdot Tr(-1, -2, -3)$$

LES MATHS

LES PRODUITS

- Scalaire $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = x * a + y * b + z * c$

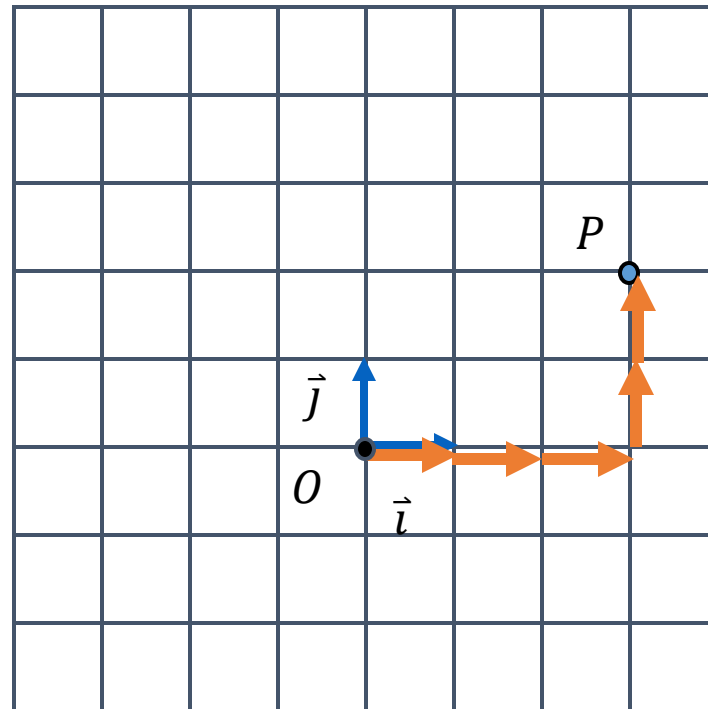
- Utilisé pour?
- Vectoriel

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$$

- Utilisé pour?

LES SYSTÈMES DE COORDONNÉES

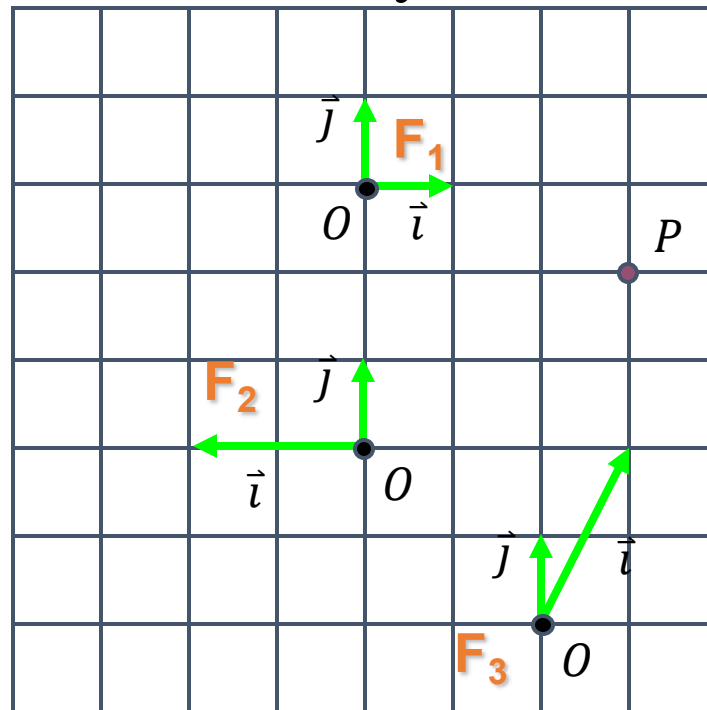
Un système de coordonnées = origine + vecteurs de base



$$P = O + x\vec{i} + y\vec{j}$$

LES MATHS

- Travailler avec des systèmes d'axe



$$P = O + x\vec{i} + y\vec{j}$$

$$\mathbf{F}_1 \quad \mathbf{P}(3,-1)$$

$$\mathbf{F}_2 \quad \mathbf{P}(-1.5,2)$$

$$\mathbf{F}_3 \quad \mathbf{P}(1,2)$$

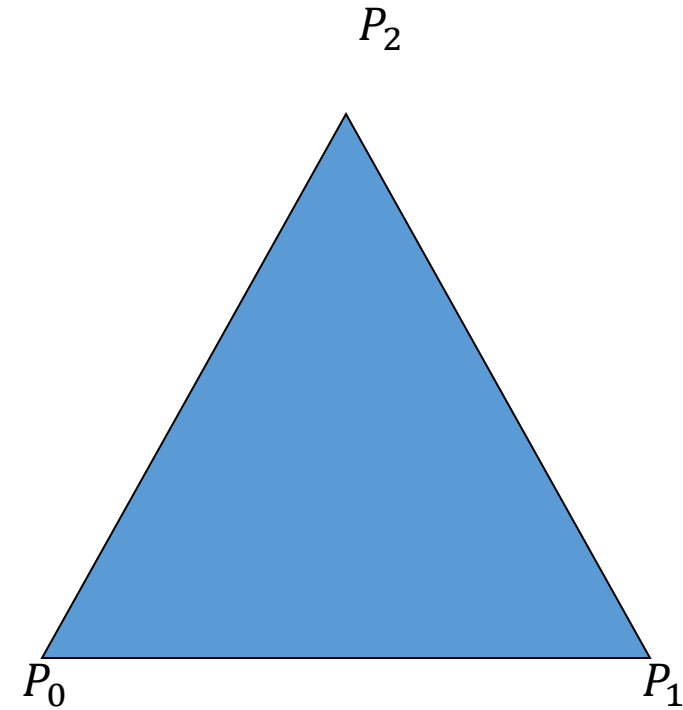
LE TRIANGLE

- La normale

$$n = \frac{(P_1 - P_0) \times (P_2 - P_0)}{\|(P_1 - P_0) \times (P_2 - P_0)\|}$$

