

IFT 6113

MESH DEFORMATION AND SKINNING

tiny.cc/ift6113

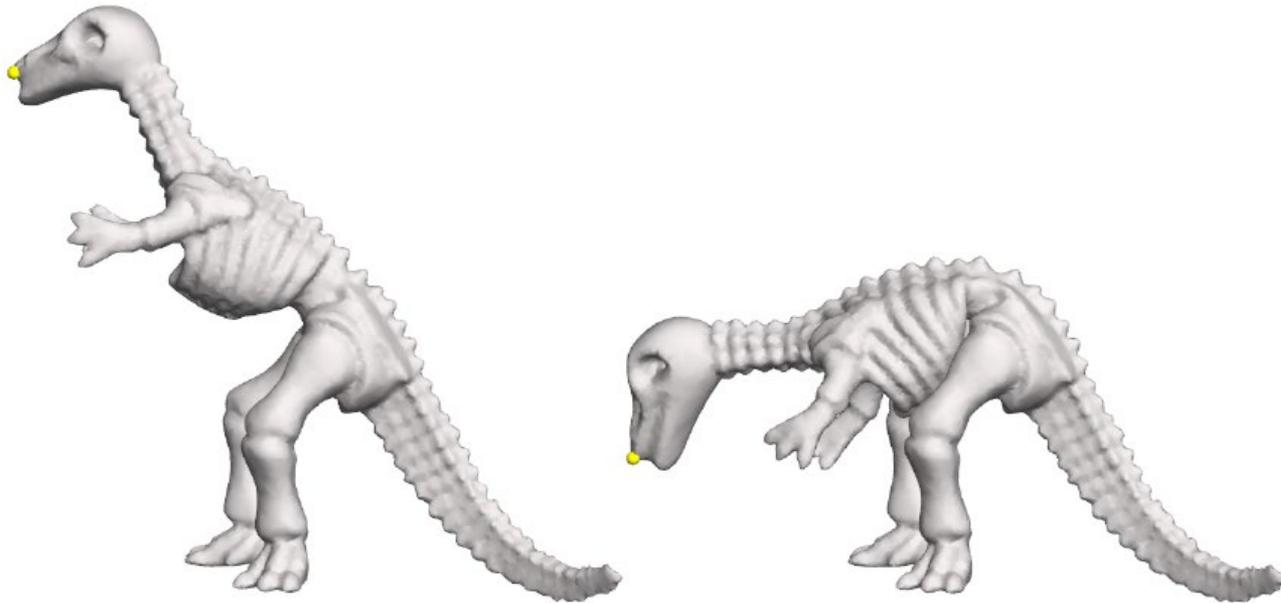


Image from 'As-Rigid-As-Possible Shape Modeling' by Sorkine & Alexa, 2007

Mikhail Bessmeltsev

What for?

- Animation!
- Mesh editing
- Image warping (2D)



This, and many other images in this presentation are from 'Polygon Mesh Processing' textbook by Botsch et al. or their website

Warning:

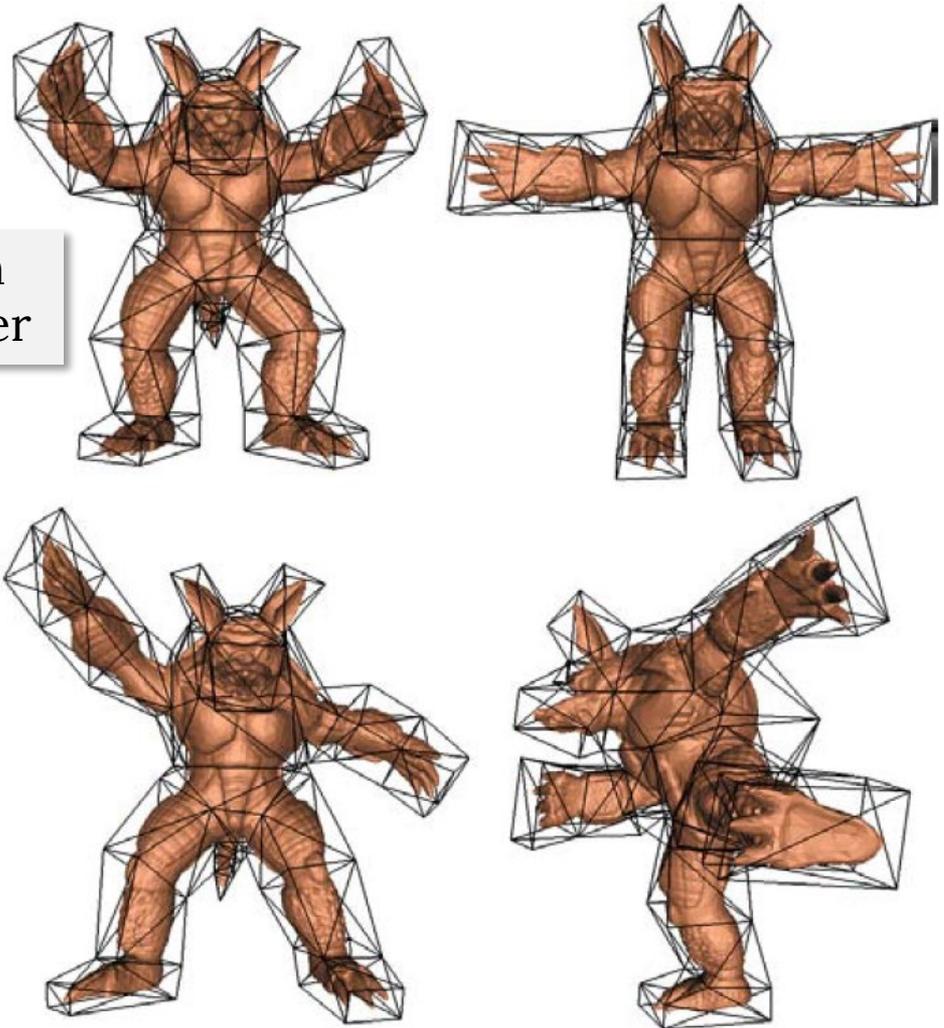
TMI

This topic is immense
We'll only see a few samples

Deformation: user interface

- Handles
- Cages
- Skeletons
- ...

More on
those later



Deformation models

Direct

$$v' = \left(\sum w_j T_j \right) v$$

- Linear Blend Skinning
- Dual Quaternion Skinning
- ...

Variational

$$v' = \operatorname{argmin}_x E(x)$$

- Multiresolution editing
- As-Rigid-As-Possible
- Laplacian Mesh Editing
- ...

Deformation models

Direct

$$v' = \left(\sum w_j T_j \right) v$$

- Linear Blend Skinning
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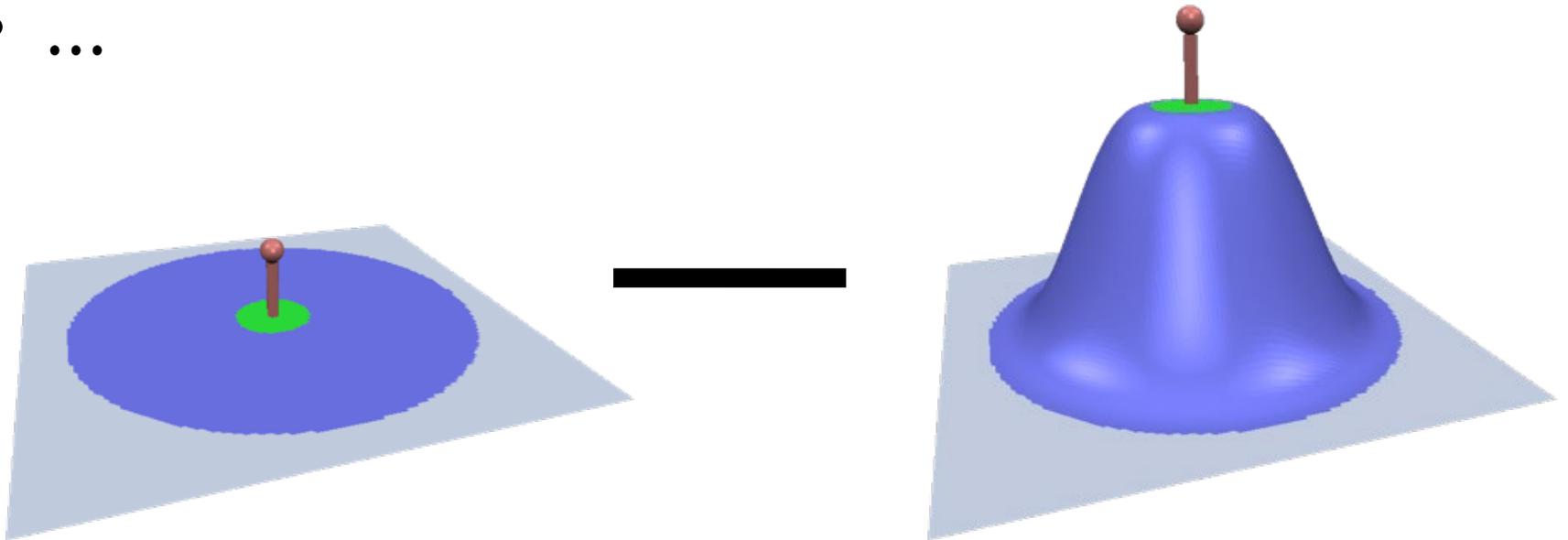
Variational

