IFT 6113
MESH DEFORMATION AND SKINNING

tiny.cc/6113

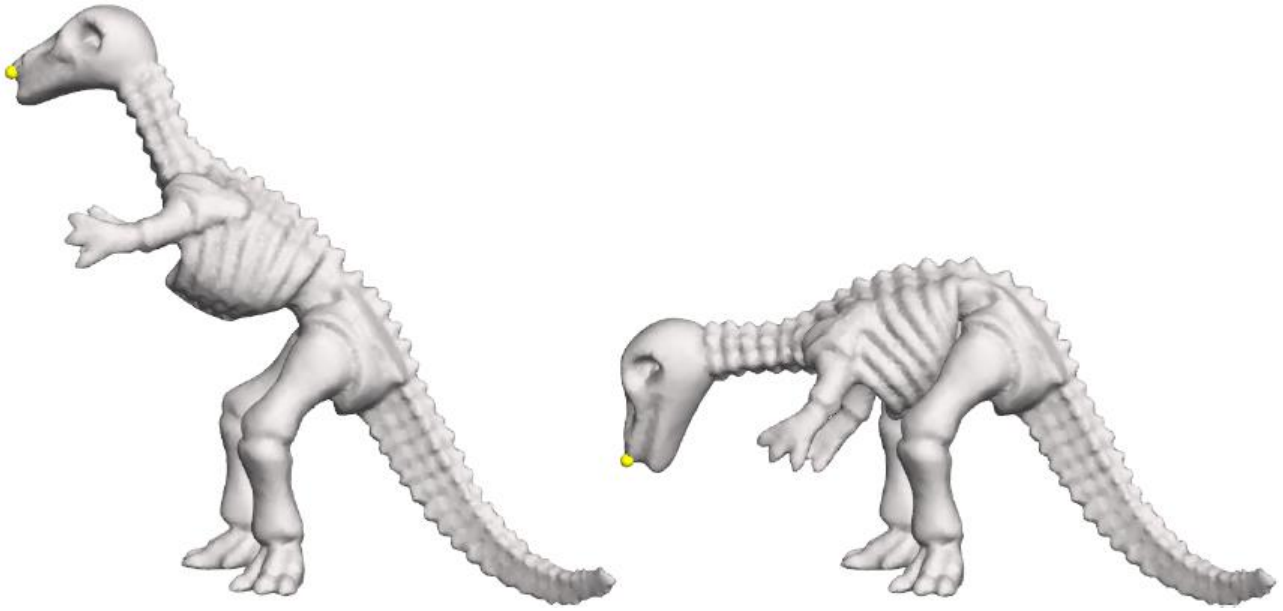


Image from ‘As-Rigid-As-Possible Shape Modeling’ by Sorkine & Alexa, 2007

Mikhail Bessmeltsev

Administrative

- Project proposal due tonight!
  - A2 due next Tuesday

What for?

- Animation!
- Mesh editing
- Image warping (2D)

This, and many other images in this presentation are from ‘Polygon Mesh Processing’ textbook by Botsch et al. or their website

Warning:

TMI

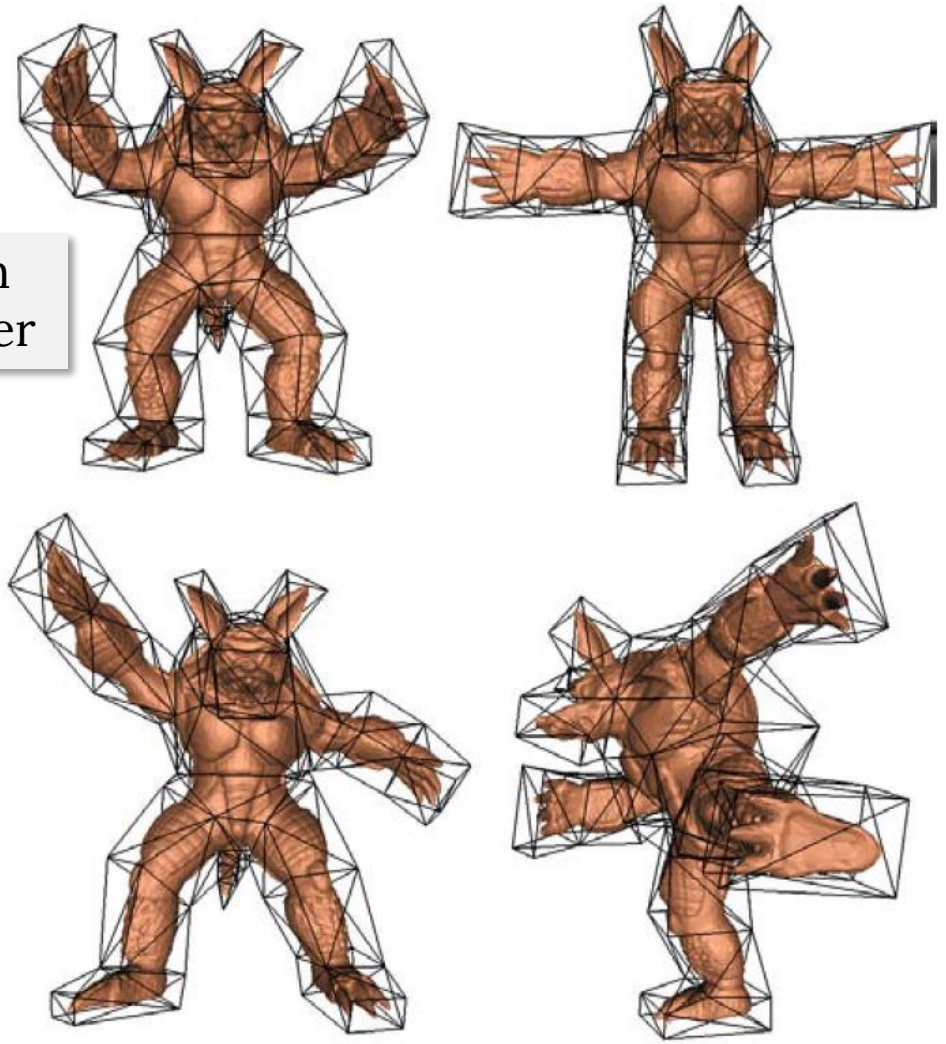
This topic is immense

We'll only see a few samples

Deformation: user interface

- Handles
- Cages
- Skeletons
- …

More on those later



Ju et al., SIGGRAPH 2007

Deformation models

Direct

$$v' = \left(\sum w_j T_j\right) v$$

- Linear Blend Skinning
- Dual Quaternion Skinning
- …

Variational

$$v' = \underset{x}{\operatorname{argmin}}\, E(x)$$

- Multiresolution editing
- As-Rigid-As-Possible
- Laplacian Mesh Editing
- …

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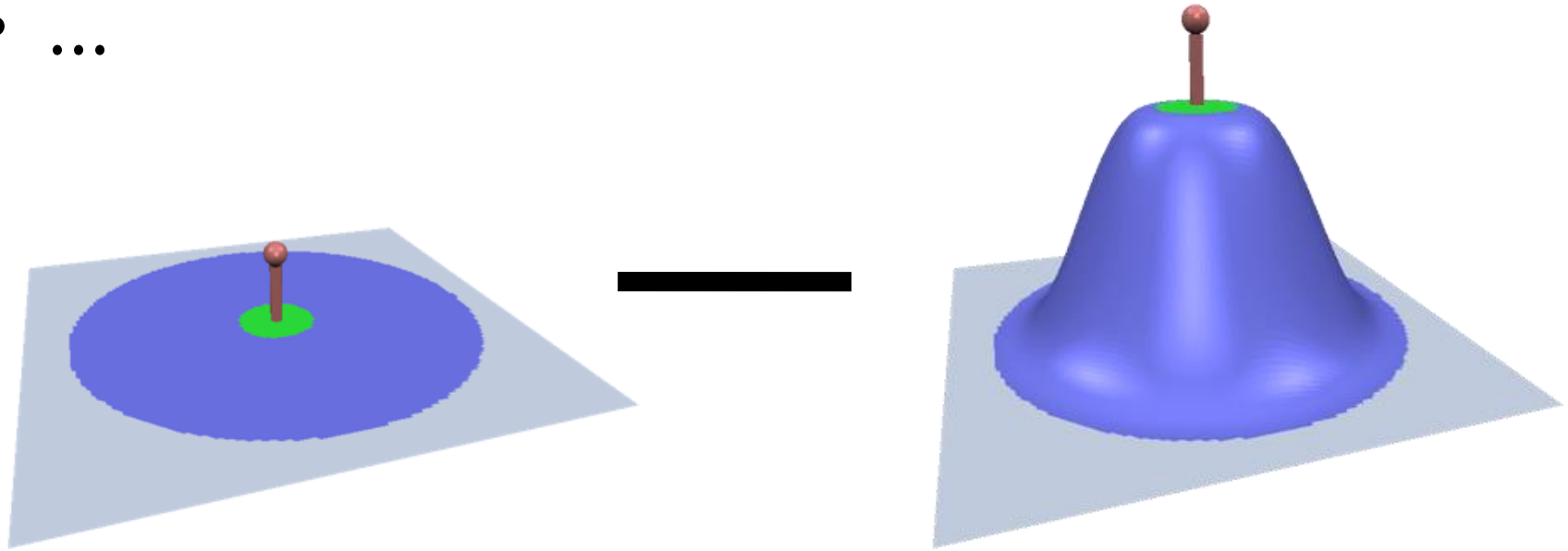
- …

Deformation: user interface

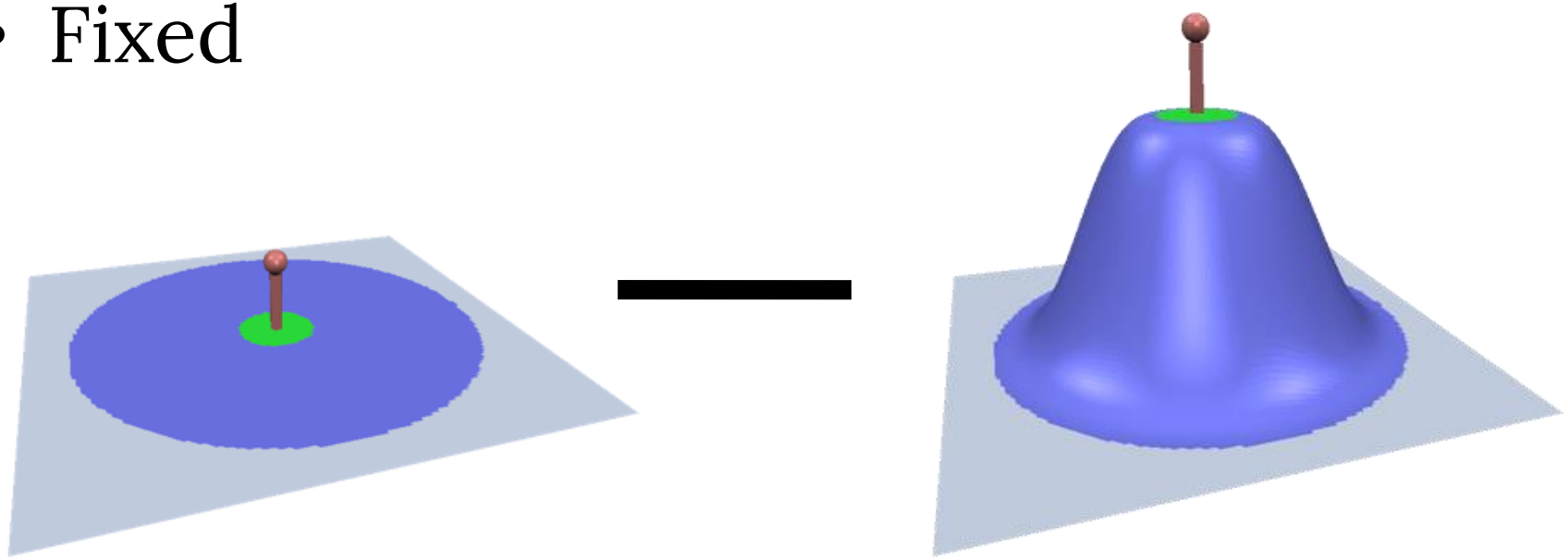
- **Handles**
- Cages
- Skeletons
- …



Modeling

Paint three surface areas:

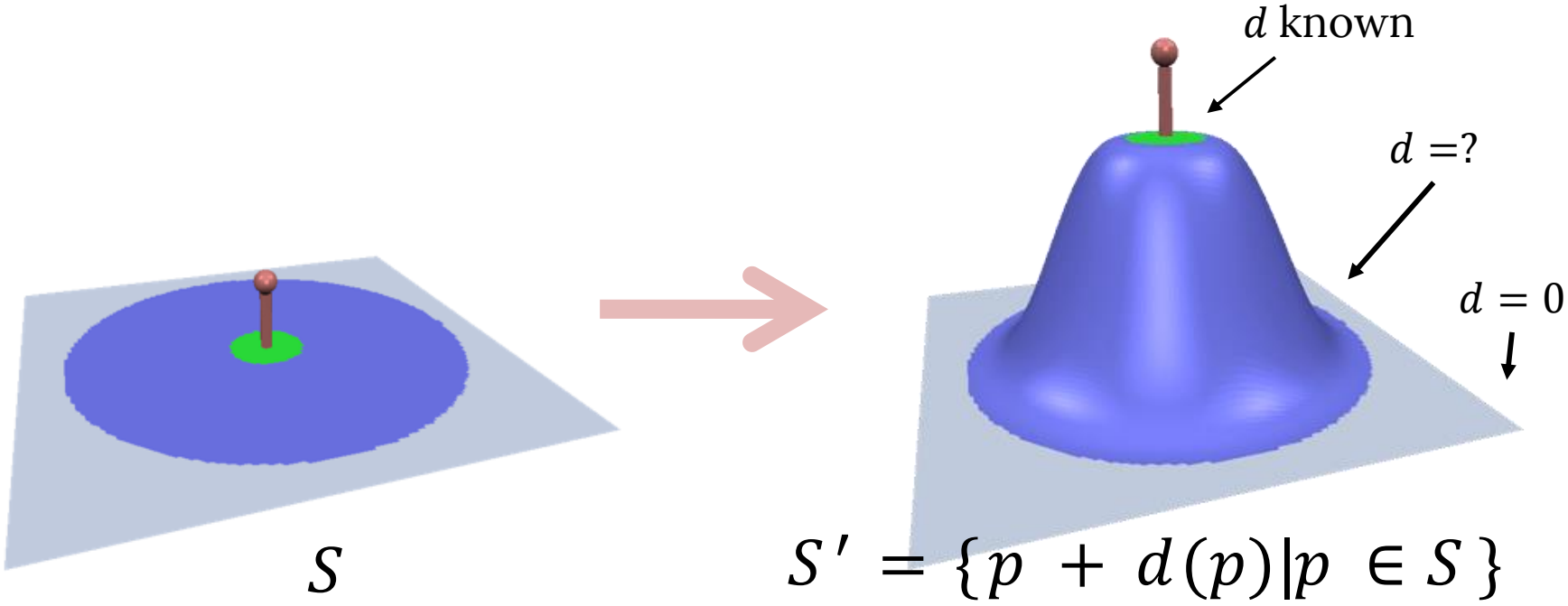
- Constrained
- Smooth falloff
- Fixed



Formulation

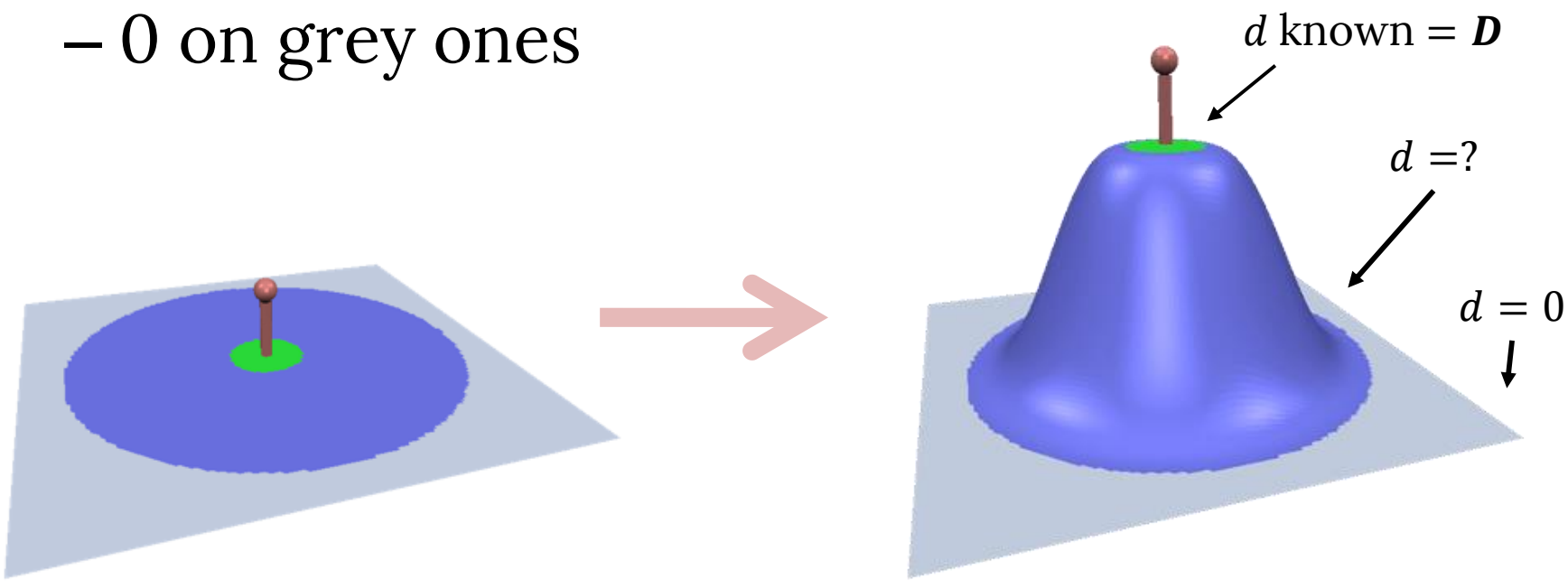
Find displacement vector field d

- Smooth
- Satisfies constraints



Simplest idea

- $d = s(p) \cdot D$
- $s(p)$ is a smooth function:
  - 1 on green vertices
  - 0 on grey ones



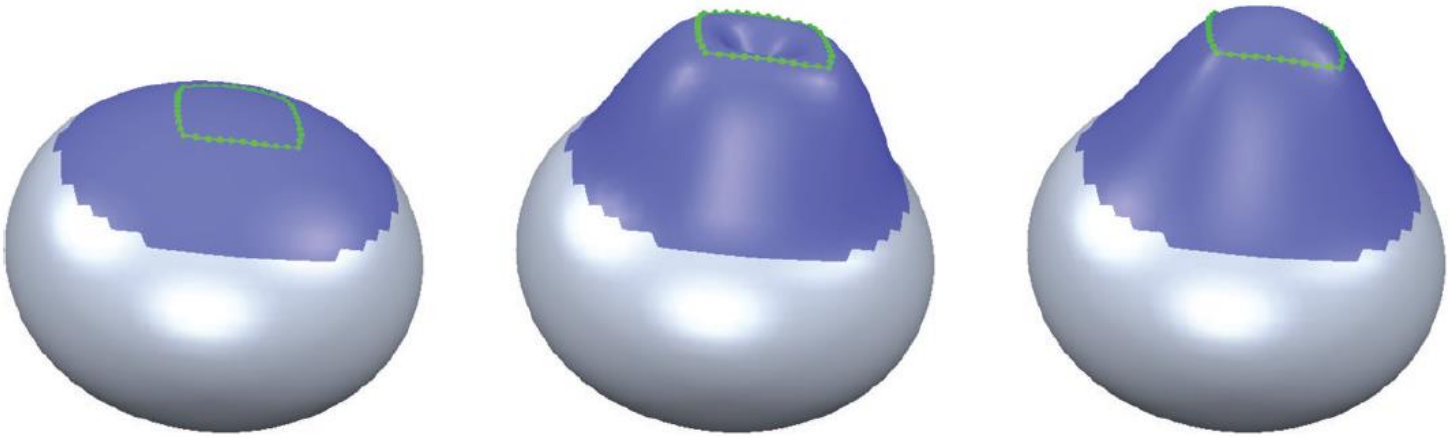
How to find $s(p)$?