- L'aire

$$A = \frac{1}{2} \|\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}\|$$

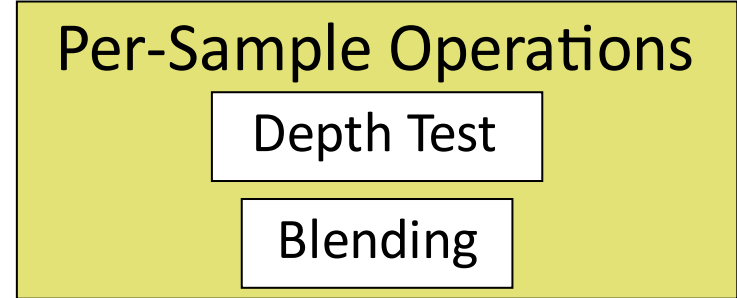
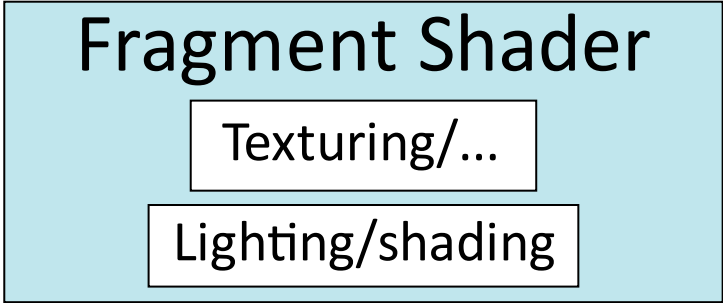
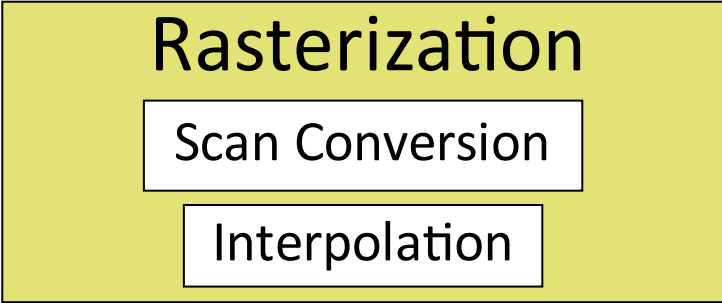
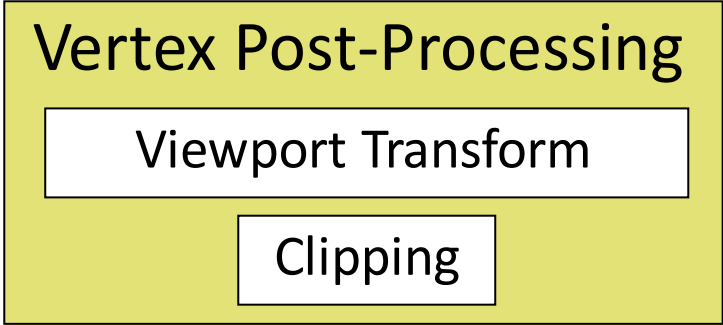
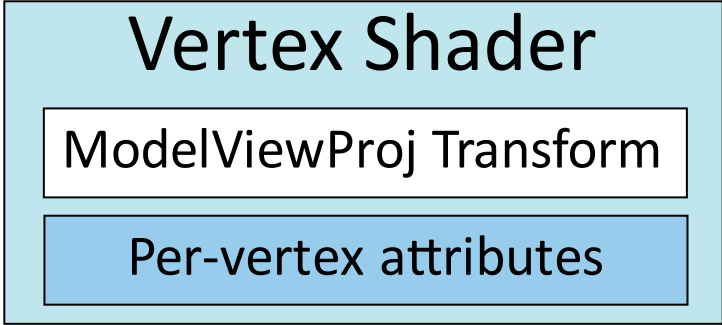


LE PLAN

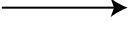
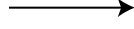
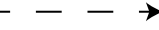
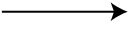
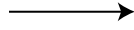
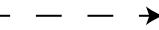
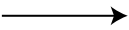
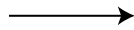
- L'équation implicite: $Ax + By + Cz + D = 0$
 - Normaliser (une option): $A^2 + B^2 + C^2 = 1$
 - (A, B, C) - normale au plan
- Comment trouver l'équation si on a trois points?