$$v' = \operatorname{argmin}_x E(x)$$

- Multiresolution editing
- As-Rigid-As-Possible
- Laplacian Mesh Editing
- ...

Deformation: user interface

- **Handles**
- Cages
- Skeletons
- ...



Modeling

Paint three surface areas:

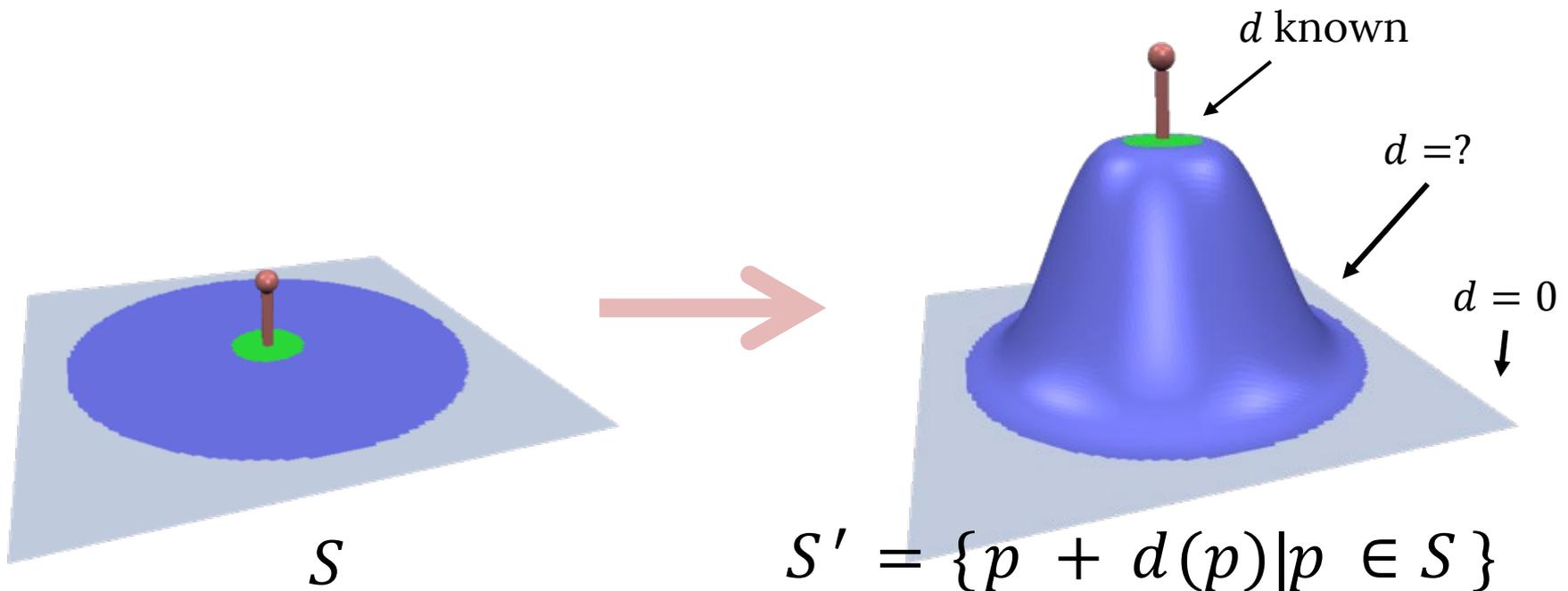
- Constrained
- Smooth falloff
- Fixed



Formulation

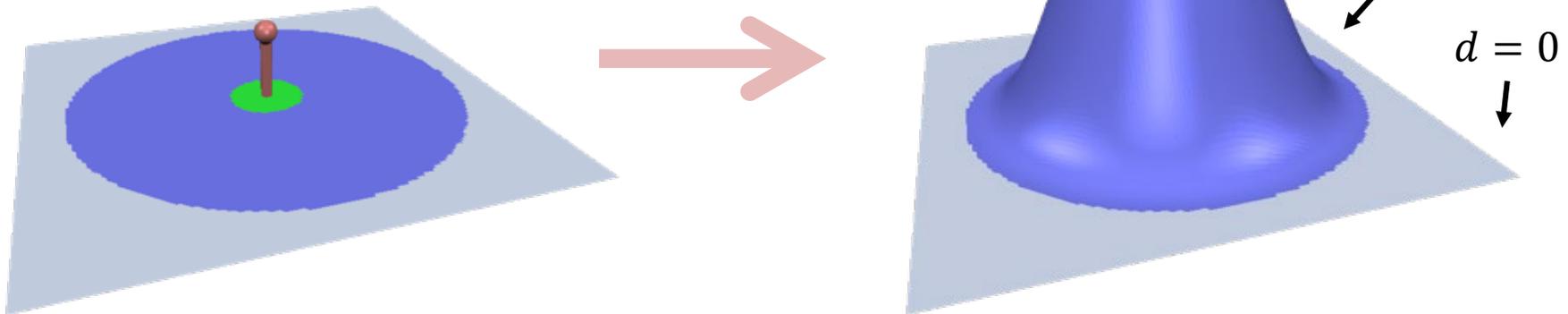