- Something inversely proportional to geodesic distance
- Or our favorite:

$$\begin{aligned} \Delta s(\mathbf{p}_i) &= 0, \quad \mathbf{p}_i \in \mathcal{R}, \\ s(\mathbf{p}_i) &= 1, \quad \mathbf{p}_i \in \mathcal{H}, \\ s(\mathbf{p}_i) &= 0, \quad \mathbf{p}_i \in \mathcal{F}. \end{aligned}$$

Solved?

Reality vs Expectation



Physically-Based

Find a deformation that preserves both fundamental forms

Express the fundamental forms of S' via vector field d

Expensive to optimize!

$$\int_\Omega k_s \left\| \mathbf{I} - \mathbf{I}' \right\|_{\mathrm{F}}^2 + k_b \left\| \mathbf{II} - \mathbf{II}' \right\|_{\mathrm{F}}^2 \, \mathrm{d}u\mathrm{d}v$$

stretching bending

Shell-Based Deformation

Find a deformation that preserves both fundamental forms

Linearize ~~Express~~ the fundamental forms of S' via vector field d

$$\int_\Omega k_s \left(\|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left(\|\mathbf{d}_{uu}\|^2 + 2\|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) \mathrm{d}u\mathrm{d}v$$

stretching

bending

Physically-Based

$$\int_\Omega k_s \left(\|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left(\|\mathbf{d}_{uu}\|^2 + 2\|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) \mathrm{d}u\mathrm{d}v$$

stretching

bending

Gateuax derivative =>

$$-k_s \Delta d + k_b \Delta^2 d = 0$$

Physically-Based

$$\int_{\Omega} k_s \left(\|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left(\|\mathbf{d}_{uu}\|^2 + 2\|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) \mathrm{d}u\mathrm{d}v$$

stretching

bending

Gateuax derivative =>

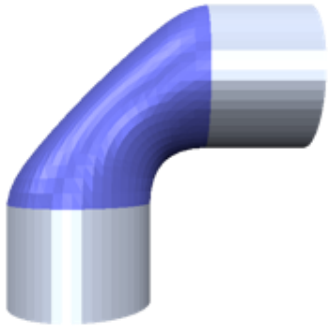
$$-k_s \Delta d + k_b \Delta^2 d = 0$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

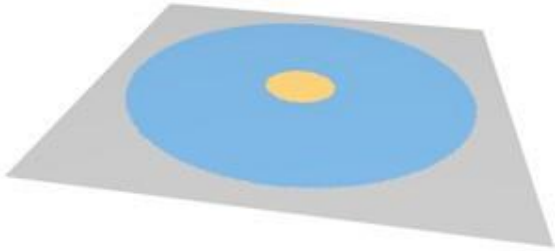
Bi-Laplacian

Deformation Energies

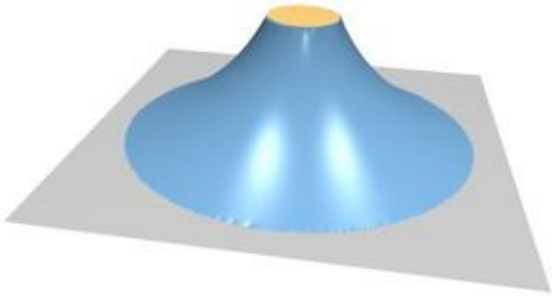
$\Delta p = 0$



$\Delta^2 p = 0$

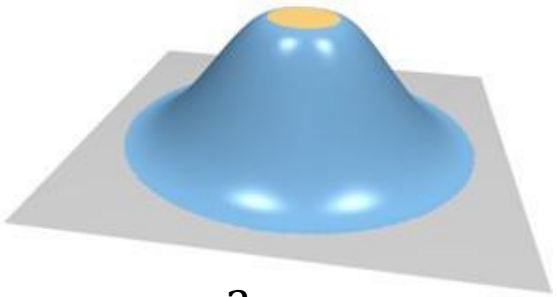


Initial state



$\Delta d = 0$

(Membrane)



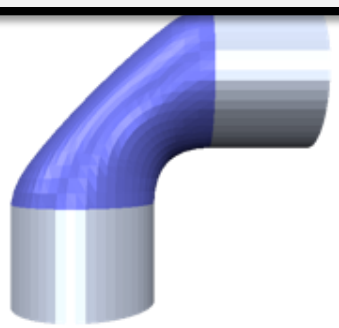
$\Delta^2 d = 0$

(Bilaplacian)

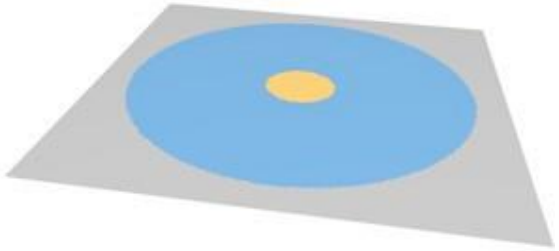
Deformation En

Higher order => more boundary conditions

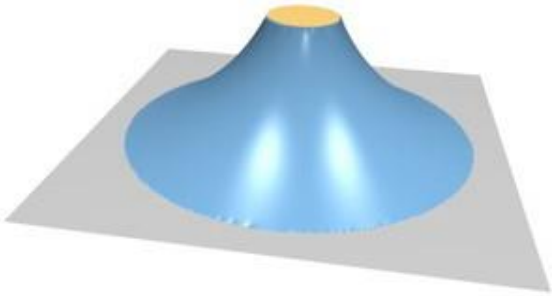
$\Delta p = 0$



$\Delta^2 p = 0$

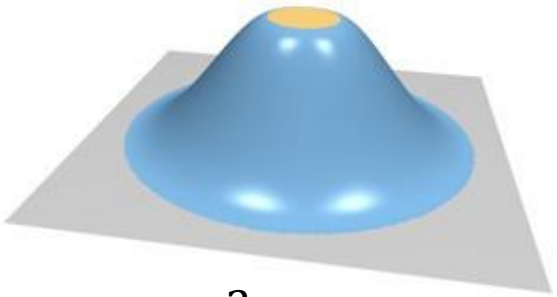


Initial state



$\Delta d = 0$

(Membrane)

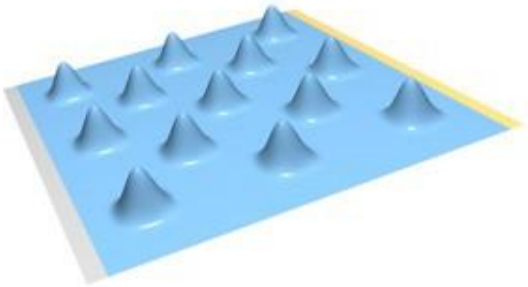


$\Delta^2 d = 0$

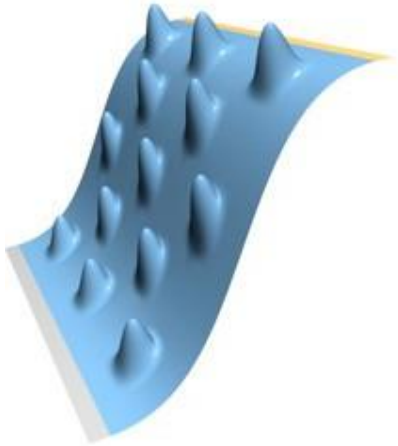
(Bilaplacian)

Solved?

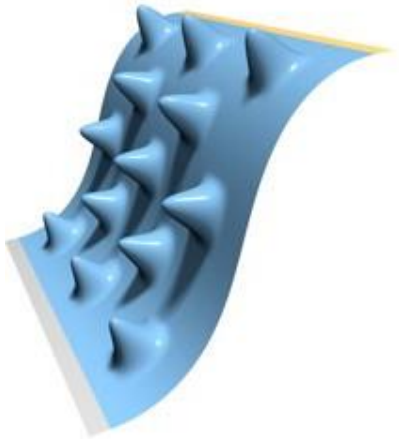
- Very fast
  - One linear solve!
- Physically-based
- Linearization => lose details



Original



Linear deformation

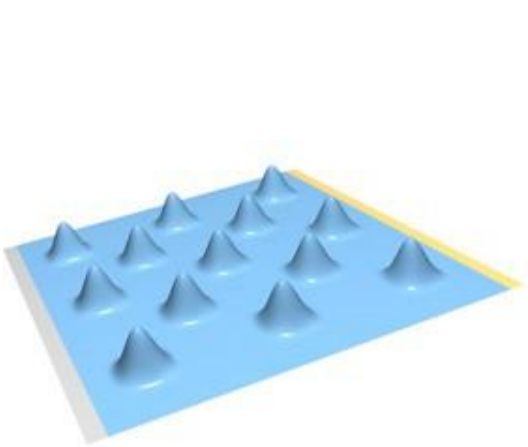


Non-linear deformation

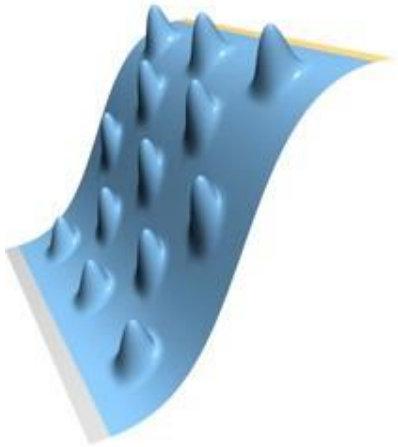
Issue

- We need to rotate details
  - Local rotation is nonlinear!

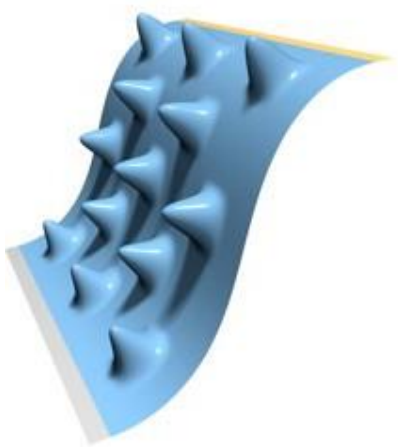
- Can we still survive with linear solves?



Original

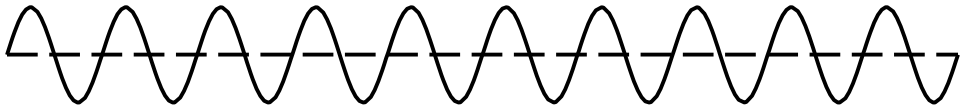


Linear deformation



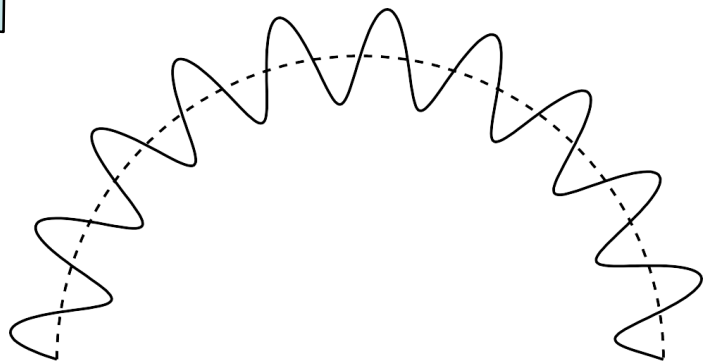
Non-linear deformation

Multiresolution Editing



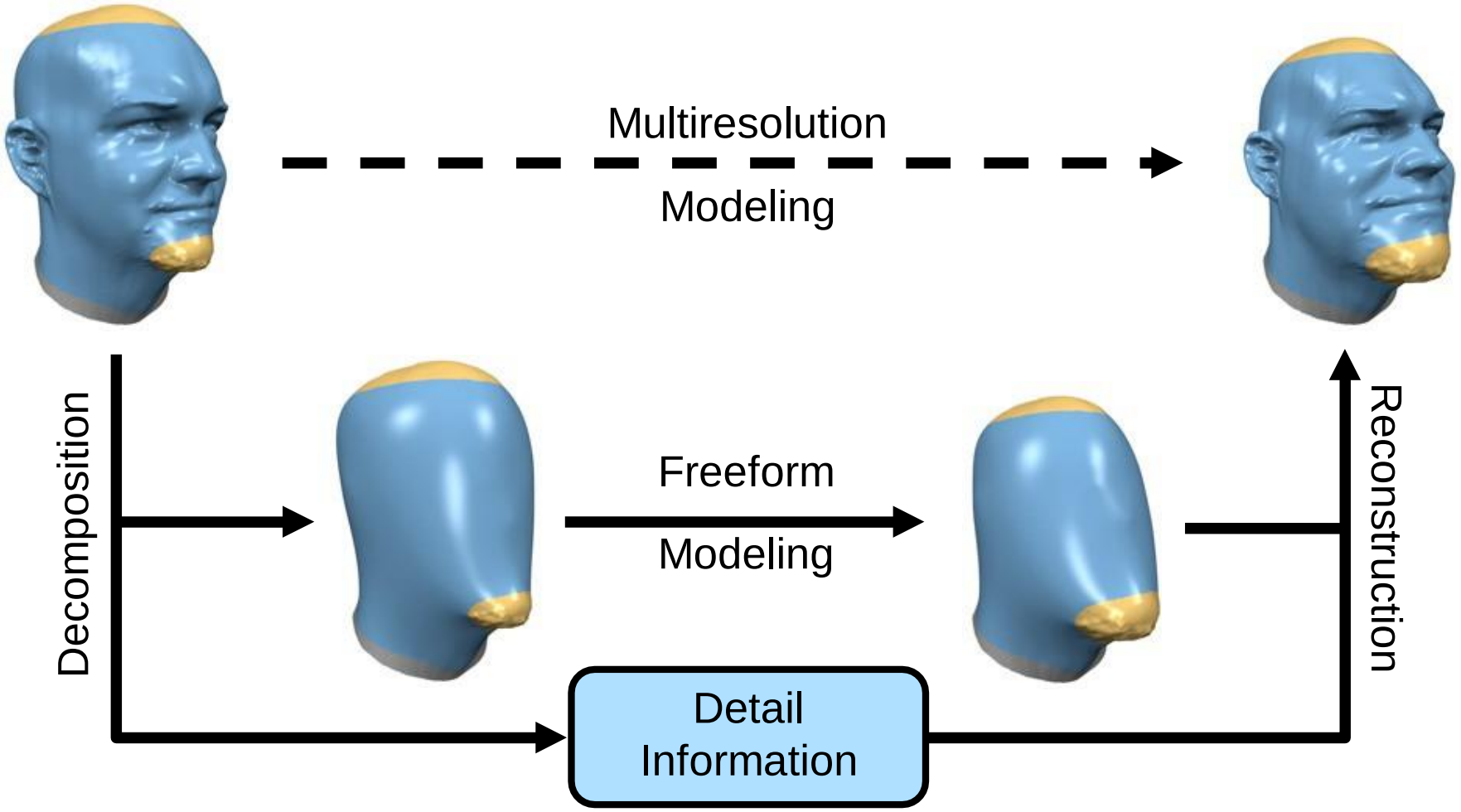
Frequency decomposition

Change low frequencies



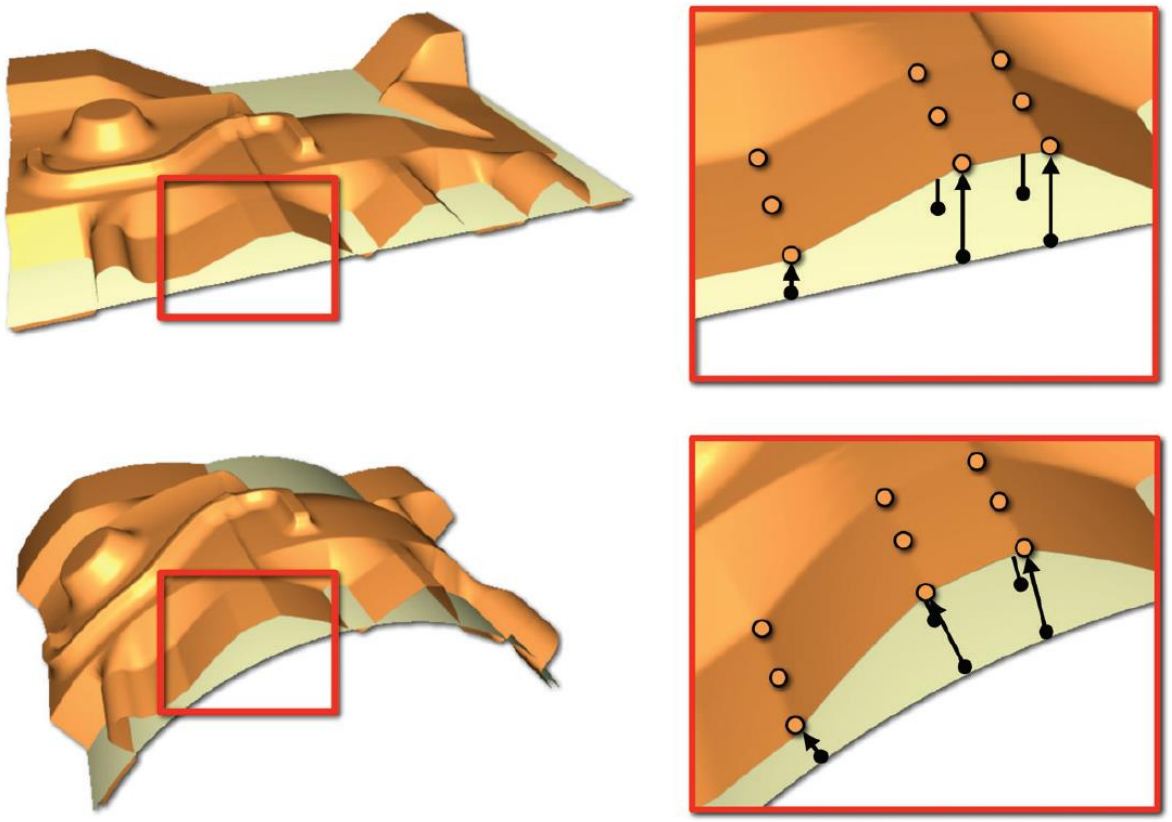
Add high frequency details, stored in local frames

Multiresolution Editing

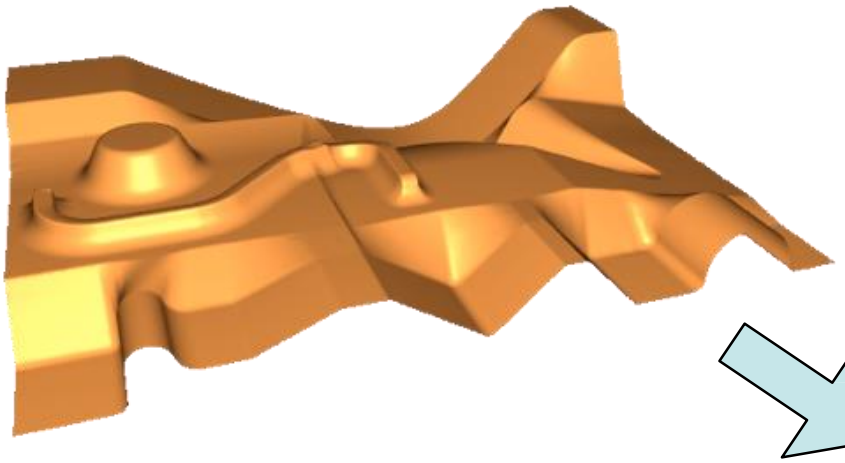


How to represent details?