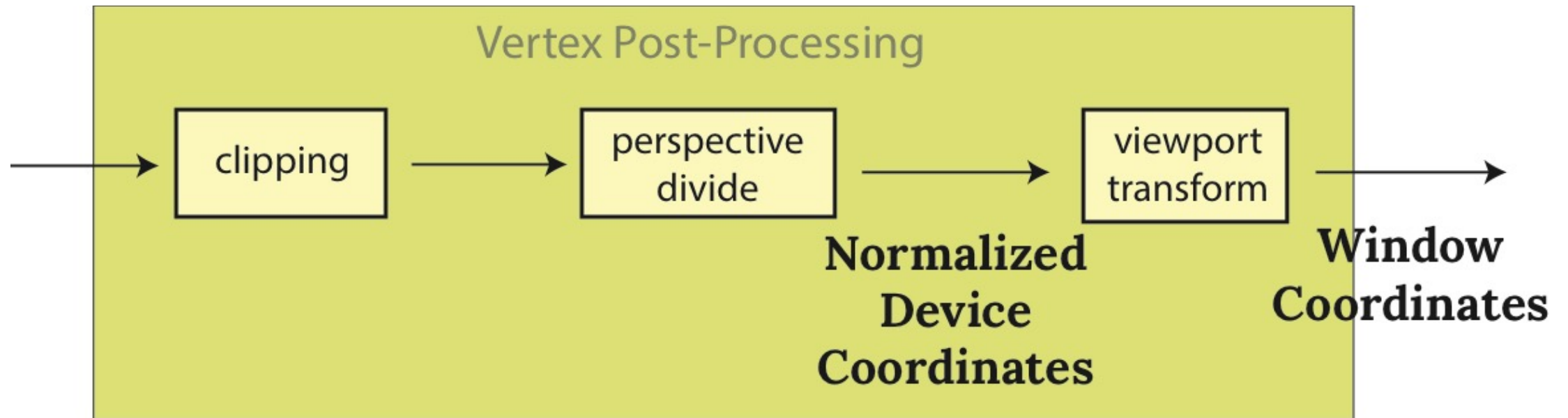
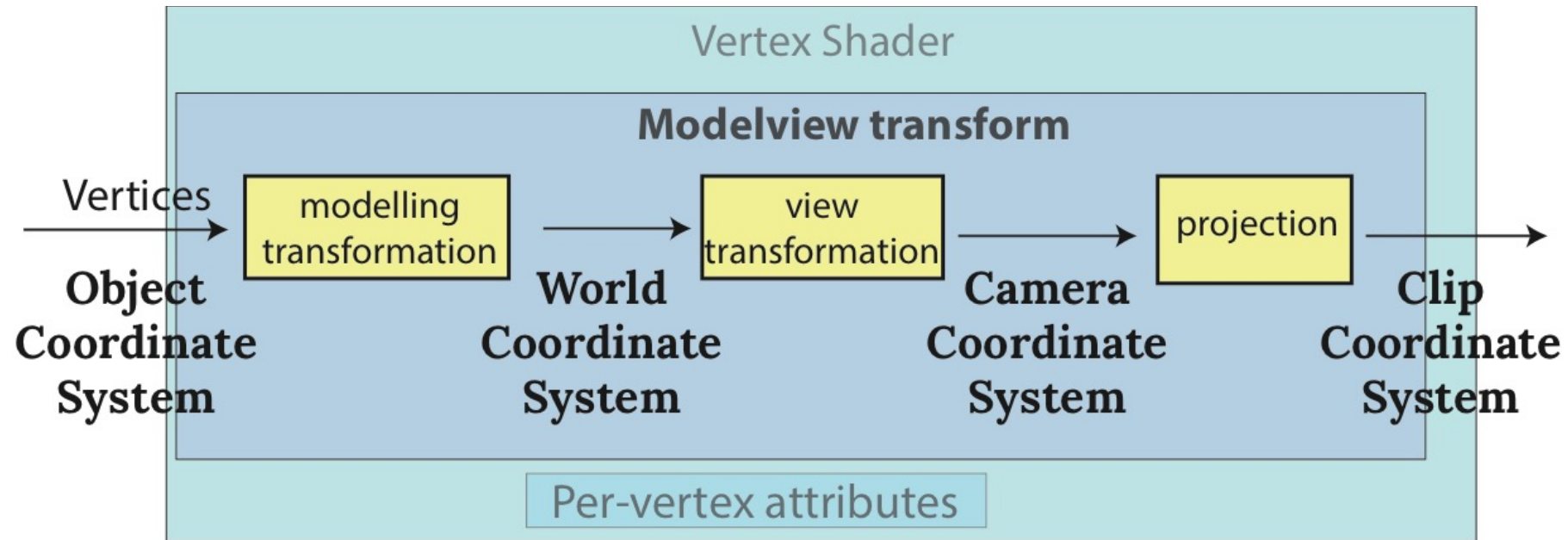
SE SOUVENIR DU PIPELINE

