IFT 6113
CLUSTERING AND SEGMENTATION
tiny.cc/ift6113

Semi-supervised Mesh Segmentation and Labeling by Lv et al., 2012

Mikhail Bessmeltsev
Task:
Break a shape into meaningful pieces.

https://doc.cgal.org/latest/Surface_mesh_segmentation/index.html
Many Applications

- Different cluster analysis results on "mouse" data set:
  - Original Data
  - K-Means Clustering
  - EM Clustering

- Unsupervised learning

- Medical imaging

- CAD

- Graphics

- http://liris.cnrs.fr/christian.wolf/graphics/anr-madras.png
- http://people.cs.umass.edu/~kalo/papers/LabelMeshes/
Task:
Break a shape into meaningful pieces.

https://doc.cgal.org/latest/Surface_mesh_segmentation/index.html
What’s meaningful?

• Semantically?
• Flat/Developable?
  • Convex?
• Voronoi-like?

http://www.cs.rug.nl/svcg/Shapes/PDE
What’s meaningful?

- Semantically?
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https://github.com/kmammou/v-hacd
Fig. 1. The pipeline of our approach. Starting from a 3D model, the user decomposes the shape into topological cylinders. Our algorithm automatically produces a single continuous curve on the shape that spirals along the cylinders. It proceeds to cut the shape along the curve and creates a developable surface that can be trivially unfolded into a single 2D shape – the so called zippable. Based on the flattening, plans for laser cutting it from fabric are generated. Finally, we attach a zipper with a single slider to the boundary of the zippable. Zipping it up reproduces a faithful approximation of the input model.
Semantic

Modeling new chairs using parts from old ones

Learning 3D Mesh Segmentation and Labeling by Evangelos Kalogerakis et al., ACM TOG 2010
k-means clustering

$$\min_{S, \mu} \sum_{i=1}^{k} \sum_{x \in S_i} \|x - \mu_i\|^2$$

https://upload.wikimedia.org/wikipedia/commons/d/d2/K_Means_Example_Step_4.svg
k-means clustering

\[ \min_{S, \mu} \sum_{i=1}^{k} \sum_{x \in S_i} \| x - \mu_i \|^2 \]

NP-hard for variable \( k \) is a even on a plane

\( O(|S_i|^{d(k+1)}) \), if \( k \) is fixed
Lloyd iterations
‘the’ k-means algorithms

\[
\min_{S, \mu} \sum_{i=1}^{k} \sum_{x \in S_i} \|x - \mu_i\|^2
\]

• **Initialization?**

• **Assignment step (S)**

\[
S_i \leftarrow \{x : \|x - \mu_i\| \leq \|x - \mu_j\| \forall j\}
\]

• **Update step (\(\mu\))**

\[
\mu_i \leftarrow \frac{1}{|S_i|} \sum_{x \in S_i} x
\]
Recall:

Voronoi Diagrams

Georgy Voronoi
Георгий Феодосьевич Вороной
1868–1908
Example
Recall:

Lloyd Iterations
for segmentation

Initialization: select random triangles = seeds
1. Grow charts around seeds greedily
2. Find new seed for each chart
   – E.g. centroid
3. Repeat
Application to Color Space

K=2

K=3

K=10

Original

4%

8%

17%

Dependence on Initial Guess

• Initialize K segment seeds, iterate:
  • Assign faces to closest seed
  • Move seed to cluster center
• Randomization: random initial seeds

“Randomized Cuts for 3D Mesh Analysis.”
Golovinskiy and Funkhouser; SIGGRAPH Asia 2008

Bug ... or feature?
Dependence on Initial Guess

“Randomized Cuts for 3D Mesh Analysis.”
Golovinskiy and Funkhouser; SIGGRAPH Asia 2008

Bug ... or feature?
Estimating the number of clusters in a data set via the gap statistic

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[Received February 2000. Final revision November 2000]

Summary. We propose a method (the 'gap statistic') for estimating the number of clusters (groups) in a set of data. The technique uses the output of any clustering algorithm (e.g. K-means or hierarchical), comparing the change in within-cluster dispersion with that expected under an appropriate reference null distribution. Some theory is developed for the proposal and a simulation study shows that the gap statistic usually outperforms other methods that have been proposed in the literature.