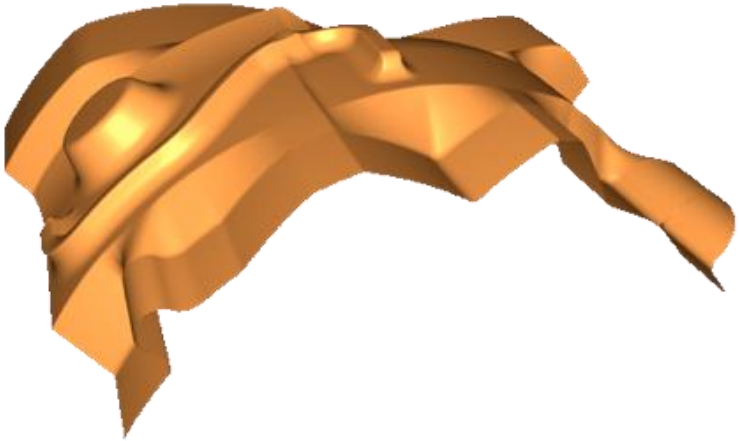
- For example, normal displacements



Result



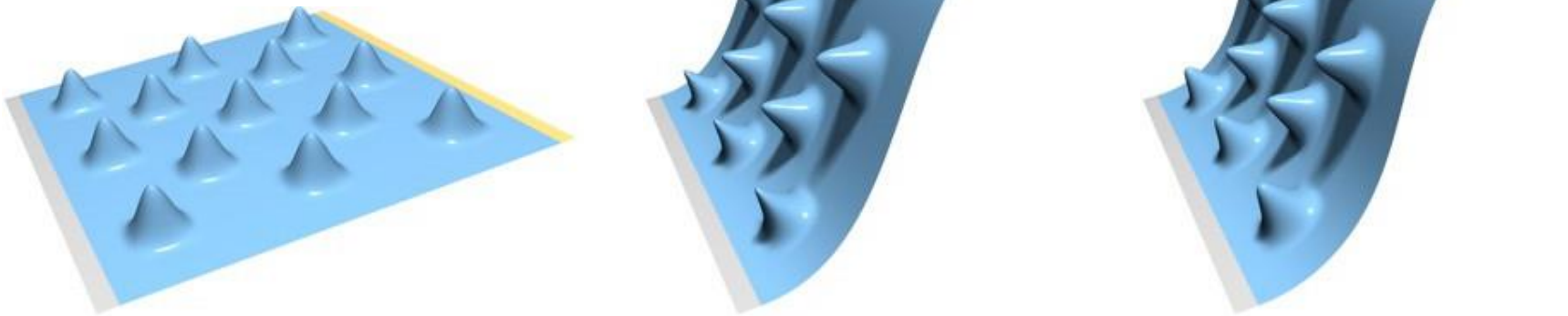
Global deformation with intuitive detail preservation



Limitations

Neighboring displacements are not coupled

- Surface bending changes their angle
- Leads to volume changes or self-intersections



Original Normal Displ. Nonlinear

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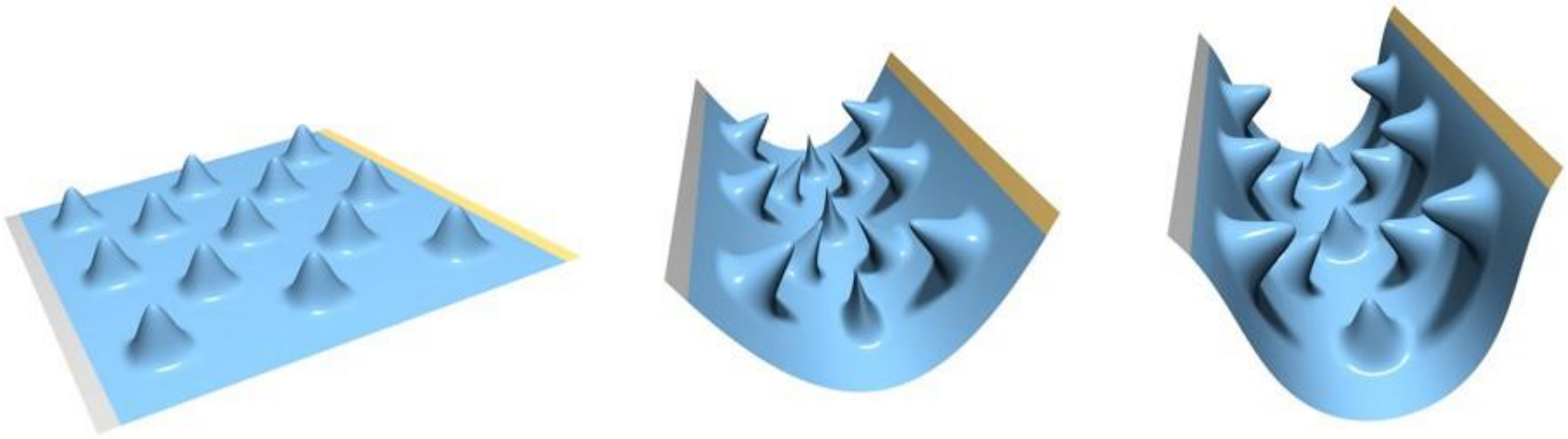


Original Normal Displ. Nonlinear

New coordinates?

Express shape in *differential coordinates*

Transform those,
then reconstruct the new shape

Long time ago:

Mean Value Property

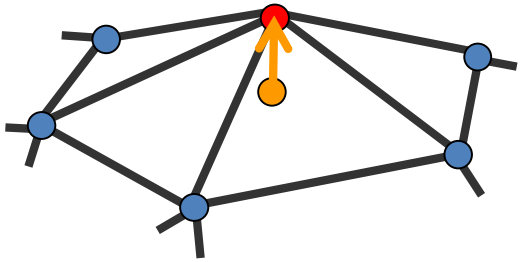
$$L_{vw} = A - D = \begin{cases} 1 & \text{if } v \sim w \\ -\text{degree}(v) & \text{if } v = w \\ 0 & \text{otherwise} \end{cases}$$

$$(Lx)_v = 0$$

Value at v is average of neighboring values

Laplacian Mesh Editing

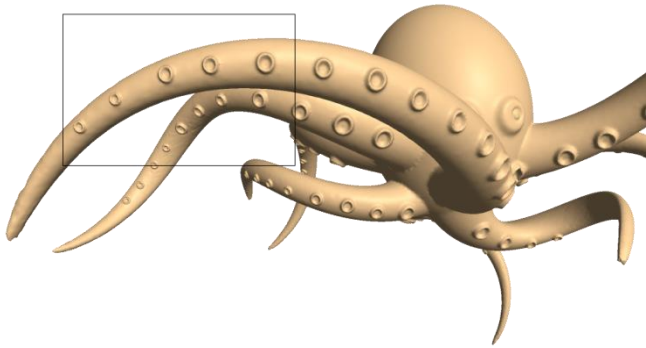
Graph Laplacian:



$$\boldsymbol{\delta}_i = \mathbf{v}_i - \frac{1}{d_i} \sum_{j \in N(i)} \mathbf{v}_j$$

Local coordinates!

$$\delta = Lv$$



Laplacian Mesh Editing

- Represent mesh using only δ
- Find a surface whose Laplacian coordinates are as close as possible to δ

$$\int_{\mathcal{S}} \left\| \Delta \mathbf{p}' - \boldsymbol{\delta}' \right\|^2 \mathrm{d}\mathcal{S} \;\rightarrow\; \min$$

$$\text{s.t. } p_i' = p_i, i \in \{point\ constraints\}$$

Laplacian Mesh Editing

Find a surface whose Laplacian coordinates are as close as possible to δ

$$\min \sum \|\delta_i - L(p'_i)\|^2 + \sum_{i \in c} \|p'_i - p_i\|^2$$

Laplacian Mesh Editing

Find a surface whose Laplacian coordinates are as close as possible to δ

$$\int_{\mathcal{S}} \left\| \Delta \mathbf{p}' - \boldsymbol{\delta}' \right\|^2 \mathrm{d}\mathcal{S} \;\rightarrow\; \min$$

s.t. $p_i' = p_i, i \in \{point\ constraints\}$

Gateaux derivative => $\Delta^2 \mathbf{p}' = \Delta \boldsymbol{\delta}'$

Before:

Physically-Based

$$\int_\Omega k_s \left(\|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left(\|\mathbf{d}_{uu}\|^2 + 2\|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) \mathrm{d}u\mathrm{d}v$$

stretching

bending

Gateuax derivative =>

$$-k_s \Delta d + k_b \Delta^2 d = 0$$

(almost) the same equation?

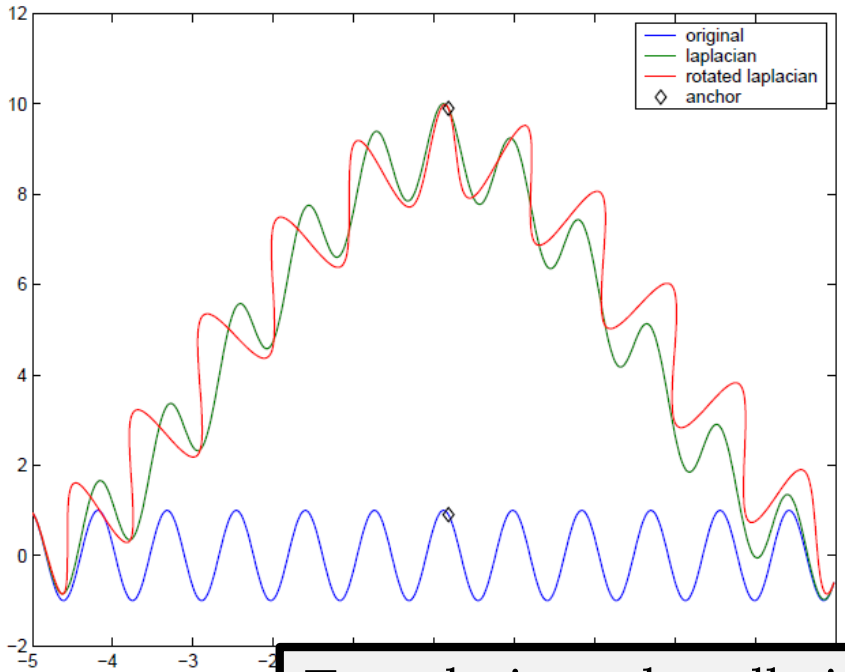
Bi-Laplacian

Issue

Reconstructing from differential coordinates makes sense only if they are

rotation and translation invariant

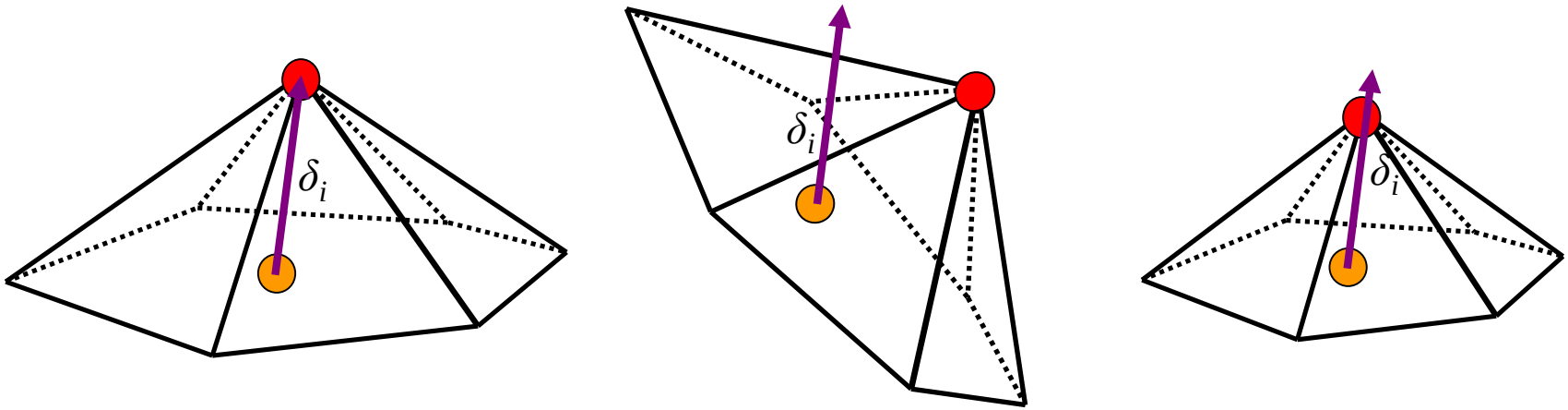
Otherwise, you get this



Translating a handle induces local rotations!

Laplacian Coordinates

- Translation invariant
- Not rotation/scale invariant



$$\delta_i = L(\mathbf{v}_i) = L(\mathbf{v}_i + \mathbf{t}); \forall \mathbf{t} \in \mathbb{R}^3$$

Solutions

1. Transform, ignoring rotations or details
2. while (not converged)
   - Estimate rotations (*from normals*)
   - Rotate differential coordinates and solve

$$E(\mathbf{V}') = \sum_{i=1}^{n} \|R_i \delta_i - L(p'_i)\|^2 + \sum_{i \in c} \|p'_i - p_i\|^2$$

[Y. Lipman, O. Sorkine, D. Cohen-Or, D. Levin, C. Rossi and H. P. Seidel, "Differential coordinates for interactive mesh editing," *Proceedings Shape Modeling Applications*, 2004]

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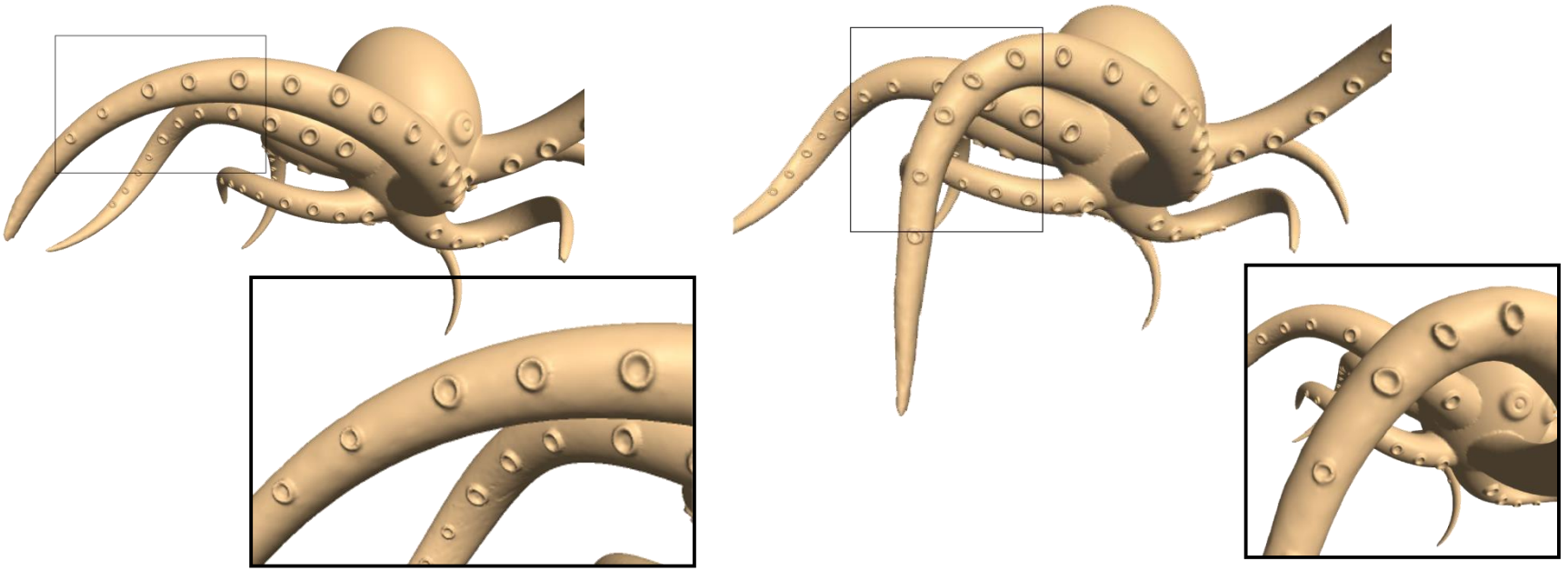


[Y. Lipman, O. Sorkine, D. Cohen-Or, D. Levin, C. Rossi and H. P. Seidel, "Differential coordinates for interactive mesh editing," *Proceedings Shape Modeling Applications*, 2004]

Rotations + **scaling** – invariant?

Add local transformations T_i as variables

$$E(\mathbf{V}') = \sum_{i=1}^{n} \|T_i \delta_i - L(p'_i)\|^2 + \sum_{i \in c} \|p'_i - p_i\|^2$$

[O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rossl, H.-P. Seidel, Laplacian Surface Editing, EUROGRAPHICS/Symposium on Geometry Processing, 2004]

Rotations + **scaling** – invariant?

Add local transformations T_i as variables

$$E(\mathbf{V}') = \sum_{i=1}^{n} \|T_i \delta_i - L(p'_i)\|^2 + \sum_{i \in c} \|p'_i - p_i\|^2$$

$$\min_{T_i} \left(\|T_i \mathbf{v}_i - \mathbf{v}'_i\|^2 + \sum_{j \in \mathcal{N}_i} \|T_i \mathbf{v}_j - \mathbf{v}'_j\|^2 \right).$$

[O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rossl, H.-P. Seidel, Laplacian Surface Editing, EUROGRAPHICS/Symposium on Geometry Processing, 2004]

Rotations + **scaling** – invariant?

Add local transformations T_i as variables

$$\min_{T_i} \left(\|T_i \mathbf{v}_i - \mathbf{v}'_i\|^2 + \sum_{j \in \mathcal{N}_i} \|T_i \mathbf{v}_j - \mathbf{v}'_j\|^2 \right).$$

T_i = translation + rotation + scaling

=> T_i is a linear function of V'

[O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rossl, H.-P. Seidel, Laplacian Surface Editing, EUROGRAPHICS/Symposium on Geometry Processing, 2004]

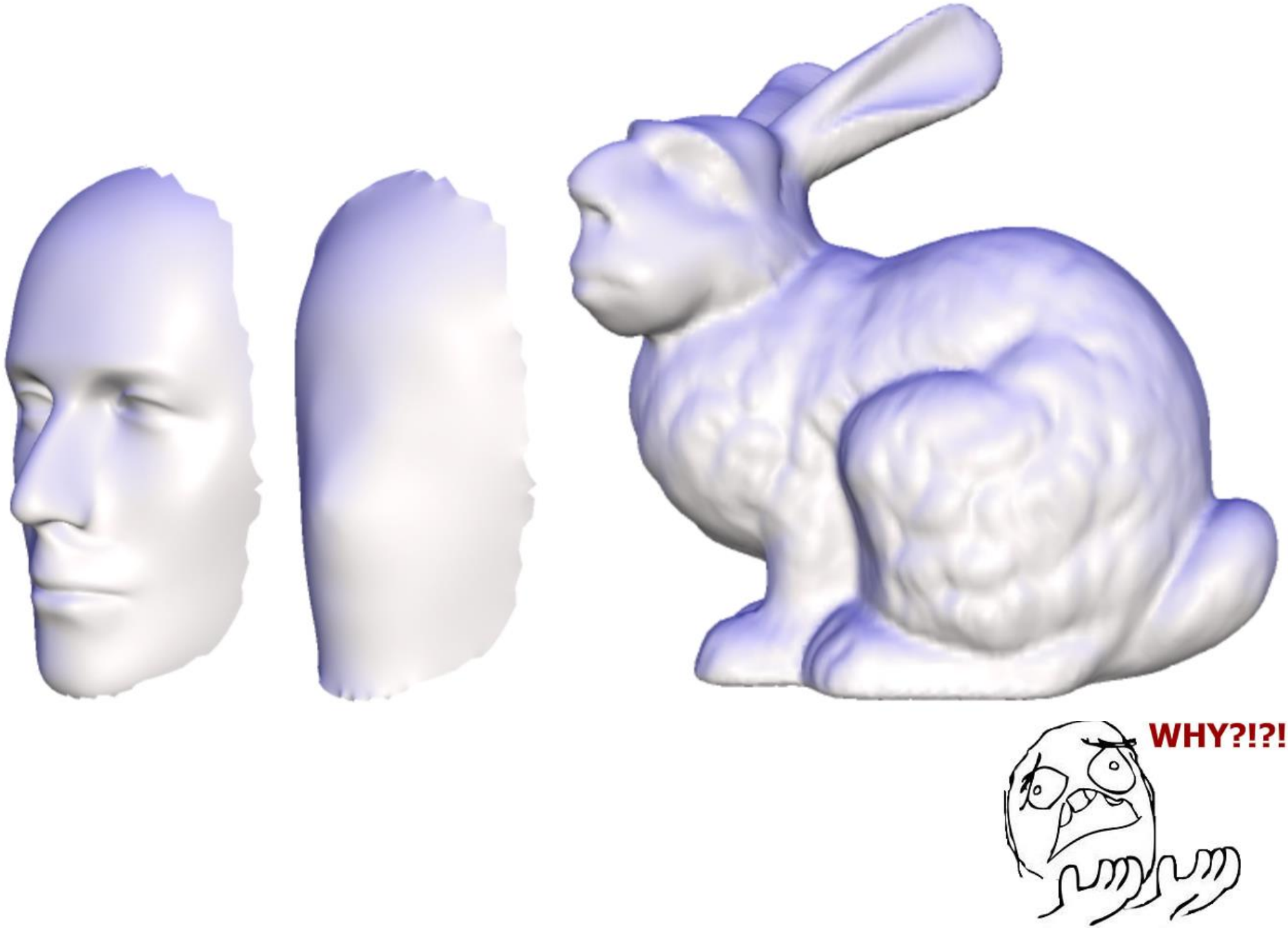
Rotations + **scaling** – invariant?