Sommets
et attributs

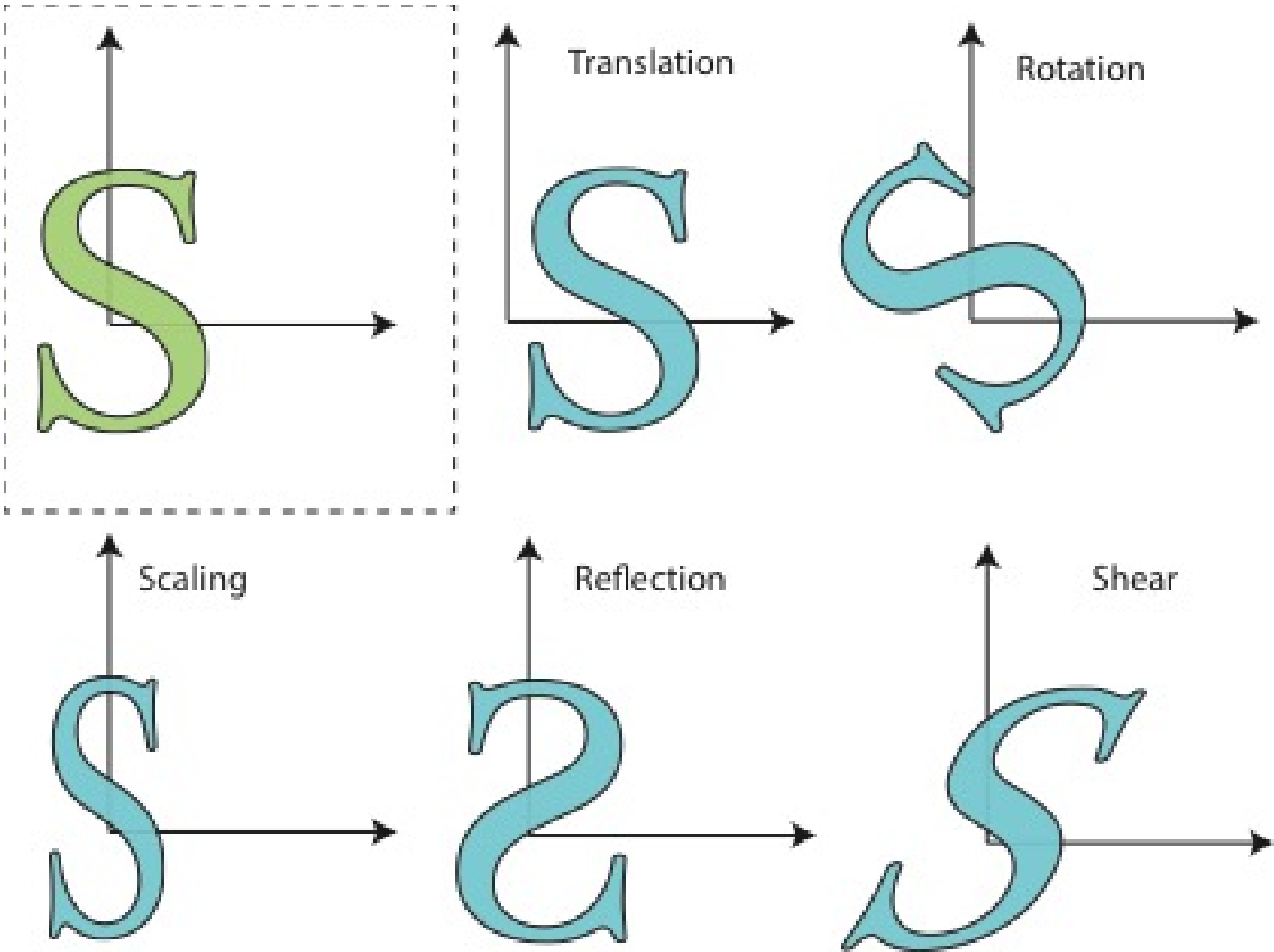


Framebuffer





LES TRANSFORMATIONS AFFINES



LES COORDONNÉES HOMOGÈNES

Les coordonnées homogènes $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \rightarrow \begin{bmatrix} x/w \\ y/w \\ z/w \end{bmatrix}$ Les coordonnées euclidiennes

Pas 1-à-1!

LA MATRICE AUGMENTÉE

La transformation linéaire

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & b_x \\ m_{21} & m_{22} & m_{23} & b_y \\ m_{31} & m_{32} & m_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation



LES TRANSFORMATIONS

Que font ces transformations affines 2D?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

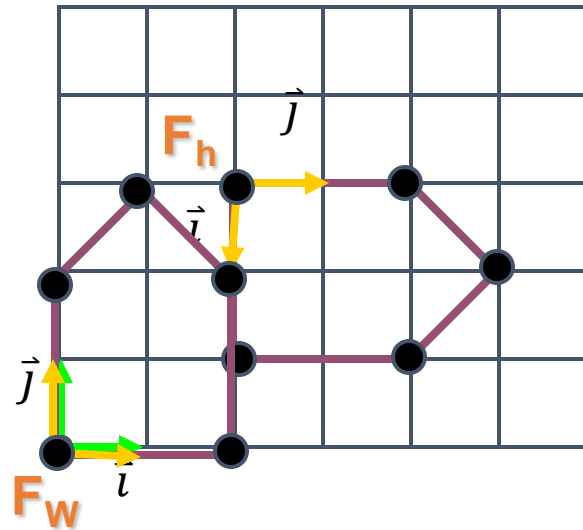
PENSER

- Que conserve chaque transformation?

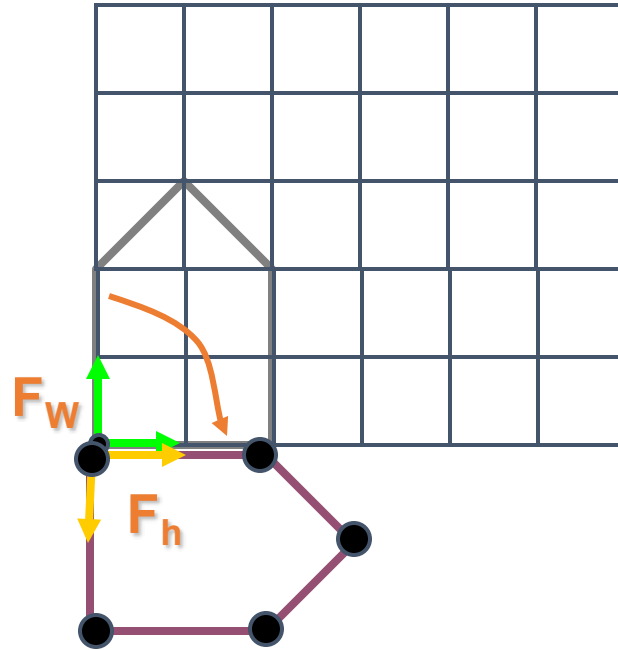
	lignes	lignes parallèles	distance	angles	normales	convexité
changement d'échelle						
rotation						
translation						
cisaillement						