Keywords: Clustering; Groups; Hierarchy; K-means; Uniform distribution

1. Introduction

Cluster analysis is an important tool for ‘unsupervised’ learning—the problem of finding groups in data without the help of a response variable. A major challenge in cluster analysis is the estimation of the optimal number of 'clusters'. Fig. 1(b) shows a typical plot of an error measure $W_k$ (the within-cluster dispersion defined below) for a clustering procedure versus the number of clusters $k$ employed: the error measure $W_k$ decreases monotonically as the number of clusters $k$ increases, but from some $k$ onwards the decrease flattens markedly. Statistical folklore has it that the location of such an 'elbow' indicates the appropriate number of clusters. The goal of this paper is to provide a statistical procedure to formalize that heuristic.

For recent studies of the elbow phenomenon, see Sugar (1998) and Sugar et al. (1999). A comprehensive survey of methods for estimating the number of clusters is given in Milligan and Cooper (1985), whereas Gordon (1999) discusses the best performers. Some of these methods are described in Sections 5 and 6, where they are compared with our method.

In this paper we propose the ‘gap’ method for estimating the number of clusters. It is designed to be applicable to virtually any clustering method. For simplicity, the theoretical part of our analysis will focus on the widely used $K$-means clustering procedure.

2. The gap statistic

Our data $\{x_{ij}\}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, p$, consist of $p$ features measured on $n$ independent observations. Let $d_{ii'}$ denote the distance between observations $i$ and $i'$. The most common choice for $d_{ii'}$ is the squared Euclidean distance $\sum (x_{ij} - x_{i'j})^2$. 
On a surface?

\[
\min_{S, \mu} \sum_{i=1}^{k} \sum_{x \in S_i} \| x - \mu_i \|^2
\]
Recall: Global Point Signature

\[
\text{GPS}(p) := \left( -\frac{1}{\sqrt{\lambda_1}} \phi_1(p), -\frac{1}{\sqrt{\lambda_2}} \phi_2(p), -\frac{1}{\sqrt{\lambda_3}} \phi_3(p), \ldots \right)
\]

“Laplace-Beltrami Eigenfunctions for Deformation Invariant Shape Representation”
Rustamov, SGP 2007
Can Apply to Features

Figure 1: The k-means clustering on the GPS coordinates results in a pose invariant segmentation.

“Laplace-Beltrami Eigenfunctions for Deformation Invariant Shape Representation.”
Rustamov; SGP 2007
Geometry of $k$-Means

• Assignment step
  – Assign point to its closest cluster center

• Update step
  – Average all points in a cluster

Doesn’t have to be Euclidean
Geometry of $k$-Means

- Assignment step
  - Assign point to its closest cluster center

- Update step
  - Average all points in a cluster
Fréchet Mean

\[ \arg \min_{p \in M} \sum_{i=1}^{N} d(p, x_i)^2 \]

"Fréchet variance"

On the board:
Generalizes Euclidean notation of "mean."
Extension to Regions on a Surface

Alternate between
1. Fitting primitive parameters
2. Assign points to patches

“Variational Shape Approximation.”
Cohen-Steiner, Alliez, and Desbrun; SIGGRAPH 2004
**k-Medioids**

- **Assignment step**
  - Assign point to its **closest** cluster center

- **Update step**
  - Replace cluster center with most **central** data point

*When Fréchet means won’t work*
Related Technique

Region Growing Algorithm
Initialize a priority queue $Q$ of elements
Loop until all elements are clustered
   Choose a seed element and insert to $Q$
   Create a cluster $C$ from seed
Loop until $Q$ is empty
   Get the next element $s$ from $Q$
   If $s$ can be clustered into $C$
      Cluster $s$ into $C$
      Insert $s$ neighbors to $Q$
   Merge small clusters into neighboring ones

“Segmentation and Shape Extraction of 3D Boundary Meshes.”
Shamir; EG STAR 2006.