Add local transformations T_i as variables

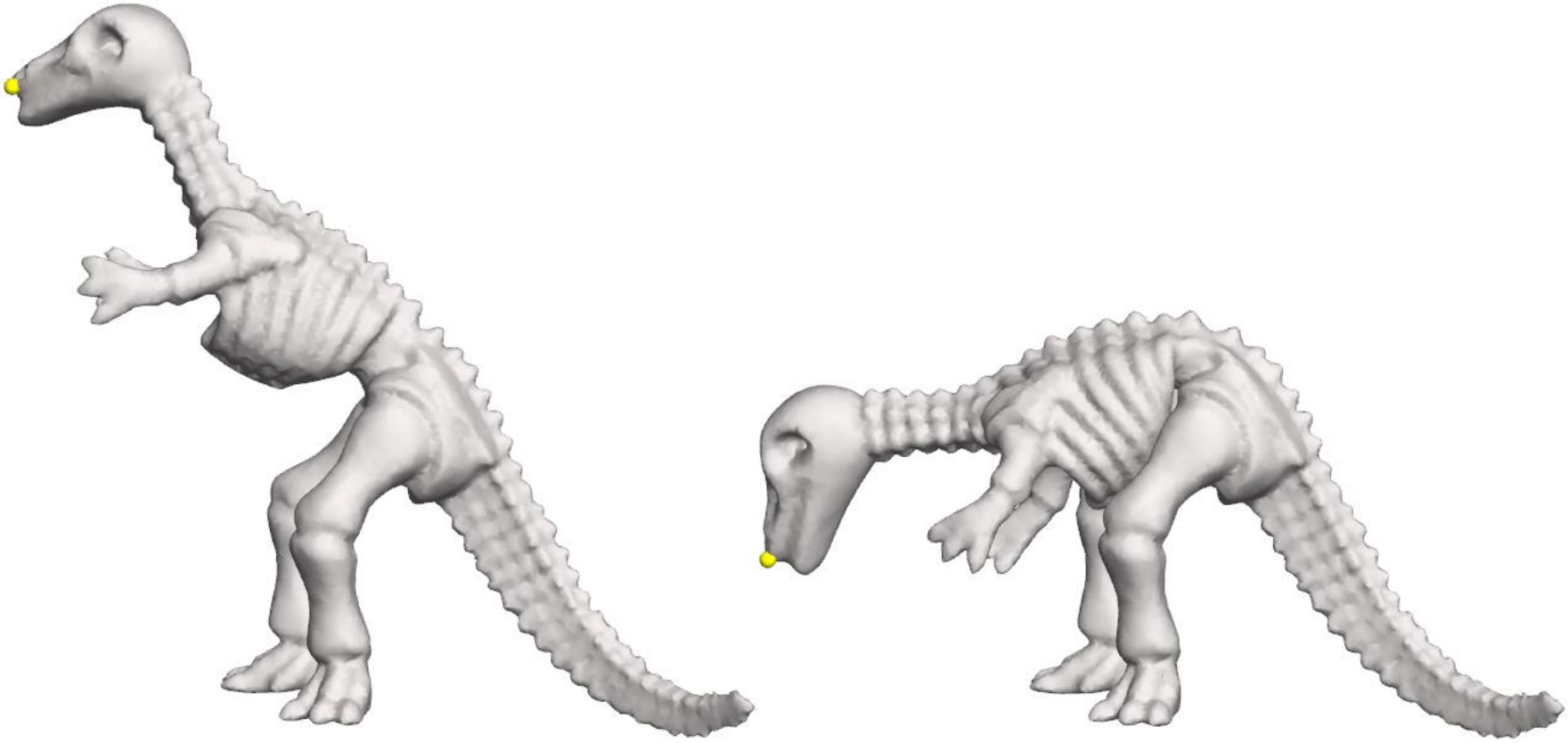
$$E(\mathbf{V}') = \sum_{i=1}^{n} \|T_i \delta_i - L(p'_i)\|^2 + \sum_{i \in c} \|p'_i - p_i\|^2$$

Quadratic optimization -> Linear solve!

[O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rossl, H.-P. Seidel, Laplacian Surface Editing, EUROGRAPHICS/Symposium on Geometry Processing, 2004]


WHY?!?!

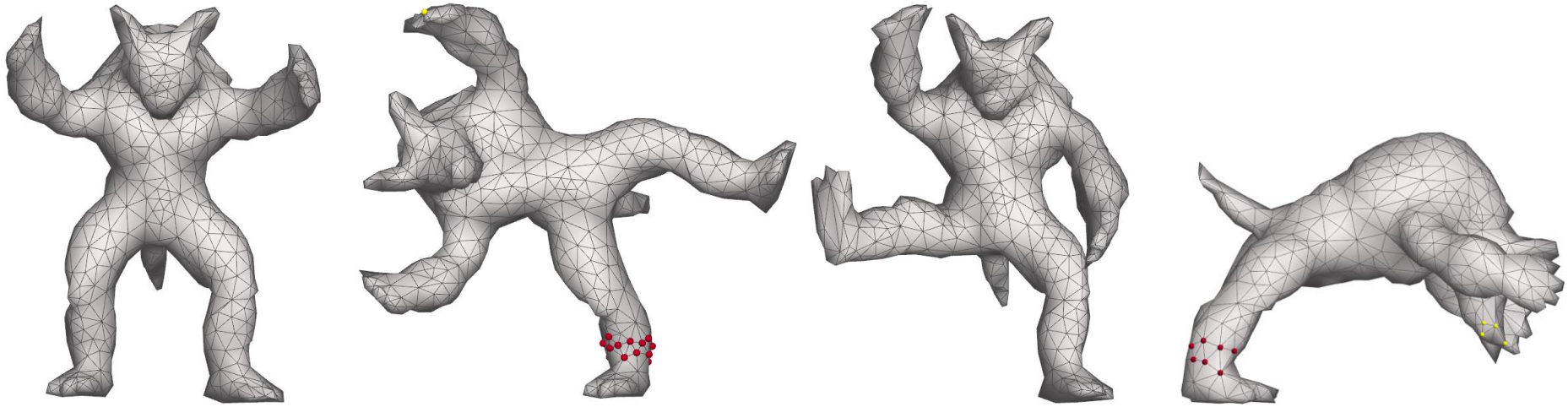
As-Rigid-As-Possible Surface Modelling



As-rigid-as-possible (ARAP)

As-rigid-as-possible (ARAP)

- "Intuitive" deformations
  - Smooth deformations at large scale
- Preserve local features
- Fast, for interactive mesh editing



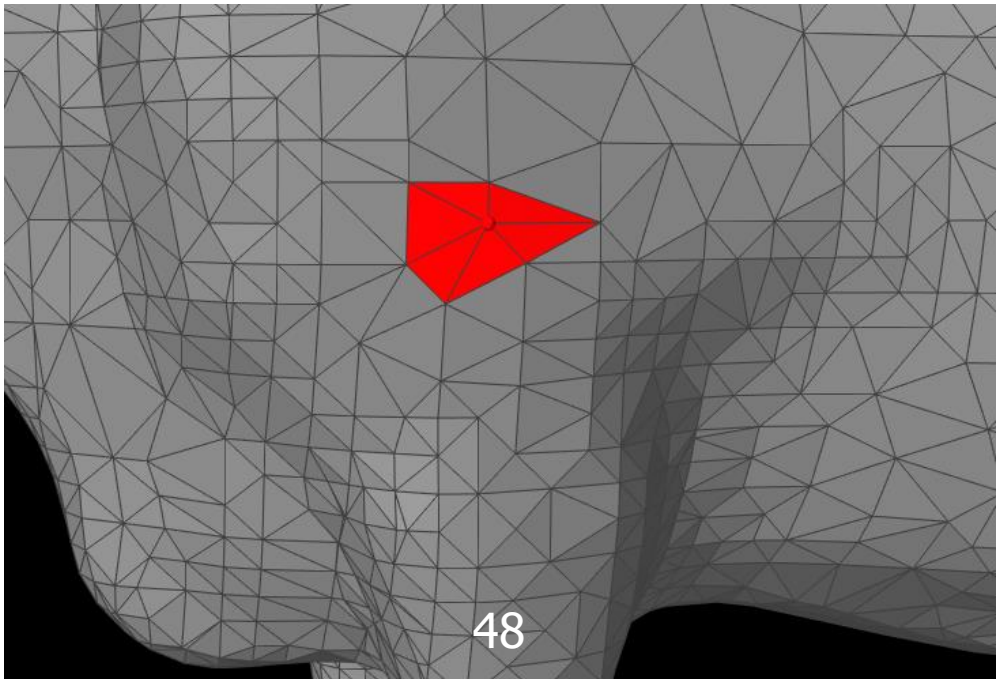
ARAP in a nutshell...

1. Break mesh into overlapping pieces
2. Try to move each piece rigidly
3. Combine all local transformations into a smooth one

Pieces

Vertex Umbrella

- Covers entire surface
- One cell per vertex
- All triangles exist in 3 cells

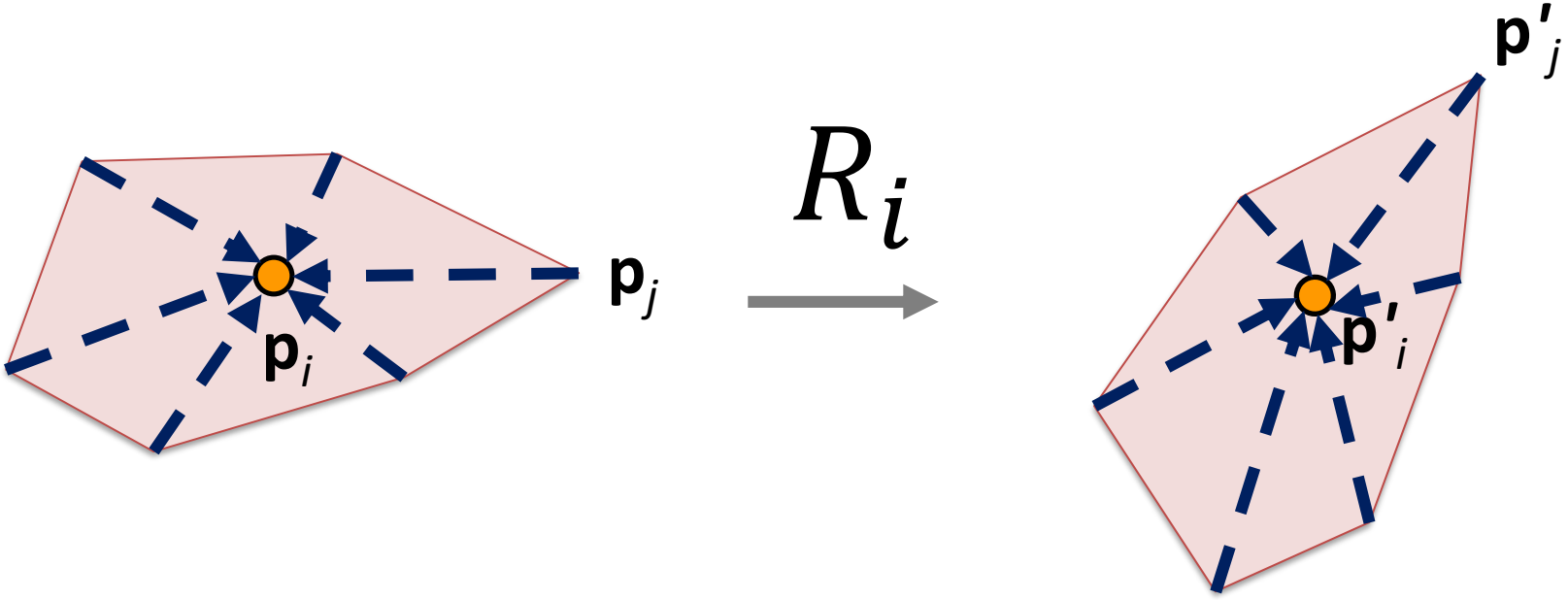


Rigid motion

If cell i moved rigidly:

$$p'_j - p'_i = R_i(p_j - p_i)$$

$$\forall j \in N(i)$$



Deviation from rigid motion

If cell i moved rigidly:

$$p'_j - p'_i = R_i(p_j - p_i)$$

$$\forall j \in N(i)$$

$$E = \sum_{j \in N(i)} \left\| p'_j - p'_i - R_i(p_j - p_i) \right\|^2$$

For the whole mesh

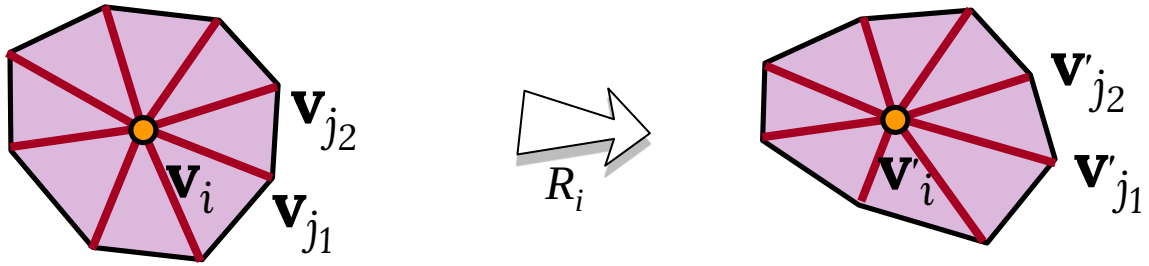
$$E = \sum_{i} \sum_{j \in N(i)} \left\| p'_j - p'_i - R_i(p_j - p_i) \right\|^2$$

For the whole mesh

$$E = \sum_{i} \sum_{j \in N(i)} \boldsymbol{w_{ij}} \left\| p'_j - p'_i - R_i(p_j - p_i) \right\|^2$$

Orthogonal Procrustes problem

How to find the best rotation matrix aligning V with V'?



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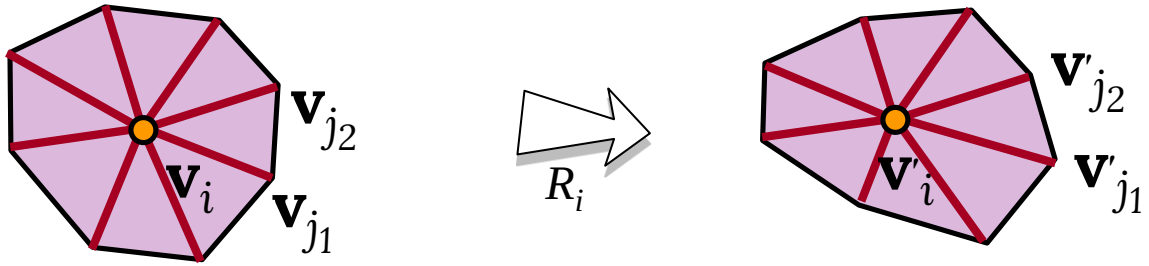
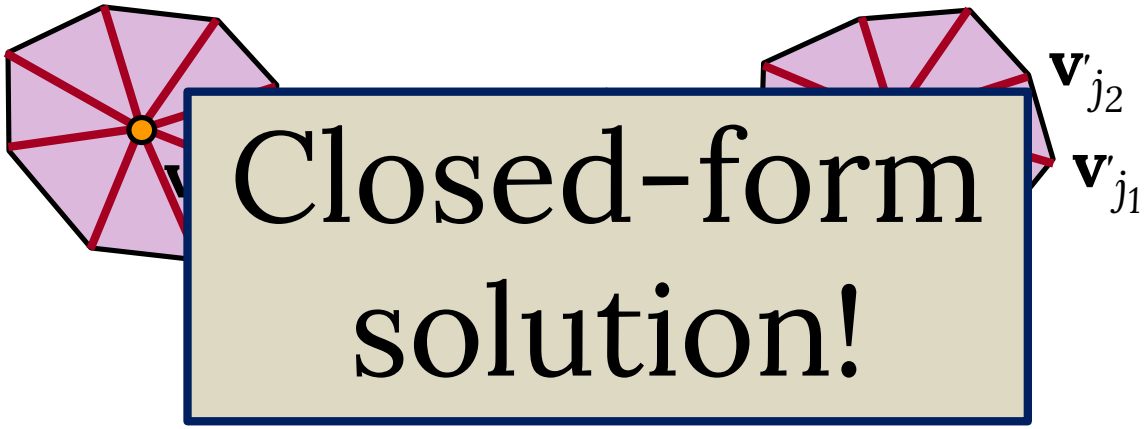


$$\underset{R}{\operatorname{argmin}} \|RA - B\|_F$$
$$\text{s. t. } R^T R = I$$

???

Procrustes problem



1. Build covariance matrix $S = VV'^T$
2. SVD: $S = U\Sigma W^T$
3. $R_i = UW^T$

Mesh Deformation

$$\min \sum_i \sum_{j \in N(i)} \boldsymbol{w_{ij}} \left\| p'_j - p'_i - R_i(p_j - p_i) \right\|^2$$

$$\text{s.t. } p'_i = \tilde{p}_i$$

point constraints

Caveats:

- $\{\boldsymbol{p'}_i\}$ and $\{R_i\}$ are unknown
- Non-linear optimization problem

Mesh Deformation

1. Start with initial guess of $\{\boldsymbol{p'}_i\}$, find $\{R_i\}$
2. Given $\{R_i\}$, minimize energy to find $\{\boldsymbol{p'}_i\}$
3. Repeat

$$\sum_{j \in N(i)} w_{ij} \left(\boldsymbol{p'}_i - \boldsymbol{p'}_j \right) = \sum_{j \in N(i)} \frac{w_{ij}}{2} \left(R_i + R_j \right) \left(\boldsymbol{p}_i - \boldsymbol{p}_j \right)$$

$$L\boldsymbol{p'} = \boldsymbol{b}$$

Advantages

Laplacian

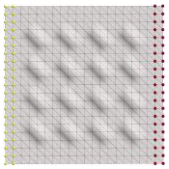
- Depends only on original mesh
- Only needs to be factored once!