LES TRANSFORMATIONS COMPOSÉES

Si on veut

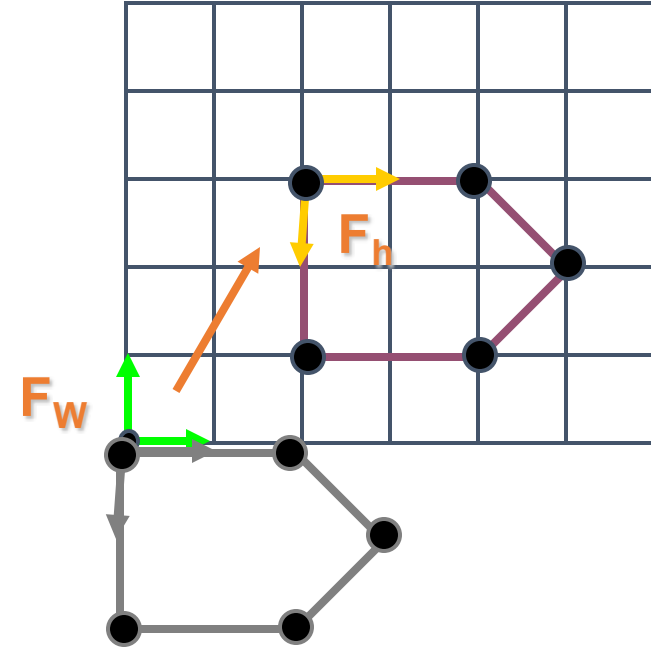


Rotate(z,-90)



$$P_A = Rot(z, -90)P_h$$

Translate(2,3,0)



$$P_W = Trans(2,3,0)P_A$$

$$P_W = Trans(2,3,0)Rot(z, -90)P_h$$

LES TRANSFORMATIONS COMPOSÉES

$$P_W = Trans(2,3,0)Rot(z, -90)P_h$$

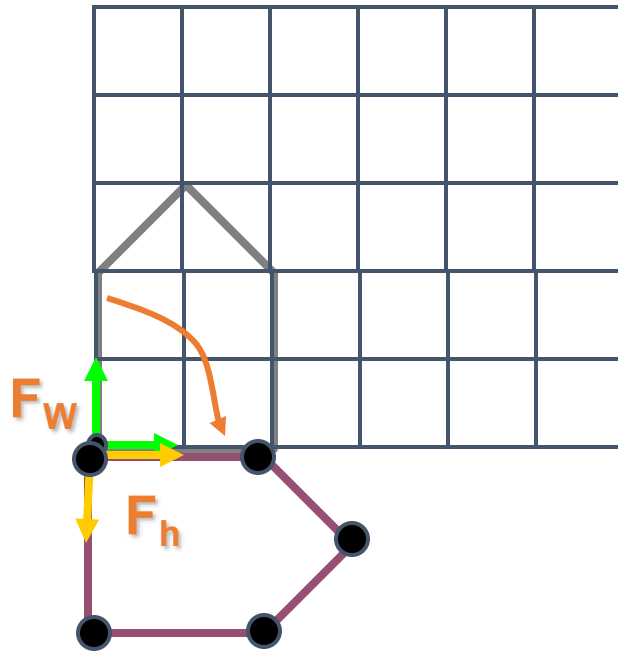
- **Lire de droite à gauche** dans le système de coordonnées fixé
 - Déplacer l'objet
- **Lire de gauche à droite:** toutes les opérations sont dans le système de coordonnées local
 - Changer le système des coordonnées

$$M_{MV} = Trans(2,3,0) \cdot M_{MV}$$

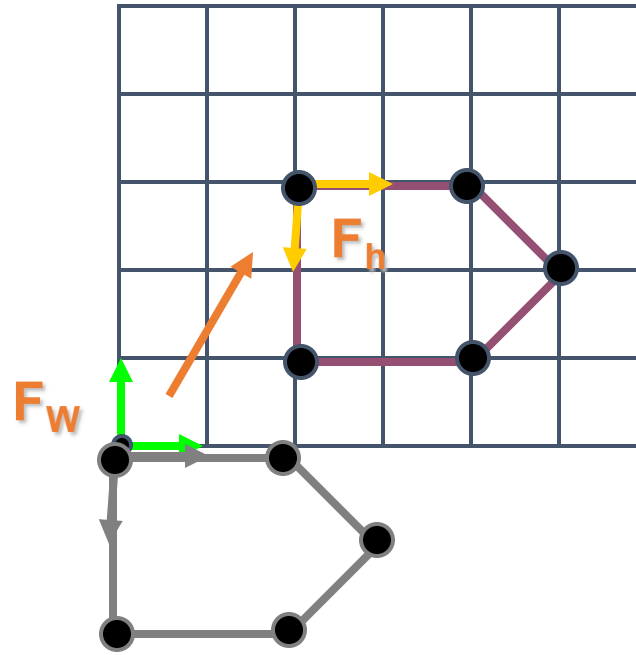
$$M_{MV} = Rot(z, -90)M_{MV}$$

LES TRANSFORMATIONS COMPOSÉES

Rotate(z,-90)



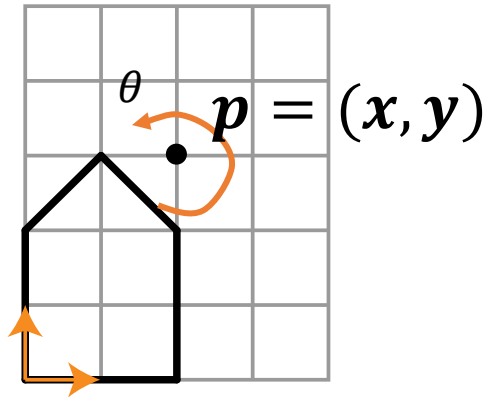
**Translate(-3,2,0) dans
le système de coordonnées local**