Find displacement vector field d

- Smooth
- Satisfies constraints



Simplest idea

- $d = s(p) \cdot D$
- $s(p)$ is a smooth function:
 - 1 on green vertices
 - 0 on grey ones



How to find $s(p)$?

- Something inversely proportional to geodesic distance
- Or our favorite:

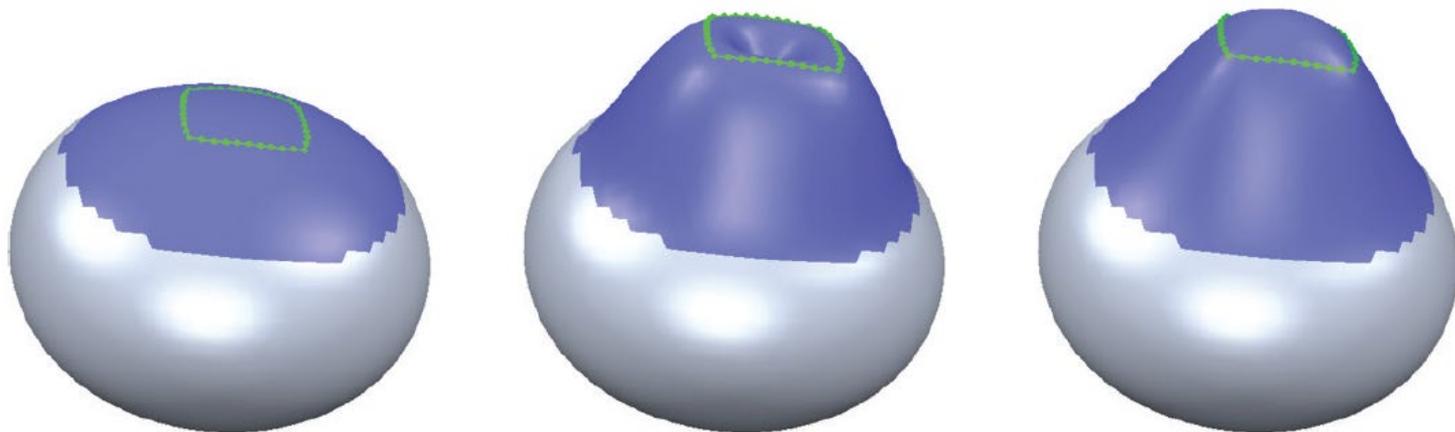
$$\Delta s(\mathbf{p}_i) = 0, \quad \mathbf{p}_i \in \mathcal{R},$$