Rotations can be computed in parallel

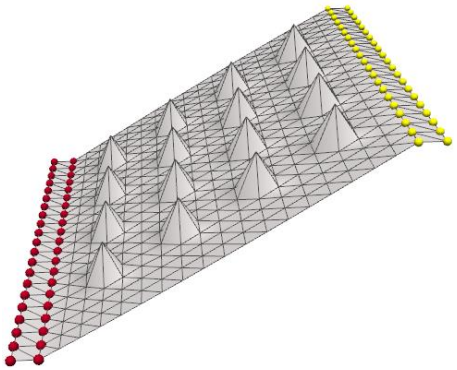
- Each iteration reduces energy
  - Updating rotations guaranteed to reduce cell-error
  - Updating positions guaranteed to reduce global error

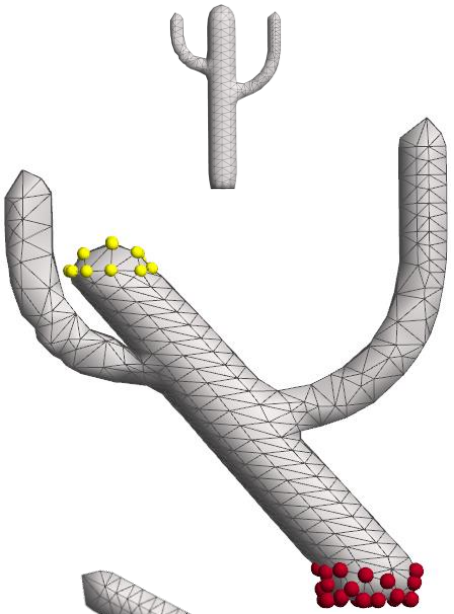
Guaranteed Convergence

Results (vs Poisson)

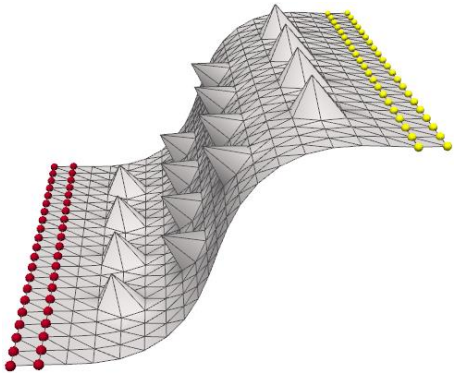


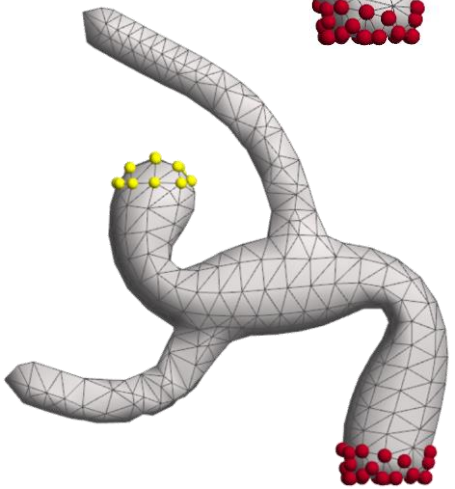
Poisson:





ARAP:





Deformation models

Direct

$$v' = \left(\sum w_j T_j\right) v$$

- Linear Blend Skinning
- Dual Quaternion Skinning
- …

Variational

$$v' = \underset{x}{\operatorname{argmin}}\, E(x)$$

- Multiresolution editing
- As-Rigid-As-Possible
- Laplacian Mesh Editing
- …

Deformation models

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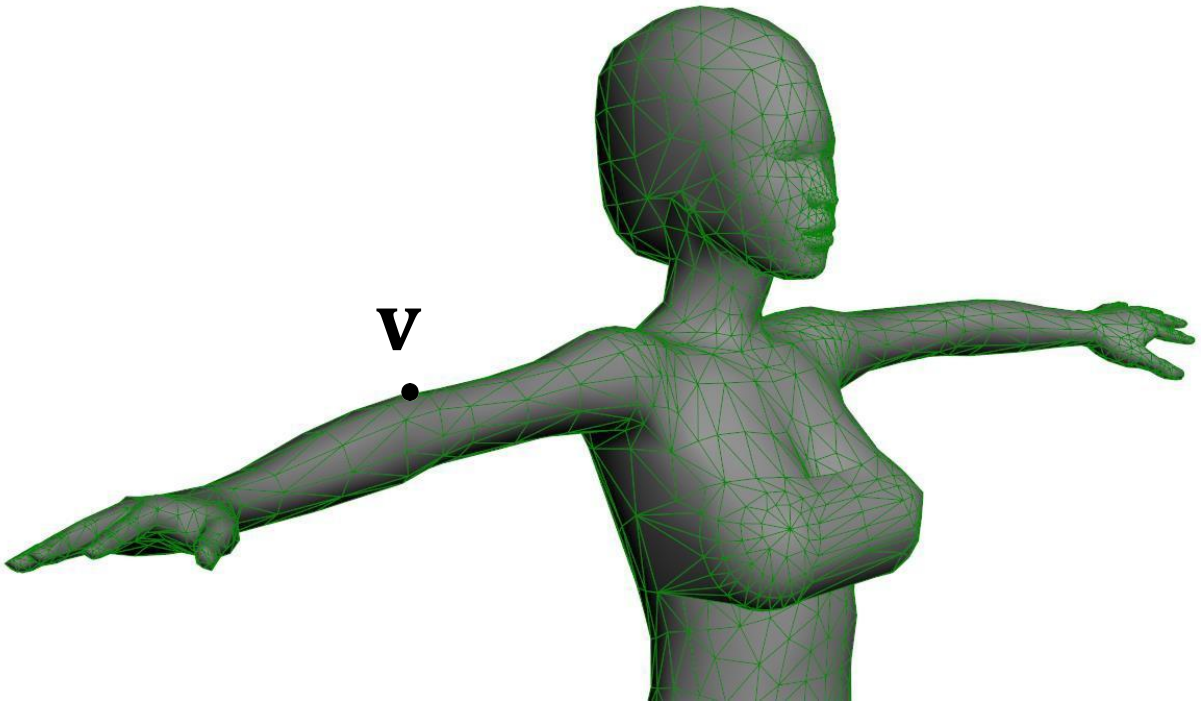
- …

1) Rest pose



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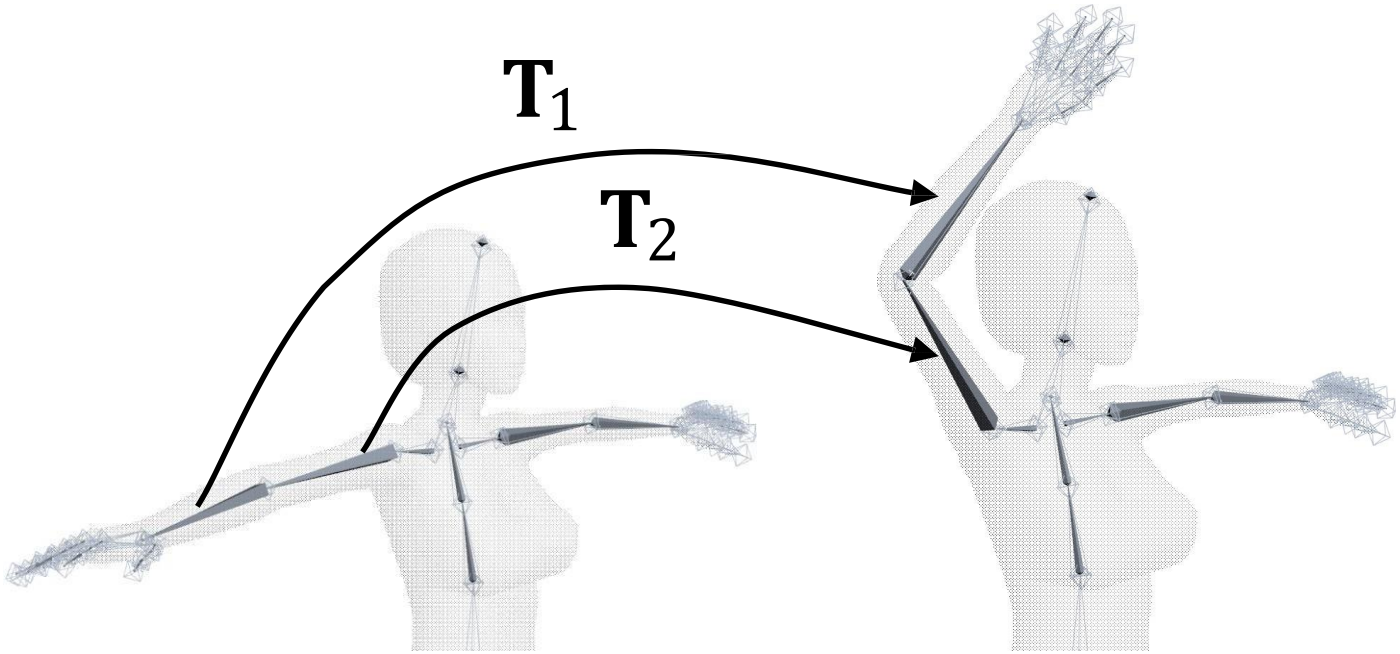
2) Skinning transformations



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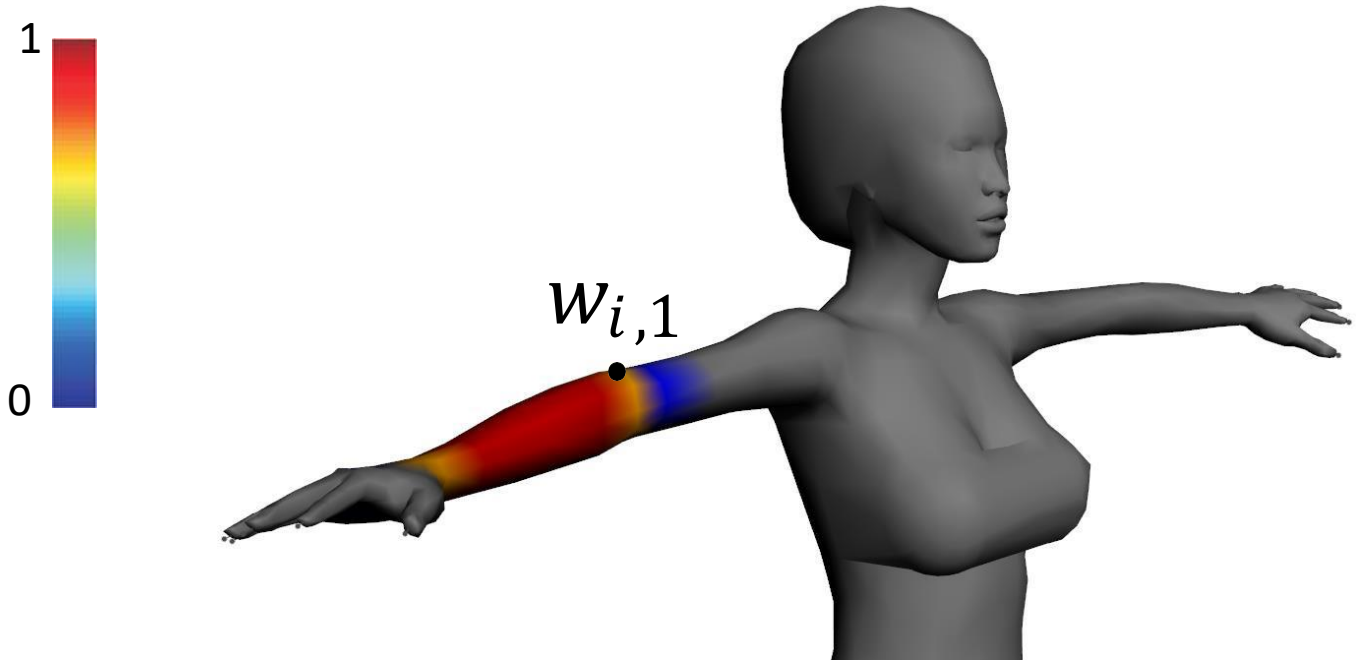
3) Skinning weights



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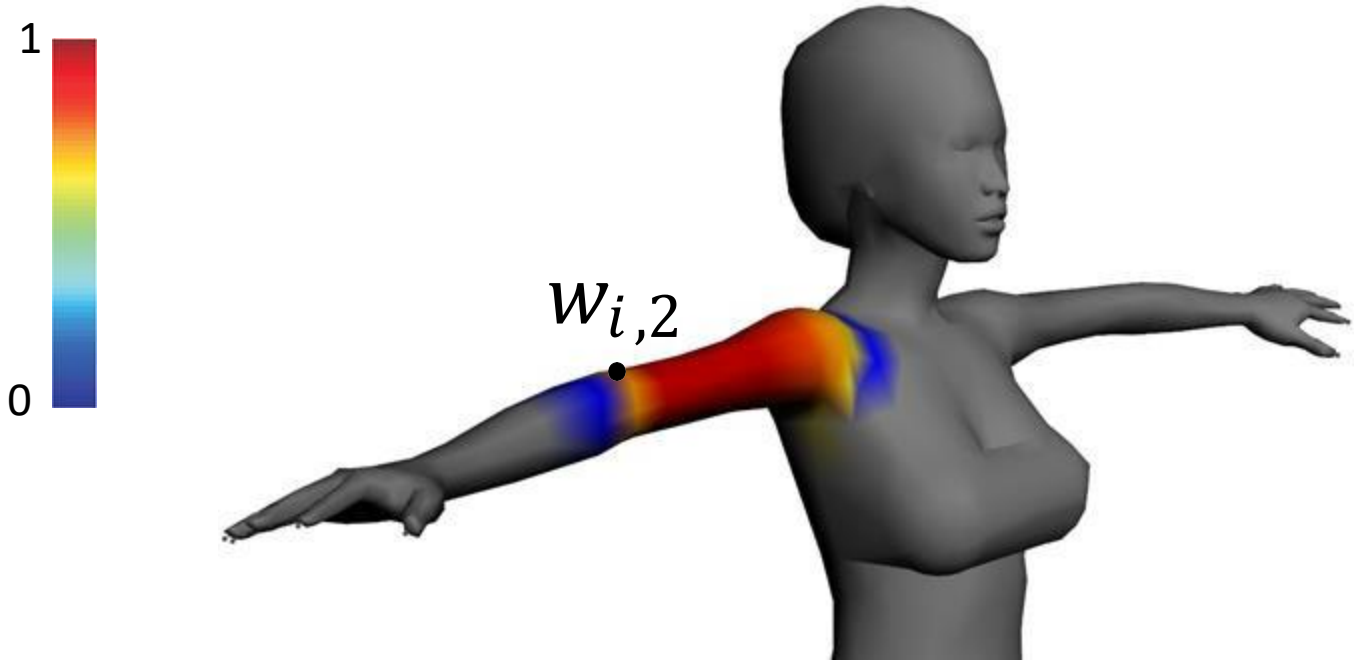

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Linear blend skinning (LBS)

$$v' = \left(\sum w_j T_j\right) v$$

LBS is used widely in the industry

Halo 3

Bolt

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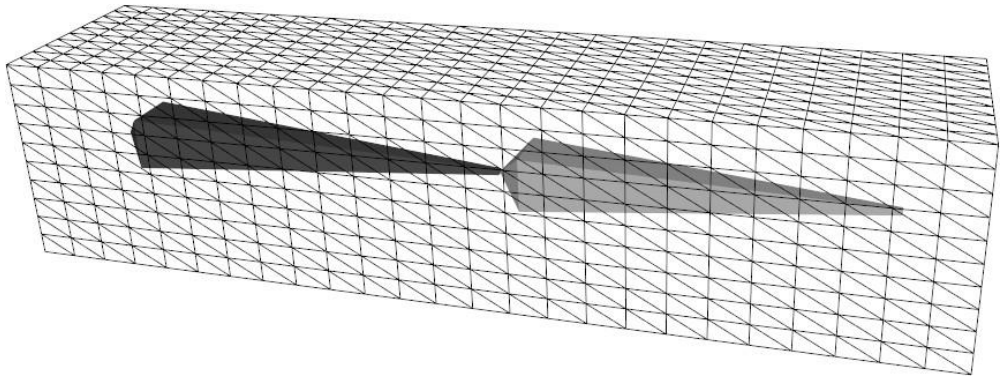
LBS: candy-wrapper artifact



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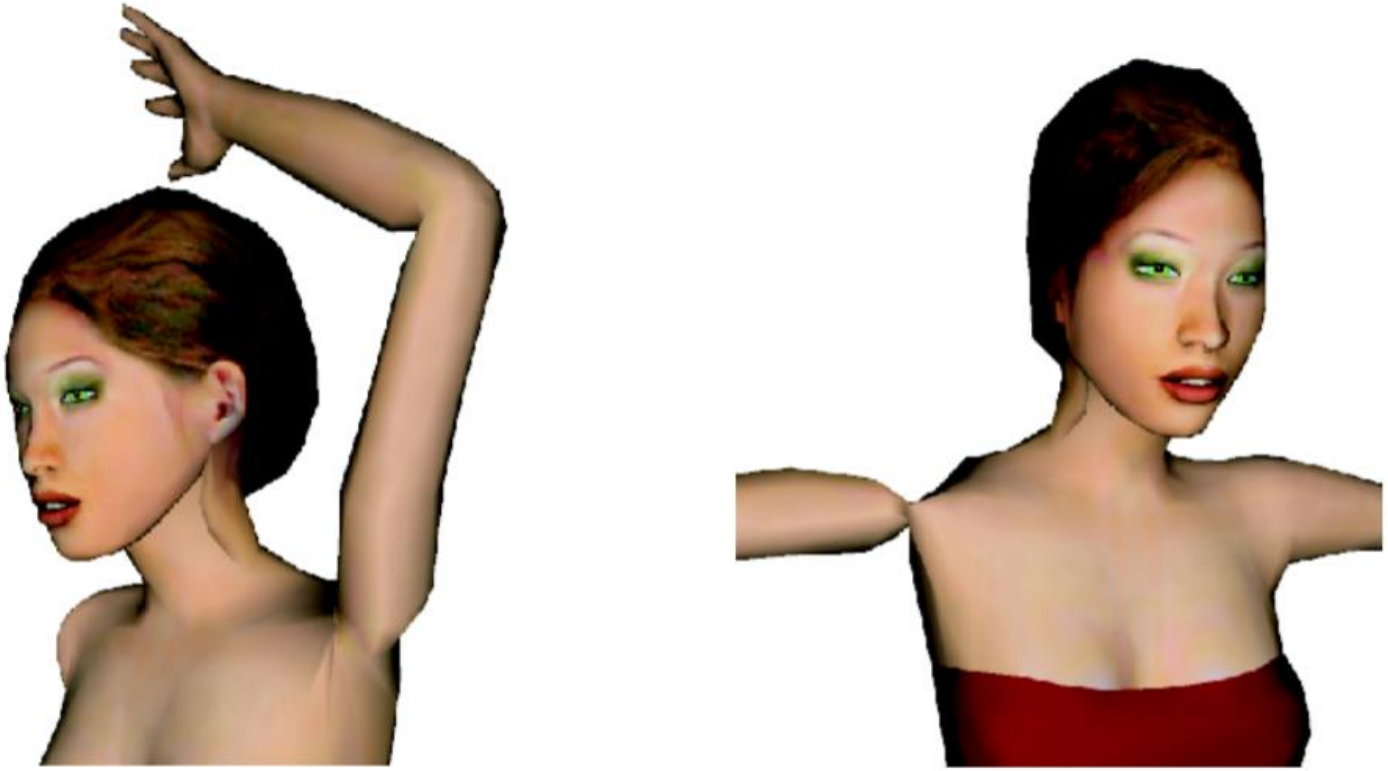

Figure 2: Typical "candy-wrapper" artifacts of linear blend skinning.

What went wrong?

$$v' = \left(\sum w_j T_j\right) v$$

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$$\boldsymbol{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{R}_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Why can't we just sum up rotation matrices?