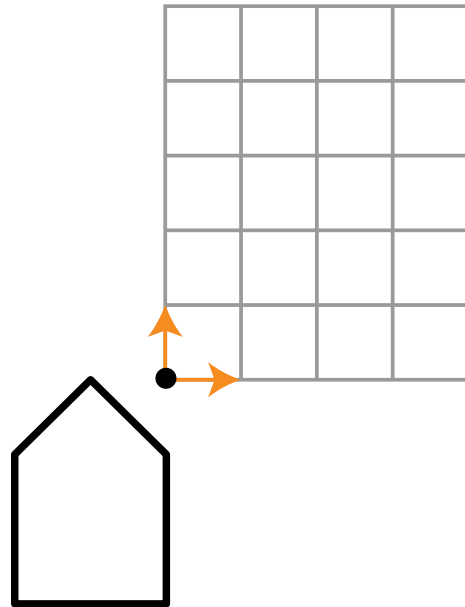
$$P_w = Rot(z, -90)Trans(-3,2,0)P_h$$

LA ROTATION AUTOUR D'UN POINT

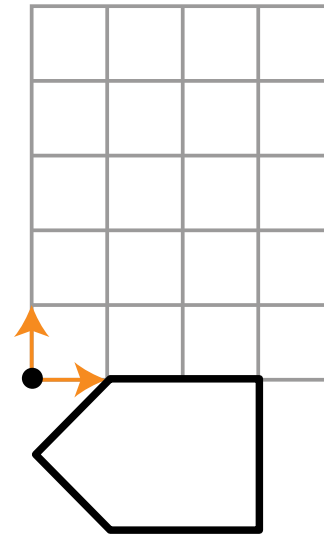
Tourner
autour p de θ :



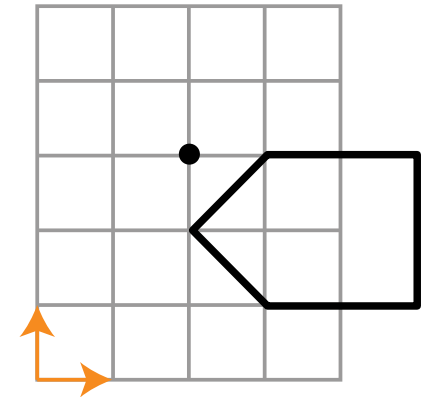
Déplacer p
à l'origine



Tourner
autour l'origine



Déplacer p
en arrière

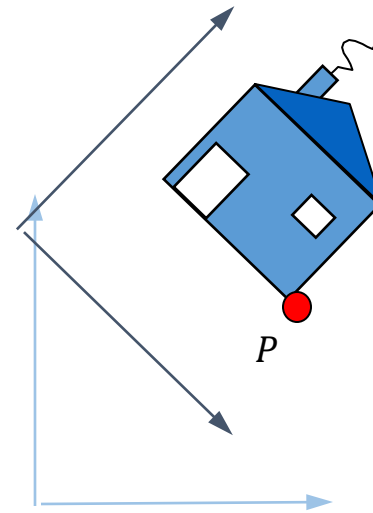
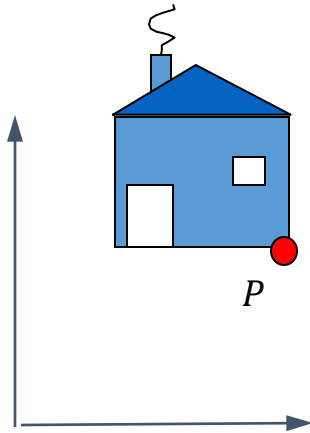


$$\mathbf{T}(x, y, z) \mathbf{R}(z, \theta) \mathbf{T}(-x, -y, -z)$$

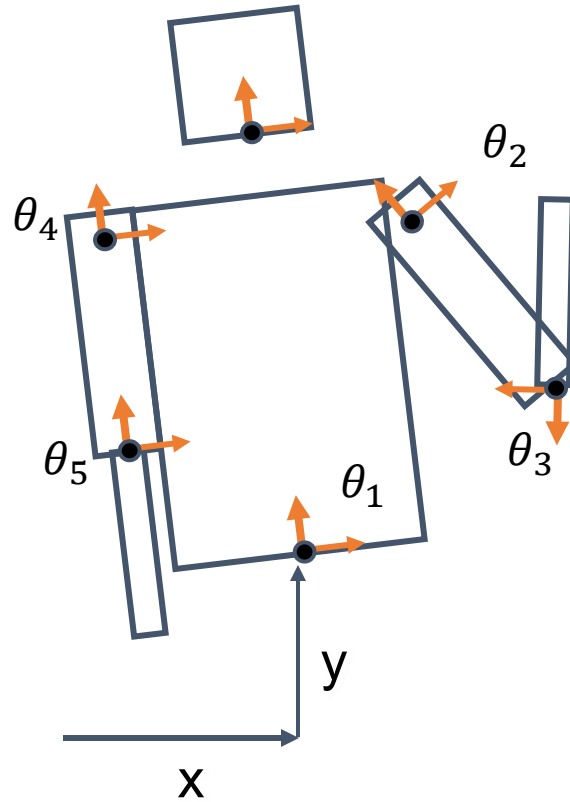
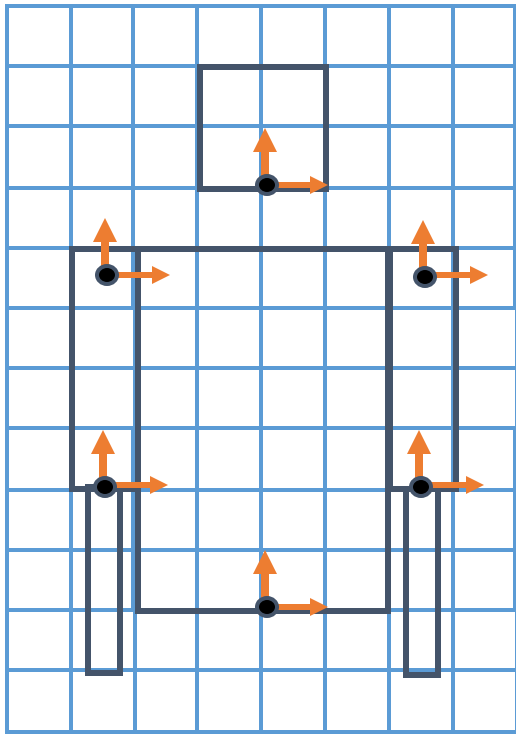
TRANSFORMATION DE COORDONNÉES