Region growing algorithm
Example Task

Clustering shapes?

https://ps.is.tuebingen.mpg.de/research_projects/3d-mesh-registration
Gromov-Hausdorff Distance

Distance between metric spaces \( X, Y \)

\[
d_{\text{GH}}(X, Y) := \inf_{\phi: X \rightarrow Y} \sup_{x, x' \in X} |d_X(x, x') - d_Y(\phi(x), \phi(x'))|
\]
On the use of Gromov-Hausdorff Distances for Shape Comparison

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Abstract
It is the purpose of this paper to propose and discuss certain modifications of the ideas concerning Hausdorff distances in order to tackle the problems of shape matching and comparison. These render these distances more amenable to practical computations without sacrificing theoretical advantages. A second goal of this paper is to establish links to several other practical methods proposed in the literature for comparing/matching shapes in precise terms. Connections with the Quadratic Assignment Problem are also established, and computational examples are presented.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modelling.

1. Introduction
Given the great advances in recent years in the fields of shape acquisition and modelling, and the resulting huge collections of digital models that have been obtained, it is of great importance to be able to define and compute meaningful notions of similarity between shapes which exhibit invariance to different deformations and/or poses of the objects represented.
Agglomerative Clustering

Merge from the bottom up

https://upload.wikimedia.org/wikipedia/commons/a/ad/Hierarchical_clustering_simple_diagram.svg

“Dendrogram”

https://upload.wikimedia.org/wikipedia/commons/a/ad/Hierarchical_clustering_simple_diagram.svg

Merge from the bottom up
Agglomerative Clustering in Geometry

“Hierarchical mesh segmentation based on fitting primitives.”
Attene, Falcidieno, and Spagnuolo; The Visual Computer 2006

Fit a primitive and measure
Typical Features

Figure 4: Example of mesh attributes used for partitioning. Left: minimum curvature, middle: average geodesic distance, right: shape diameter function.

Additional Desirable Properties

- Cardinality
  - Not too small and not too large or a given number (of segment or elements)
  - Overall balanced partition
- Geometry
  - Size: area, diameter, radius
  - Convexity, Roundness
  - Boundary smoothness
- Topology
  - Connectivity (single component)
  - Disk topology
  - a given number (of segment or elements)

“Segmentation and Shape Extraction of 3D Boundary Meshes.”
Shamir; EG STAR 2006.
via Q. Huang, Stanford CS 468, 2012
Issue So Far

No notion of optimality.

No use of local relationships.
Spectral Clustering


- Rough notion of optimality
- Assembles local relationships
Normalized Cuts for Two Cuts
Normalized Cuts for Two Cuts

Symmetric similarity matrix $W$

Cut score $C(A, B) := \sum_{i \in A} \sum_{j \in B} w_{ij}$

Volume $V(A) := \sum_{i \in A} \sum_{j} w_{ij}$

Normalized cut score

$N(A, B) := C(A, B)(V(A)^{-1} + V(B)^{-1})$
Normalized Cuts

\[ x_i \ := \ \begin{cases} 
V(A)^{-1} & \text{if } i \in A \\
-V(B)^{-1} & \text{if } i \in B 
\end{cases} \]

On the board:

\[ x^\top L x = \sum_{\substack{i \in A \\
j \in B}} w_{ij}(V(A)^{-1} + V(B)^{-1})^2 \]

\[ x^\top D x = V(A)^{-1} + V(B)^{-1} \]

\[ N(A, B) = \frac{x^\top L x}{x^\top D x} \]

\[ x^\top D 1 = 0 \]
Eigenvalue Problem

On the board:
- Relaxation of normalized cuts
- Eigenvalue problem

\[
\begin{align*}
\min_x & \quad \frac{x^T L x}{x^T D x} \\
\text{s.t.} & \quad x^T D 1 = 0
\end{align*}
\]
Example on kNN Graph

For $\geq 2$ Clusters

- **Recursive bi-partitioning** (Hagen et al. 1991)
  - Analogy: Agglomerative clustering
  - Potentially slow/unstable

- **Cluster multiple eigenvectors**
  - Analogy: $k$-means after dimension reduction
  - More popular approach

Recall:
Second-Smallest Eigenvector

\[ Lx = \lambda x \]

Fiedler vector ("algebraic connectivity")

Used for graph partitioning
Back to the Laplacian
Nodal domain
A connected region where a Laplacian eigenfunction has constant sign
Courant’s Theorem

The \( k \)-th Laplacian eigenfunction has at most \( k \) nodal domains.
Issue

Image courtesy Q. Huang

Inconsistent!
What’s meaningful?