Geometry of linear blending

$SE(3)$

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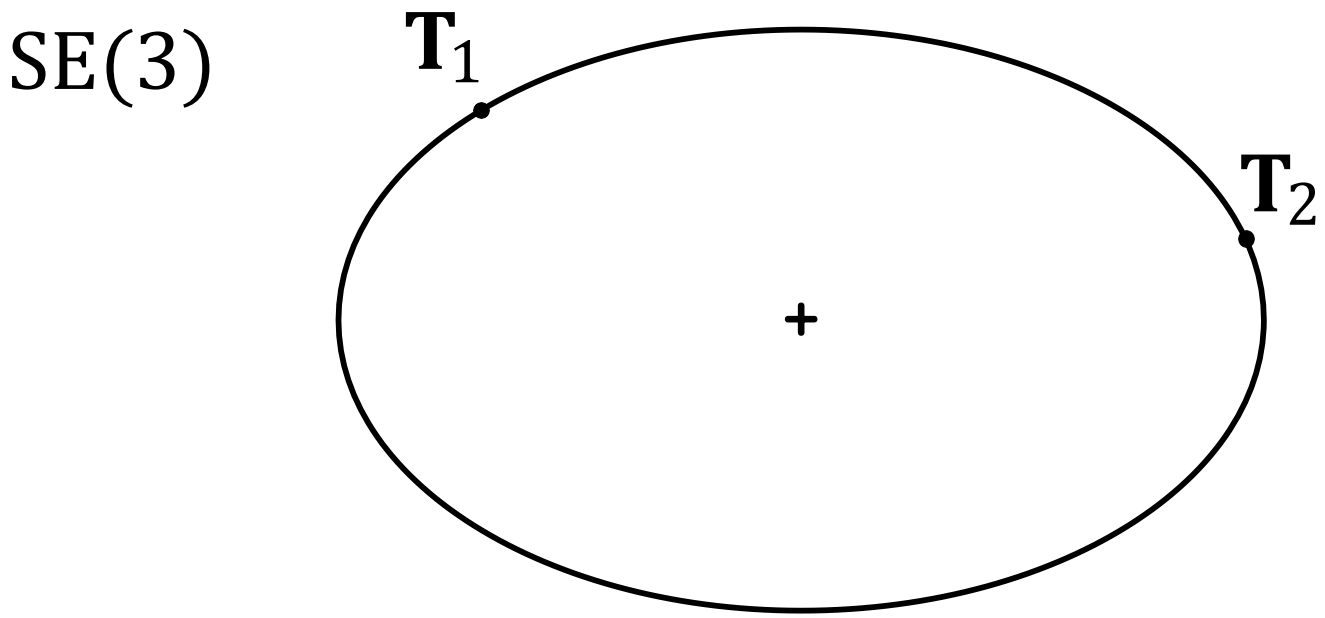
Geometry of linear blending



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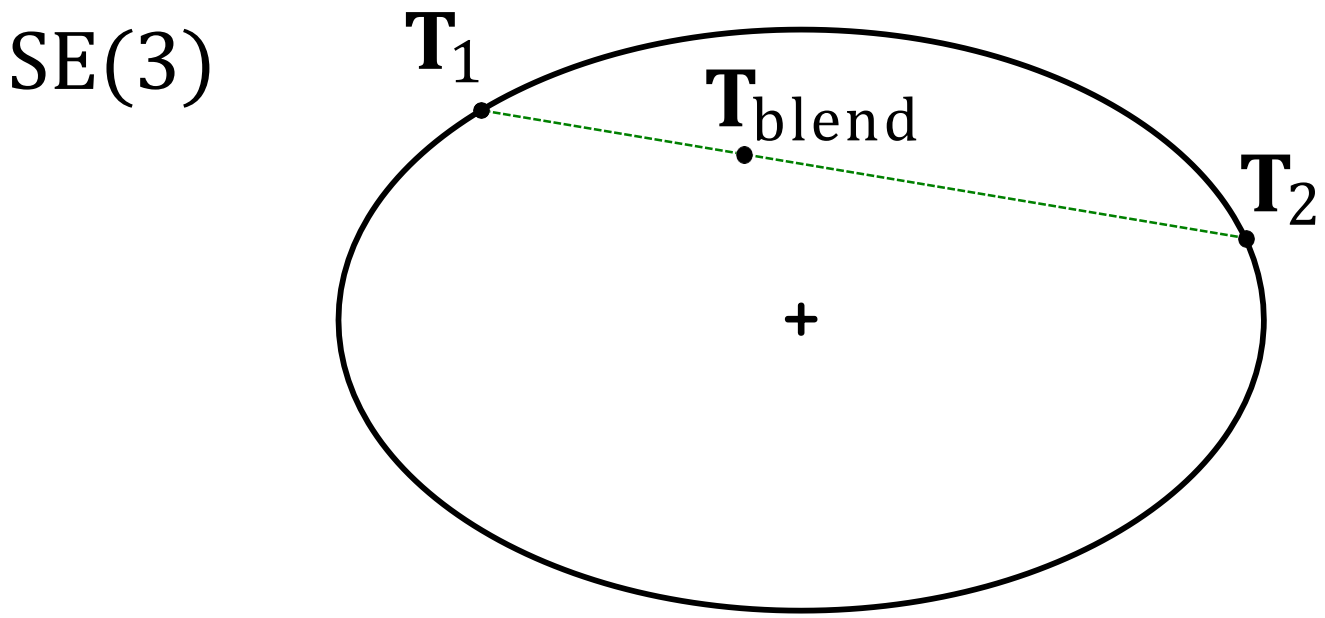
Geometry of linear blending



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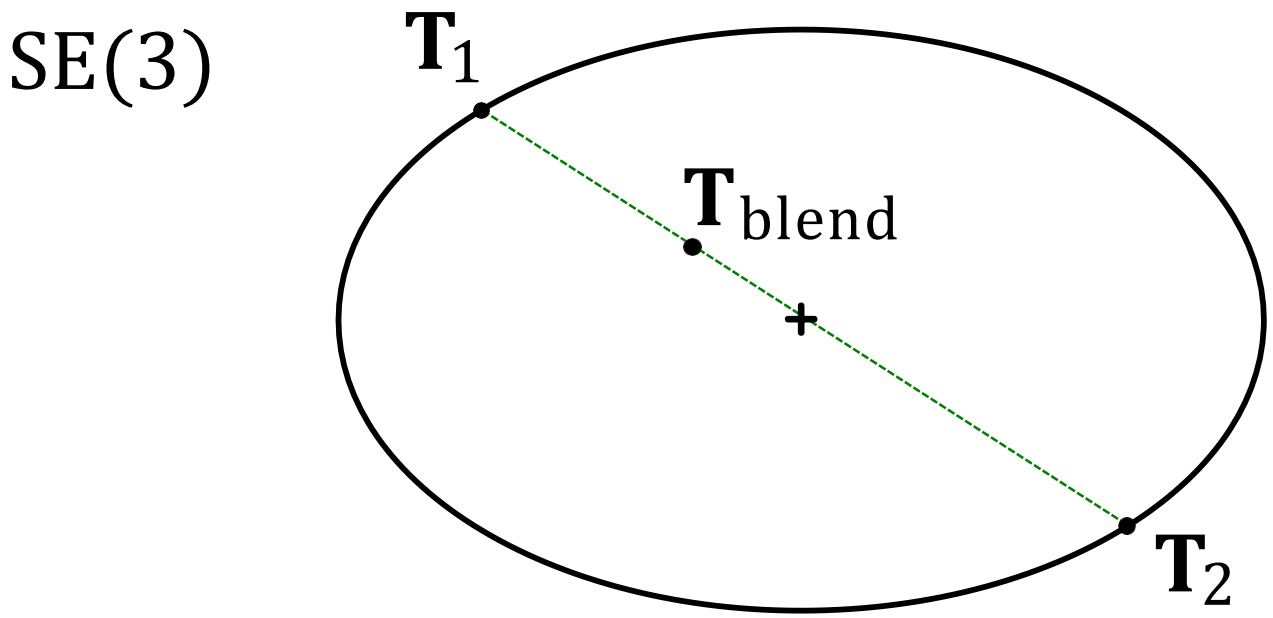
Geometry of linear blending



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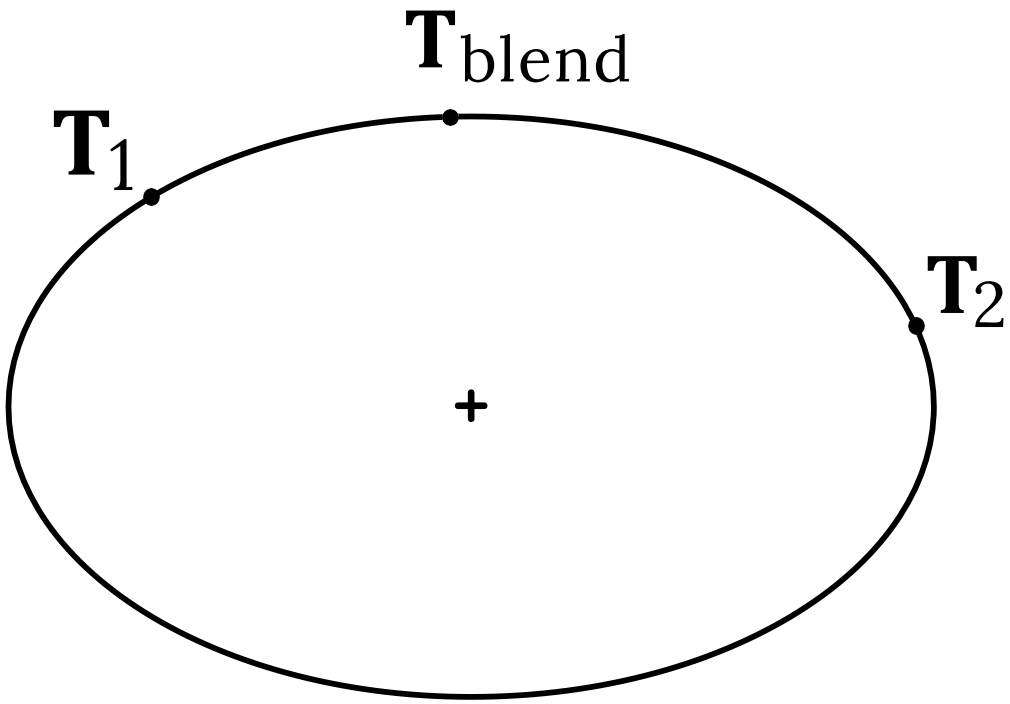
Intrinsic blending



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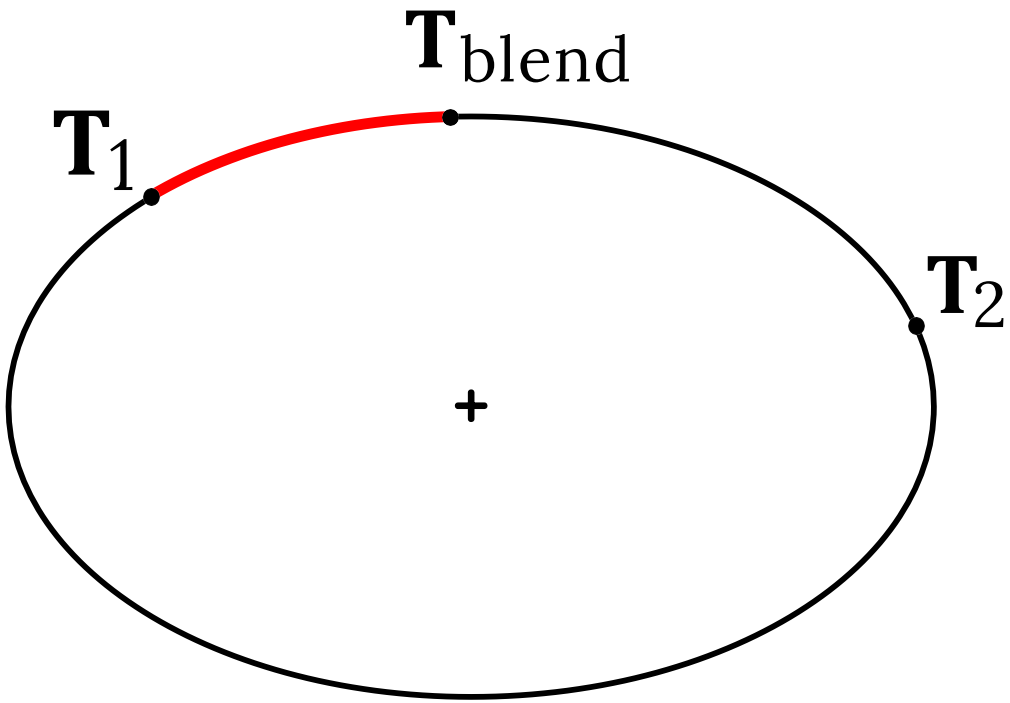

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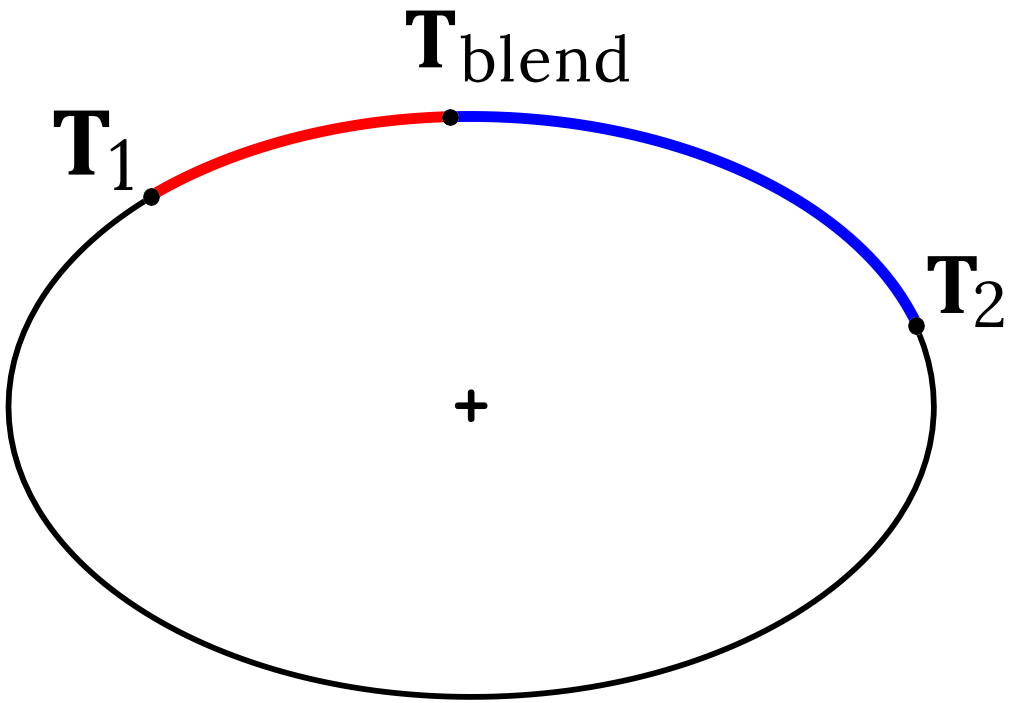

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Intrinsic blending using Lie algebras

[Buss and Fillmore 2001, Alexa 2002, Govindu 2004, Rossignac and Vinacua 2011]

$$\underset{X}{\operatorname{argmin}} \sum_{X} w_j d\left(X, T_j\right)$$

$$d(\mathbf{X}, \mathbf{Y}) = \|\log(\mathbf{Y}\mathbf{X}^{-1})\|^2$$

Karcher / Frechet mean

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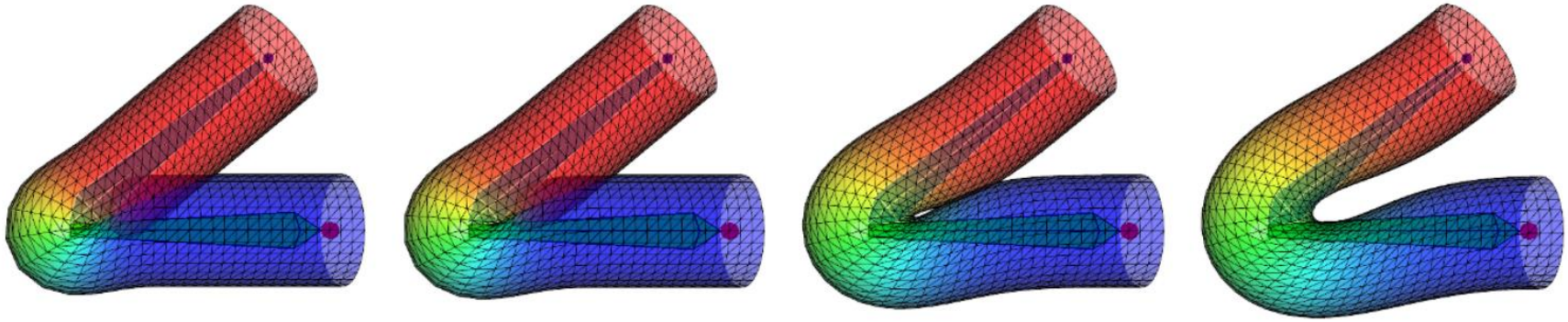
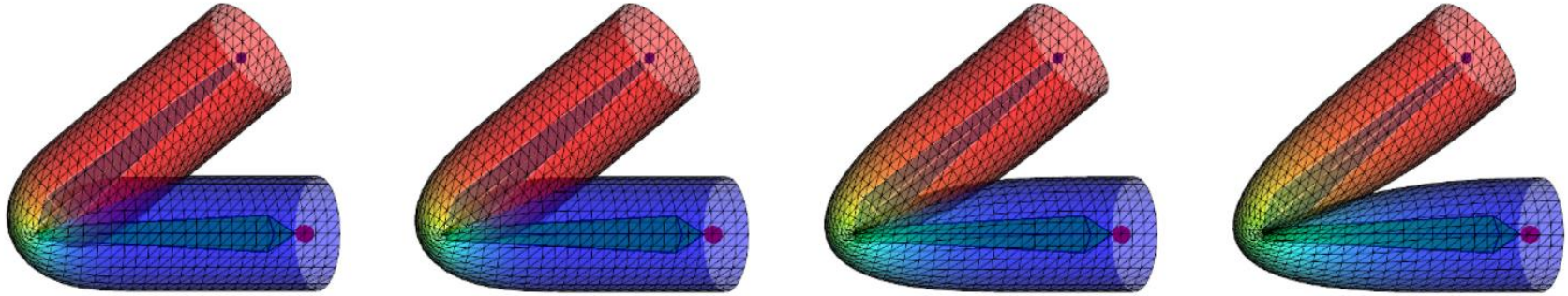
Dual Quaternion Skinning

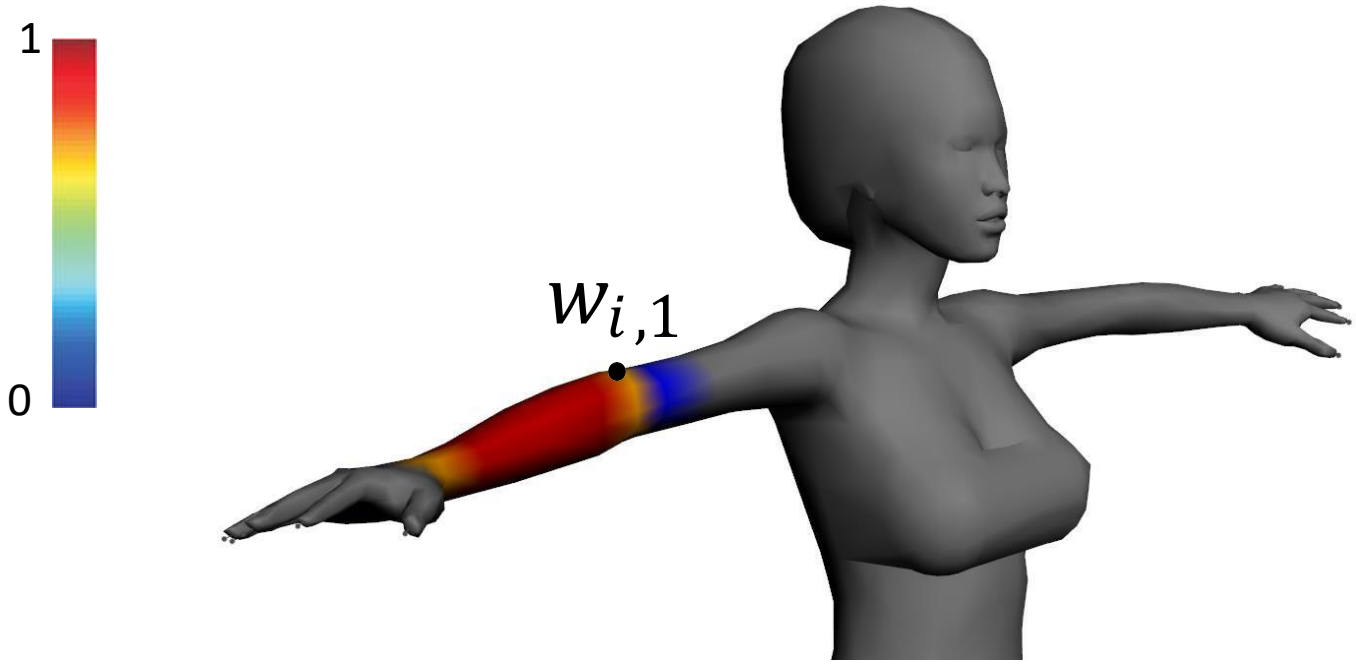
Dual Quats



Linear Blend



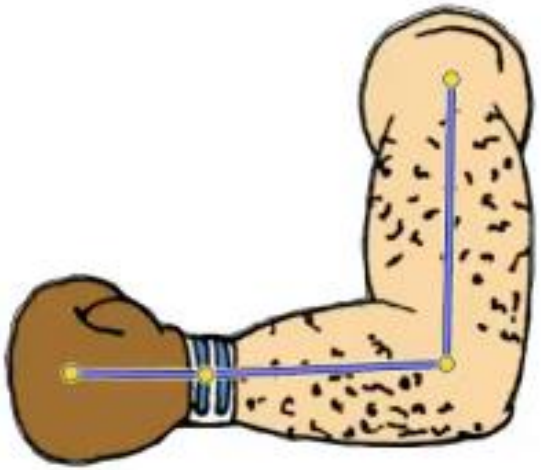
Where do the weights come from?