Les colonnes sont les nouveaux vecteurs de base (et la nouvelle origine)!

$$\begin{pmatrix} \begin{array}{c} \cos \theta \\ \sin \theta \\ 0 \end{array} & \begin{array}{c} -\sin \theta \\ \cos \theta \\ 0 \end{array} & \begin{array}{c} p_x \cdot (1 - \cos \theta) + p_y \cdot \sin \theta \\ p_y \cdot (1 - \cos \theta) + p_x \cdot \sin \theta \\ 1 \end{array} \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$



LA HIÉRARCHIE DE TRANSFORMATIONS



$$M_1 = Tr_{(x,y)} \cdot Rot \theta_1$$

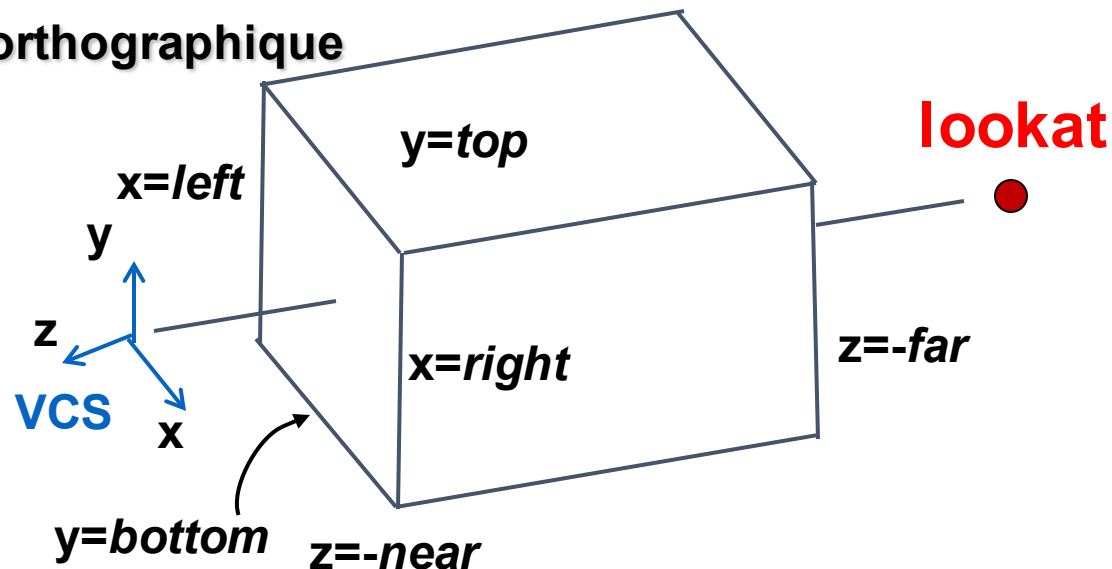
$$M_2 = M_1 \cdot Tr_{(2.5,5.5)} \cdot Rot \theta_2$$