• Semantically?
• Flat/Developable?
  • Convex?
• Voronoi-like?

http://www.cs.rug.nl/svcg/Shapes/PDE
Obvious Counterexample

http://www.erflow.eu/brain-segmentation-science-case

Shape provides only a clue
Supervised Learning

“Learning 3D Mesh Segmentation and Labeling.”
Kalogerakis, Hertzmann, and Singh; SIGGRAPH 2010

Use example data to help
Conditional Random Field

\[ c^* := \arg \min_c \left[ \sum_i \alpha_i E_1(c_i; x_i) + \sum_{ij} \ell_{ij} E_2(c_i, c_j; y_{ij}) \right] \]

Unary descriptor term

Binary label compatibility term
Before Someone Asks

3D Shape Segmentation with Projective Convolutional Networks

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Abstract

This paper introduces a deep architecture for segmenting 3D objects into their labeled semantic parts. Our architecture combines image-based Fully Convolutional Networks (FCNs) and surface-based Conditional Random Fields (CRFs) to yield coherent segmentations of 3D shapes. The image-based FCNs are used for efficient view-based reasoning about 3D object parts. Through a special projection layer, FCN outputs are effectively aggregated across multiple views and scales, then are projected onto the 3D object surfaces. Finally, a surface-based CRF combines the projected outputs with geometric consistency cues to yield coherent segmentations. The whole architecture (multi-view FCNs and CRF) is trained end-to-end. Our approach significantly outperforms the existing state-of-the-art methods in the currently largest segmentation benchmark (ShapeNet). Finally, we demonstrate promising segmentation results on noisy 3D shapes acquired from consumer-grade depth cameras.

1. Introduction

In recent years there has been an explosion of 3D shape data on the web. In addition to the increasing number of community-curated CAD models, depth sensors deployed on a wide range of platforms are able to acquire 3D geometric representations of objects in the form of polygonal meshes or point clouds. From the computer vision community, the availability of large annotated datasets (e.g., ShapeNet\textsuperscript{\textregistered}) has enabled state-of-the-art learning algorithms to be trained on large amounts of data. However, 3D object segmentation remains a challenging task due to the presence of ambiguities in the ground truth annotation and variations in the appearance of the shapes. This paper introduces a deep learning architecture that is able to segment 3D objects into their labeled semantic parts. The architecture is based on a multi-view representation of the 3D object and combines image-based reasoning with surface-based reasoning to yield coherent segmentations. Our approach is able to outperform the state-of-the-art on a number of publicly available datasets and demonstrates the potential of deep learning for 3D object segmentation.

The shape segmentation task, while fundamental, is challenging because of the variety and ambiguity of shape parts that must be assigned the same semantic label; because accurately detecting boundaries between parts can involve extremely subtle cues; because local and global features must be jointly examined; and because the analysis must be robust to noise and undersampling.

We propose a deep architecture for segmenting and labeling 3D shapes that simply and effectively addresses these challenges, and significantly outperforms prior methods. The key insights of our technique are to repurpose image-based deep networks for view-based reasoning, and aggregate their outputs onto the surface representation of the shape in a geometrically consistent manner. We make no geometric or topological assumptions about the shape, nor exploit any hand-tuned geometric descriptors.

Our view-based approach is motivated by the success of deep networks on image segmentation tasks. Using rendered shapes lets us initialize our network with layers that have been trained on large image datasets, allowing better generalization. Since images depict shapes of photographed objects (along with texture), we expect such pre-trained layers to already encode some information about parts and their relationships. Recent work on view-based 3D shape classification [43, 35] and RGB-D recognition [13, 42] have shown the benefits of transferring learned representations.