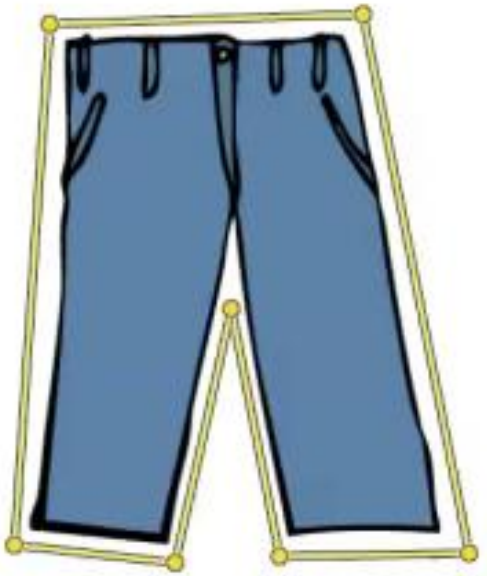


Manual?

Automatic skinning weight computation

Weights should obtain a few basic qualities





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Inverse Euclidean distance weights are too crude, show obvious artifacts

$$w_j(\mathbf{v}) = \frac{1}{\|\mathbf{c}_i - \mathbf{v}\|^2}$$

[Shepard 1968],
[Schaefer et al. 2006], etc.

weights optimized *inside* shape

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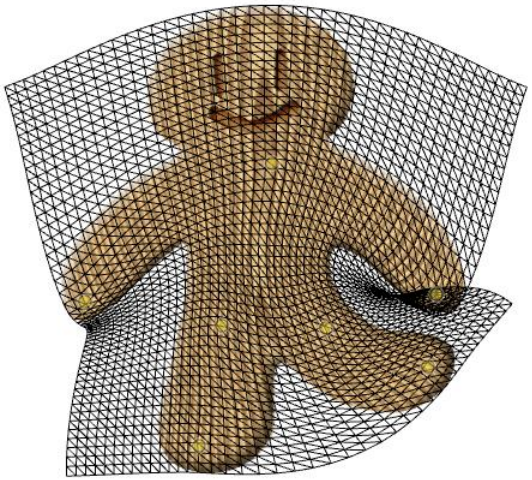

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[Shepard 1968],
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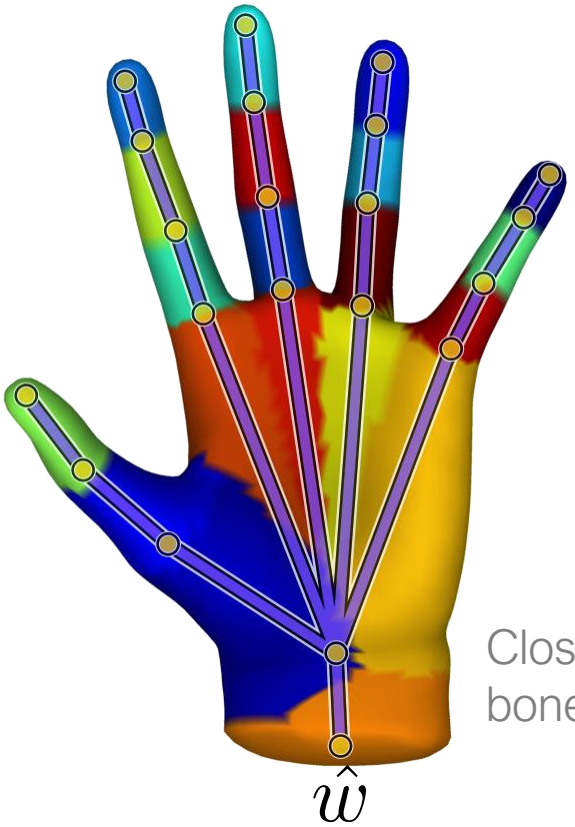
Discontinuous projection onto surface can be smoothed out



$\hat{w}$

Closest *visible* bone

[Baran & Popović 2007]

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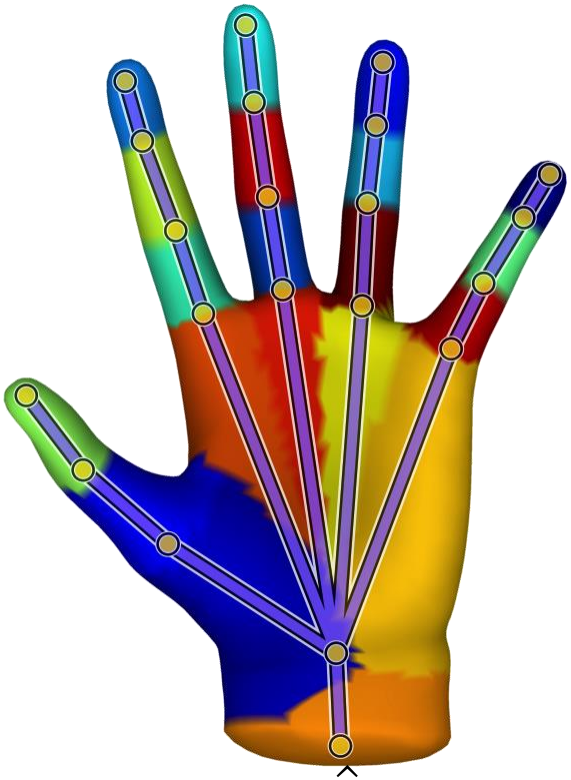
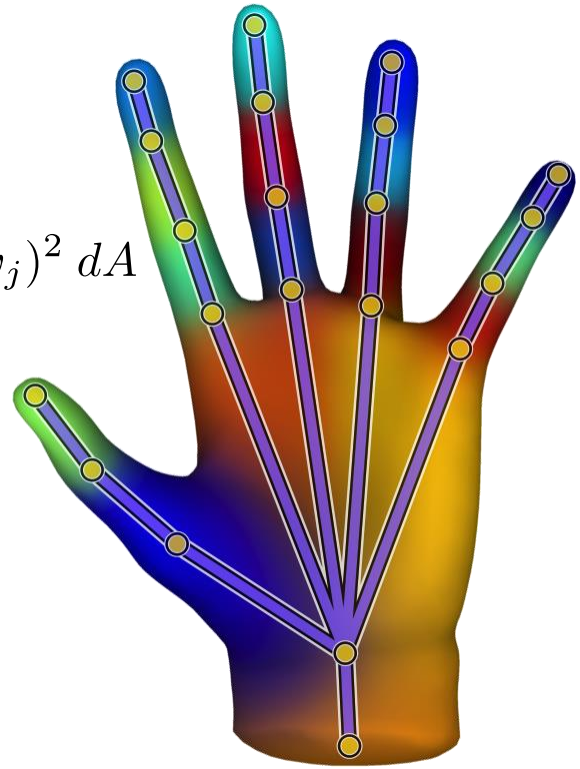
Discontinuous projection onto surface can be smoothed out



$\hat{w}$

$$\underset{w_j}{\text{argmin}} \int_\Omega \|\nabla w_j\|^2 + h_j(w_j - \hat{w}_j)^2 \, dA$$



w

[Baran & Popović 2007]

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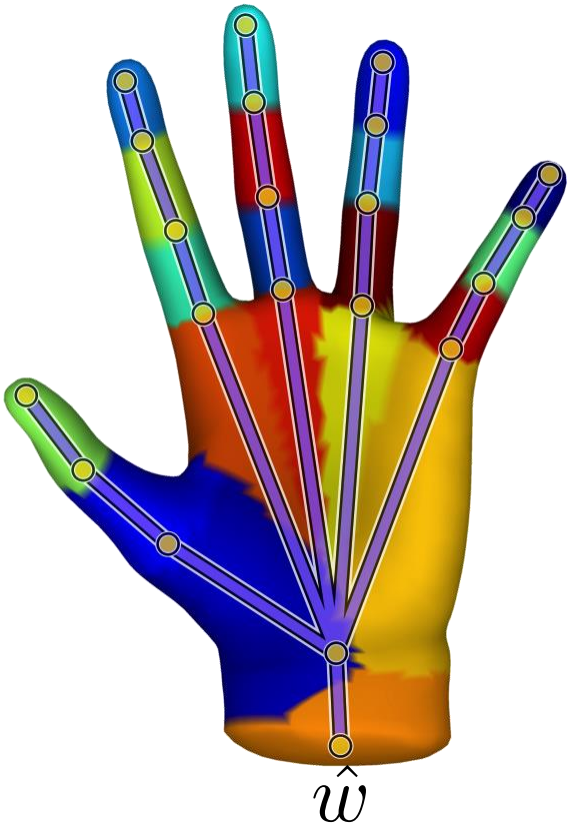
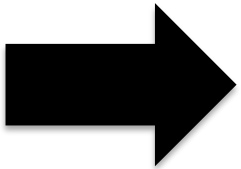
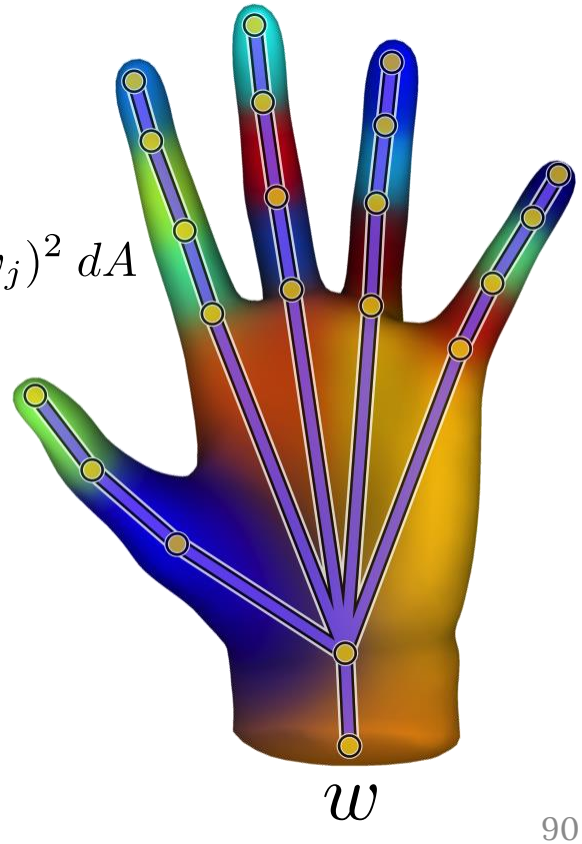


$\hat{w}$

smoothness

$$\underset{w_j}{\text{argmin}} \int_\Omega \|\nabla w_j\|^2 + h_j(w_j - \hat{w}_j)^2 \, dA$$





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[Baran & Popović 2007]

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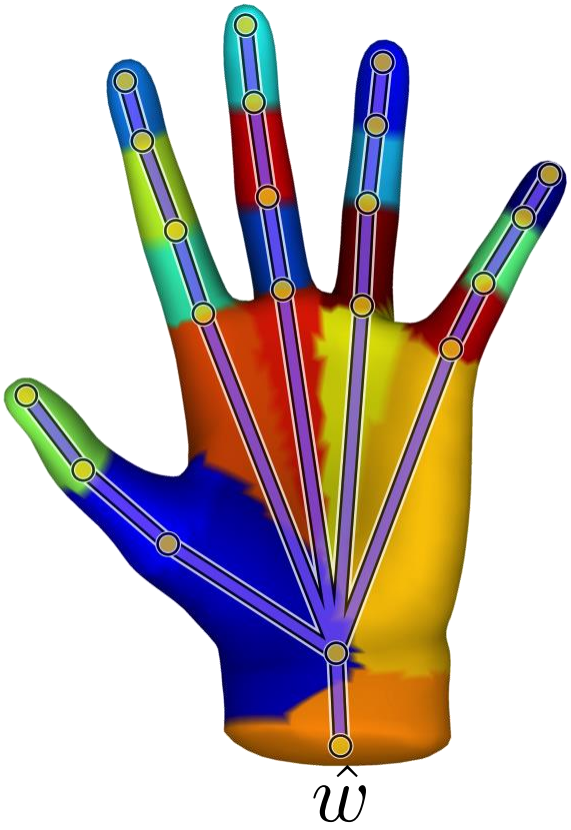

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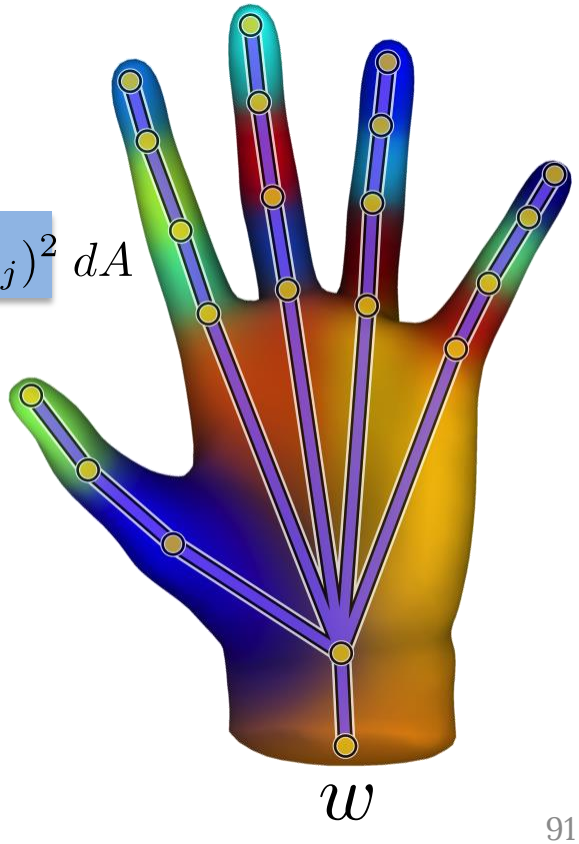
$$\underset{w_j}{\text{argmin}} \int_\Omega \|\nabla w_j\|^2 + h_j(w_j - \hat{w}_j)^2 \, dA$$

"data"



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[Baran & Popović 2007]

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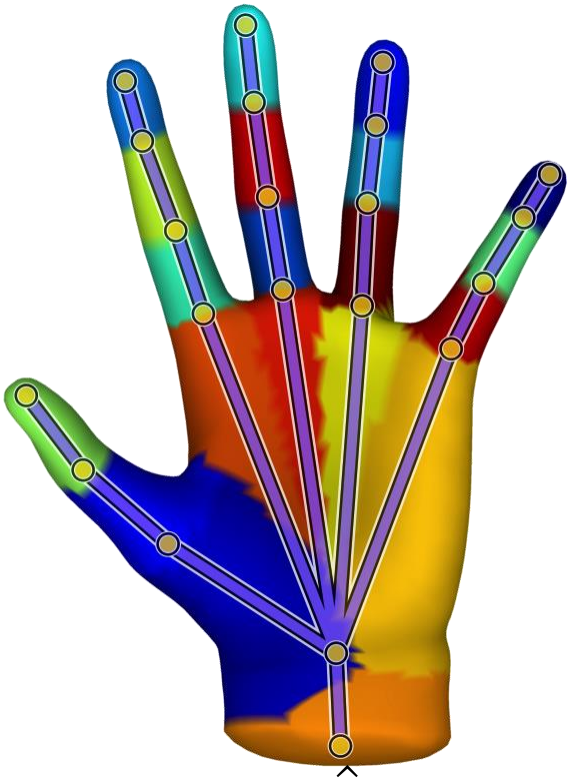

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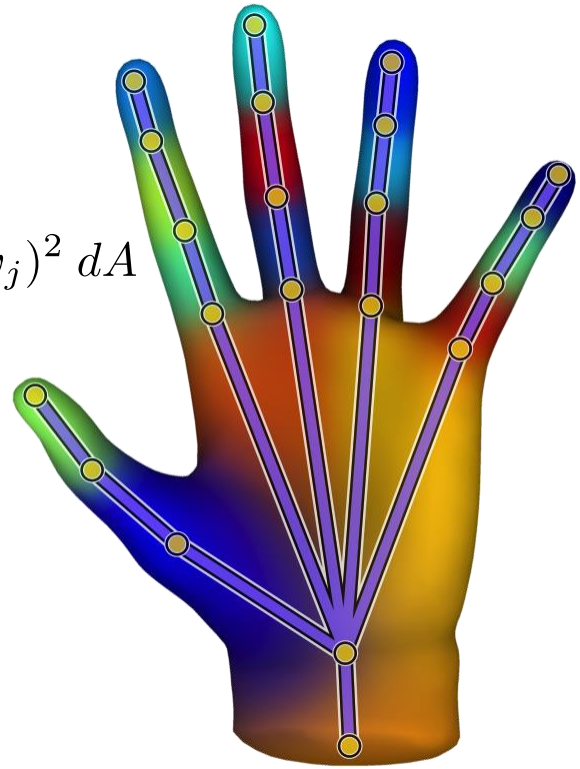

w

[Baran & Popović 2007]