$$M_3 = M_2 \cdot Tr_{(0,-3.5)} \cdot Rot \theta_3$$

LE VOLUME DE VUE PROJECTION ORTHOGRAPHIQUE

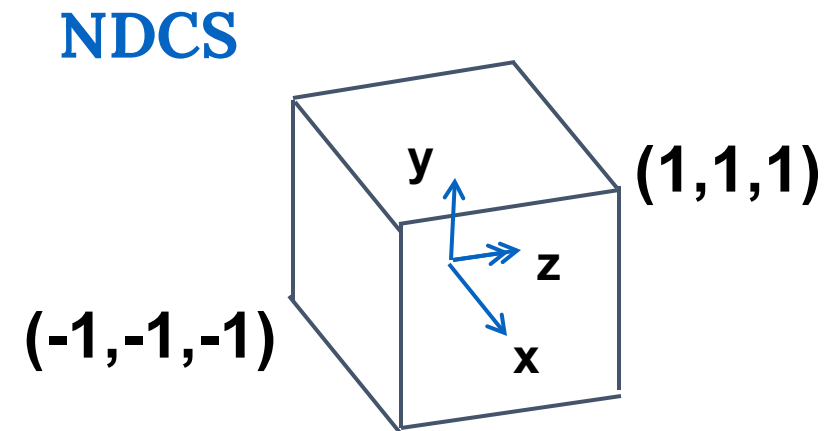
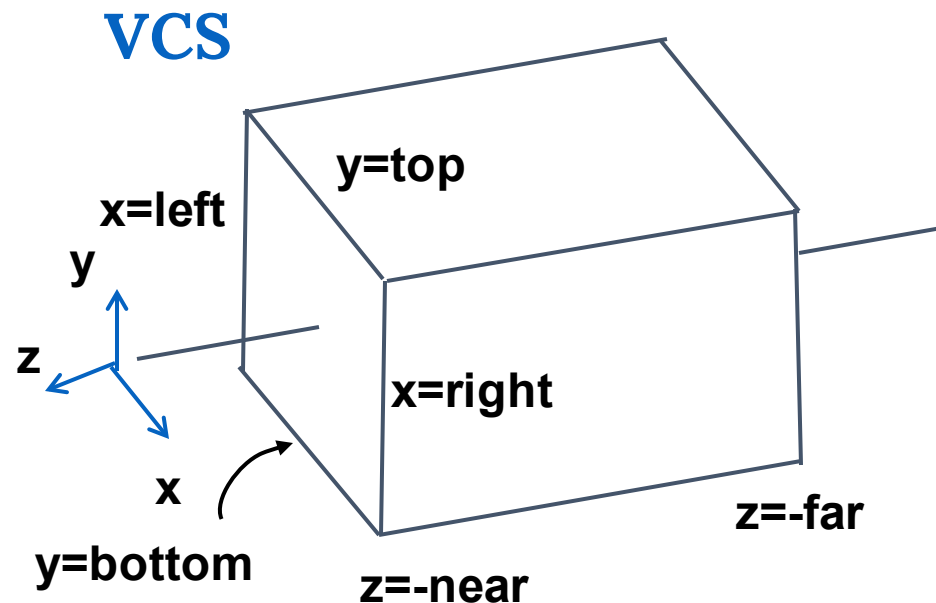
- Est une projection parallèle dans la direction perpendiculaire au plan de projection
- Il spécifie le champ de vue, utilisé pour le *clipping*
- Il limite le domaine de z stocké pour le test de profondeur

le volume de vue orthographique



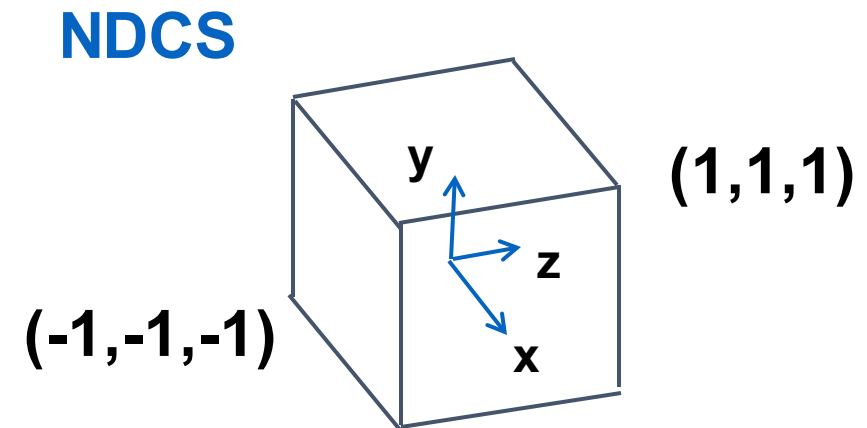
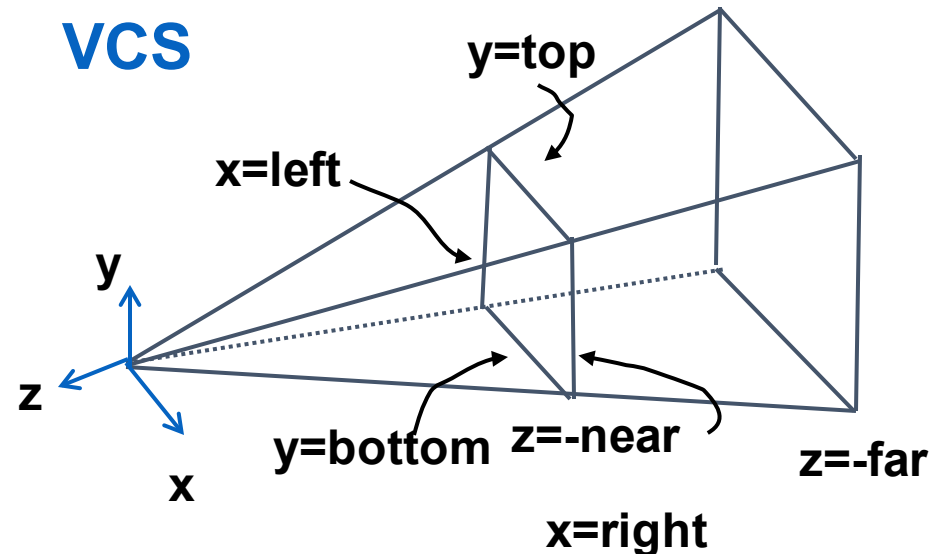
L'AXE Z

- Un flip de l'axe z change le *handedness* du système des coordonnées
 - RHS (règle de la main droite) avant la projection
 - les coordonnées de la vue/caméra, du monde
 - LHS (règle de la main gauche) après
 - clip, les coordonnées de l'appareil normalisées



LA MATRICE DE LA PROJECTION PERSPECTIVE OPENGL

Mapper (faire correspondre) la pyramide tronquée à un cube NDCS. Après, la coordonnée z sera ignorée. On doit changer l'échelle/déplacer/faire cisaillement → une transformation générique



BONNE CHANCE!