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Gradient energy weights not smooth at handles

$$\underset{w_j}{\text{argmin}} \int_\Omega (\Delta w_j)^2 \, dA$$

$$\underset{w_j}{\text{argmin}} \int_\Omega \|\nabla w_j\|^2 \, dA$$

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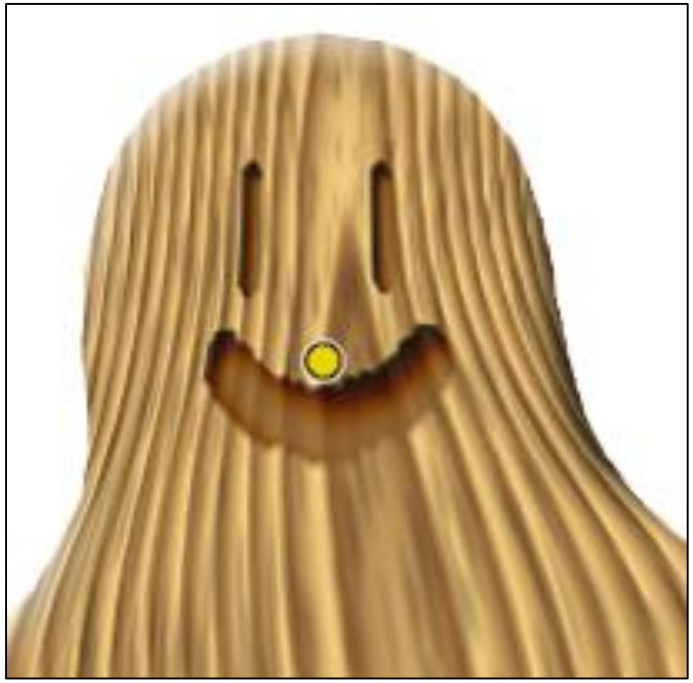
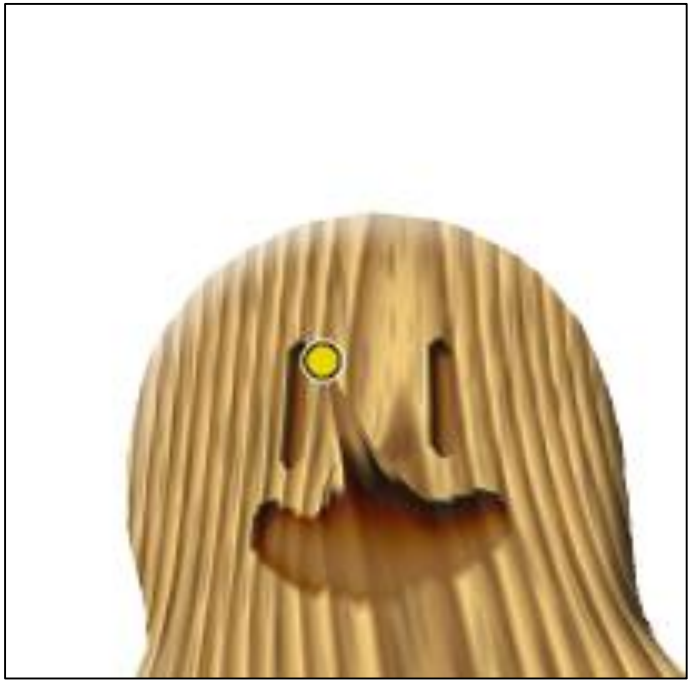
Gradient energy weights not smooth at handles



$$\underset{w_j}{\text{argmin}} \int_\Omega (\Delta w_j)^2 \, dA$$



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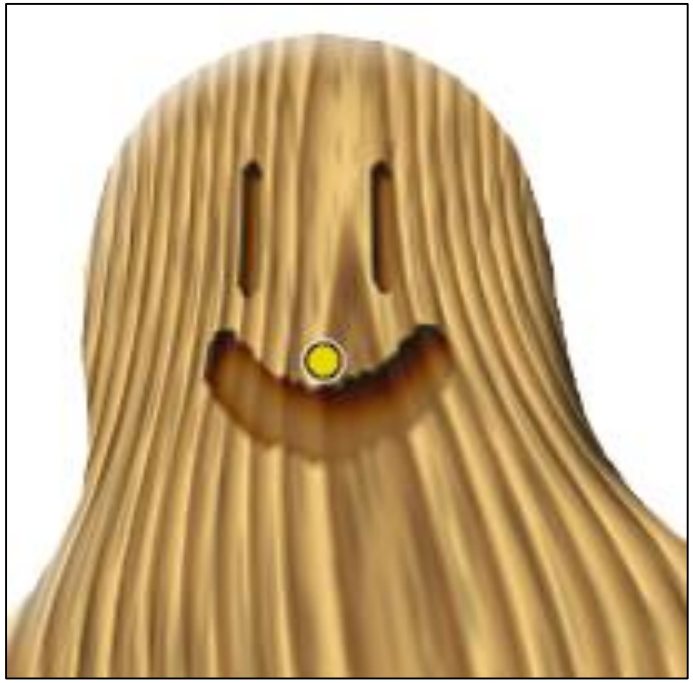
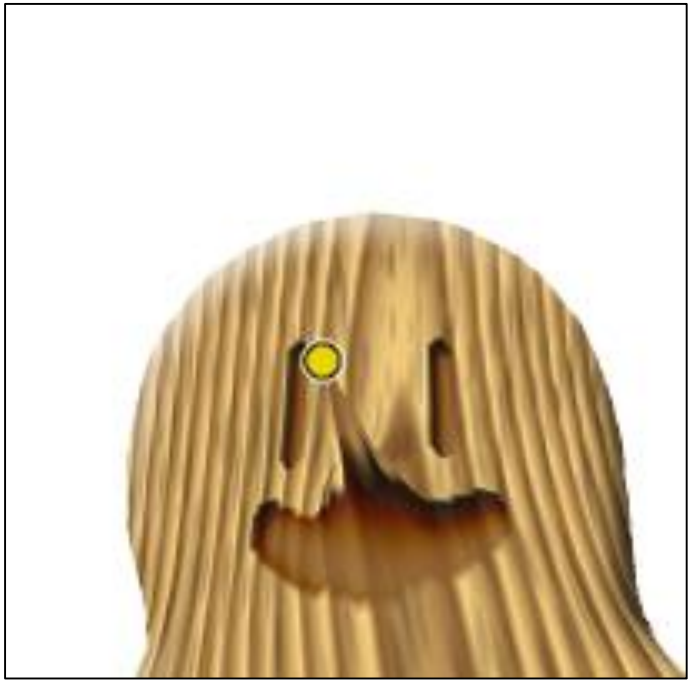


$$\Delta^2 w_j = 0$$



$$\Delta w_j = 0$$

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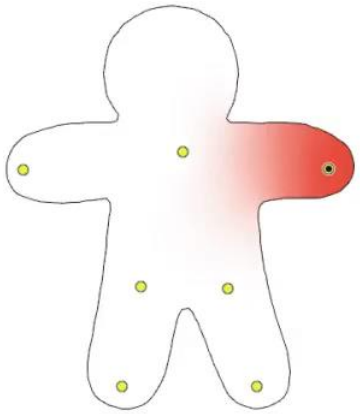
Point constraints for Laplace equation

https://www.facebook.com/521399544544480/photos/a.523048724379562/800968259920939/?type=1&theater , Facebook group “Circus tents and circus equipment”

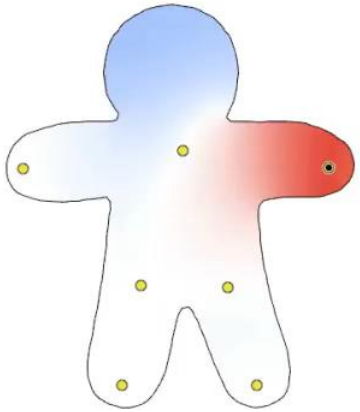
Non-negative, local weights are mandatory

$$0 \leq w_j \leq 1$$



$$\underset{w_j}{\operatorname{argmin}} \int_{\Omega} (\Delta w_j)^2 \, dA$$

[Botsch & Kobbelt 2004]



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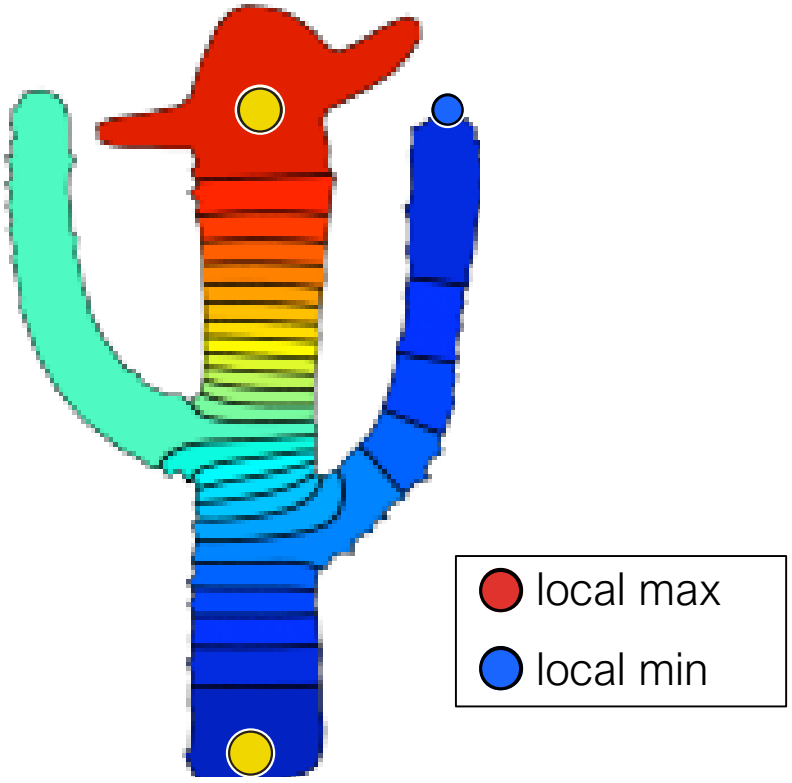
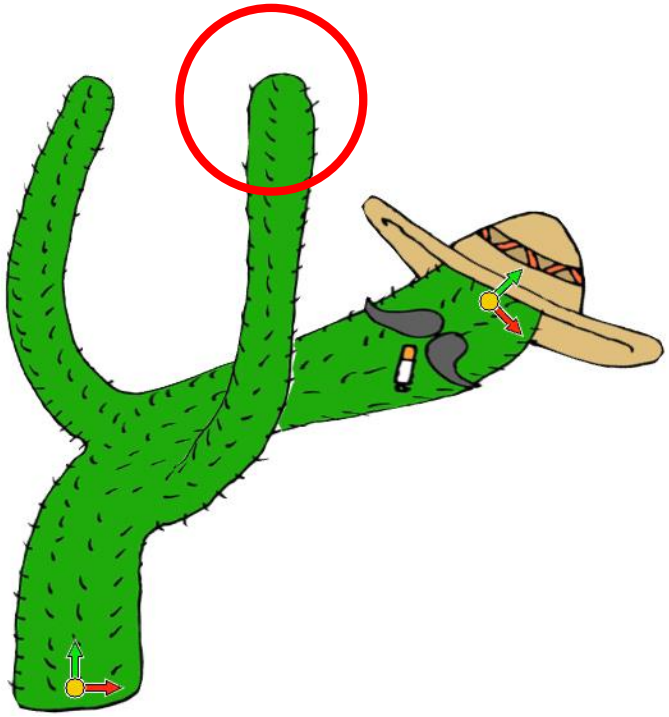
Spurious extrema cause distracting artifacts



$$0 \leq w_j \leq 1$$



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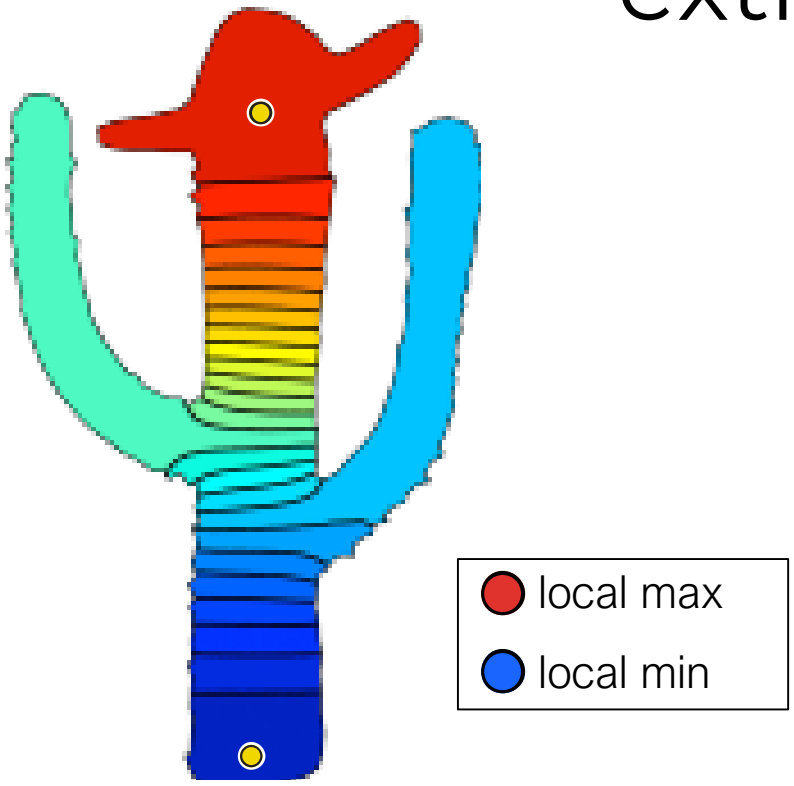
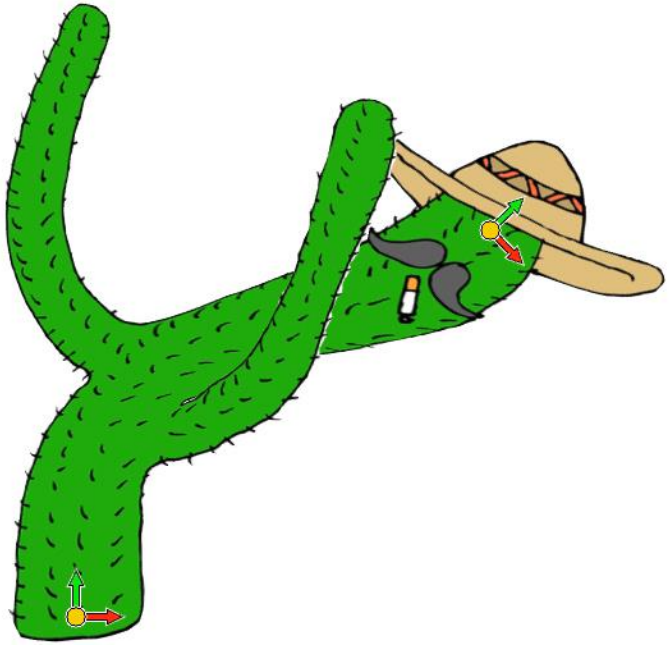
Must explicitly prohibit spurious extrema





w_j is “monotonic”

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Previous methods fail in one way or another

	Euclidean	Δw_j	$\Delta^2 w_j$
smooth	✓	–	✓
non-negative	✓	✓	-
shape-aware	–	✓	✓
local	-/✓	–	-
monotonic	-	✓	-
arbitrary handles	-	✓	✓
	[Shepard 1968, Sibson 1980, Schaefer et al. 2006]	[Baran & Popovic 2007, Joshi et al. 2007]	[Botsch & Kobbelt 2004, Sorkine et al. 2004, Finch et al. 2011]

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Constrained optimization ensures satisfaction of all properties

$$\underset{w_j, j=1,\ldots,m}{\operatorname{argmin}} \sum_{j=1}^{m} \int_{\Omega} (\Delta w_j)^2 \, dV$$

+ shape-aware
+ smoothness

[Botsch & Kobbelt 2004, Sorkine et al. 2004, Joshi & Carr 2008, Jacobson et al. 2010, Finch et al. 2011, Andrews et al. 2011]

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$$\underset{w_j, j=1,\ldots,m}{\operatorname{argmin}} \sum_{j=1}^{m} \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$w_j(\mathbf{v}) = \begin{cases} 1 & \mathbf{v} \in h_j, \\ 0 & \mathbf{v} \in h_k \\ \text{linear on cage facets} \end{cases}$$

+ shape-aware
+ smoothness
+ arbitrary handles

[Botsch & Kobbelt 2004, Sorkine et al. 2004, Joshi & Carr 2008, Jacobson et al. 2010, Finch et al. 2011, Andrews et al. 2011]

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$$0 \leq w_j \leq 1,$$

$$\sum_{j=1}^{m} w_j = 1$$

+ shape-aware
+ smoothness
+ arbitrary handles
+ non-negativity

[Jacobson et al. 2011]

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$$\underset{w_j, j=1,\ldots,m}{\operatorname{argmin}} \sum_{j=1}^{m} \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$\|w\|_1 = 1$$

+ shape-aware
+ smoothness
+ arbitrary handles
+ non-negativity
+ locality

[Rustamov 2011]

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Constrained optimization ensures satisfaction of all properties

$$\underset{w_j, j=1,\ldots,m}{\operatorname{argmin}} \sum_{j=1}^{m} \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$\|w\|_1 = 1 \rightarrow \sum_{j=1}^{m} |w_j| = 1$$

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[Rustamov 2011]

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[Rustamov 2011]

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Constrained optimization ensures satisfaction of all properties

$$\underset{w_j, j=1,\ldots,m}{\operatorname{argmin}} \sum_{j=1}^{m} \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$\nabla w_j \cdot \nabla u_j > 0$$

+ shape-aware
+ smoothness
+ arbitrary handles
+ non-negativity
+ locality
+ monotonicity

[Weinkauf et al. 2011, Jacobson et al. 2012, Günther et al. 2014]

Previous methods fail in one way or another

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	[Shepard 1968, Sibson 1980, Schaefer et al. 2006]	[Baran & Popovic 2007, Joshi et al. 2007]	[Botsch & Kobbelt 2004, Sorkine et al. 2004, Finch et al. 2011]

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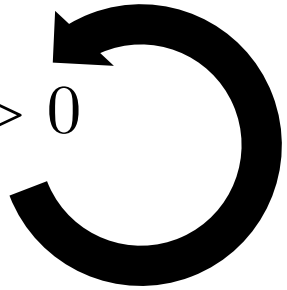

+ shape-aware
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[Weinkauf et al. 2011, Jacobson et al. 2012, Günther et al. 2014]

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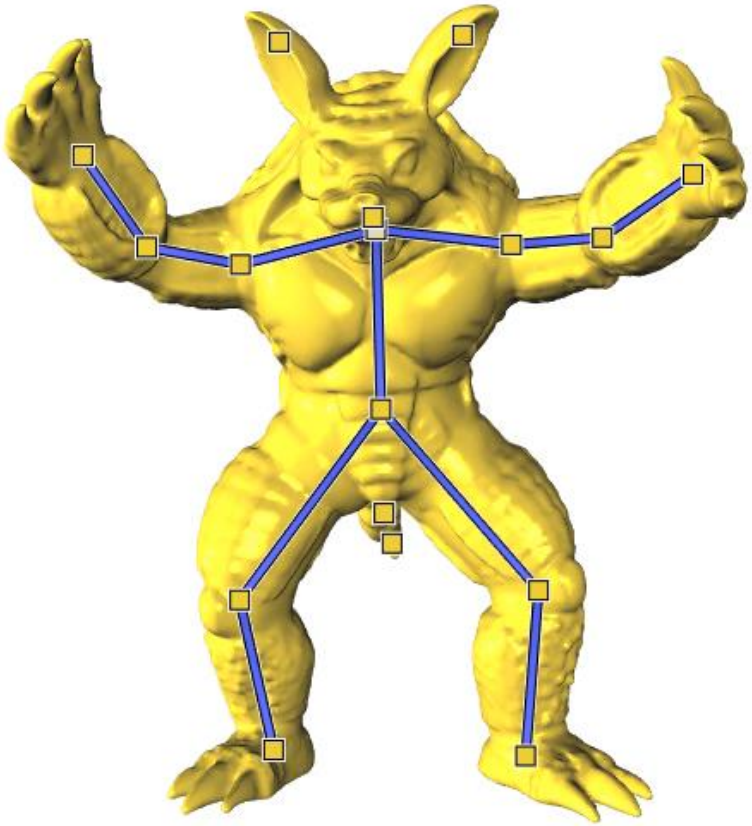
Weights retain nice properties in 3D



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