$$s(\mathbf{p}_i) = 1, \quad \mathbf{p}_i \in \mathcal{H},$$

$$s(\mathbf{p}_i) = 0, \quad \mathbf{p}_i \in \mathcal{F}.$$

Solved?

Reality vs Expectation



Physically-Based

Find a deformation that preserves both fundamental forms

Express the fundamental forms of S' via vector field d

$$\int_{\Omega} k_s \left\| \mathbf{I} - \mathbf{I}' \right\|_F^2 + k_b \left\| \mathbf{II} - \mathbf{II}' \right\|_F^2 \, dudv$$

Expensive to optimize!

stretching bending

Physically-Based

$$\int_{\Omega} k_s \left(\|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left(\|\mathbf{d}_{uu}\|^2 + 2 \|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) dudv$$

stretching bending

Gateaux derivative =>

$$-k_s \Delta d + k_b \Delta^2 d = 0$$

Physically-Based

$$\int_{\Omega} k_s \left(\|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left(\|\mathbf{d}_{uu}\|^2 + 2 \|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) dudv$$

stretching bending

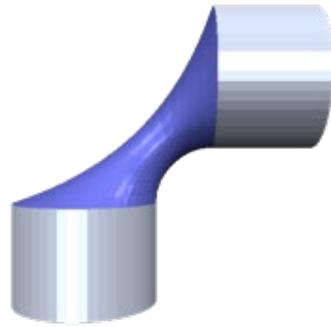
Gateaux derivative =>

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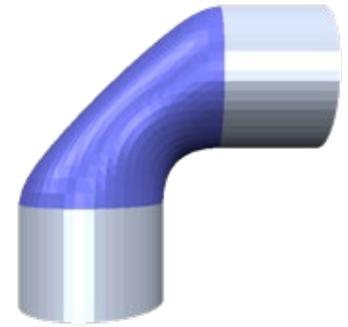
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Bi-Laplacian

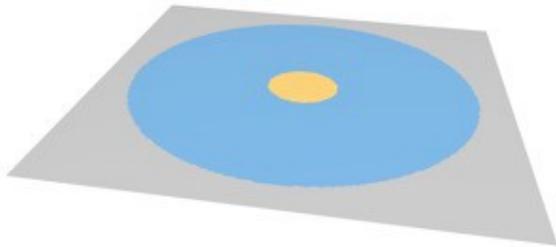
Deformation Energies



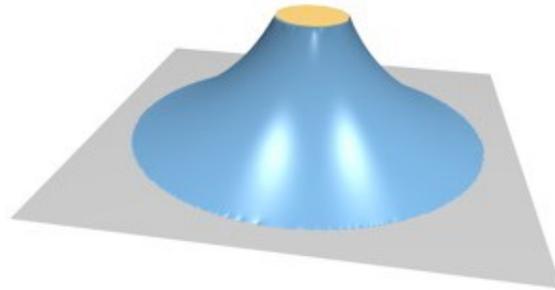
$$\Delta p = 0$$



$$\Delta^2 p = 0$$

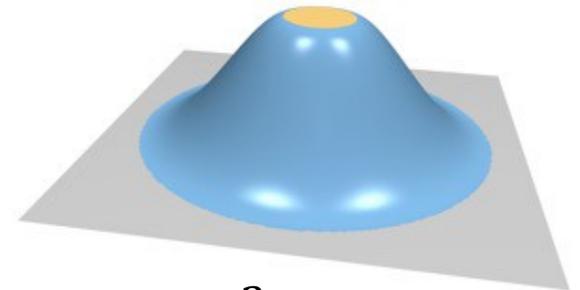


Initial state



$$\Delta d = 0$$

(Membrane)

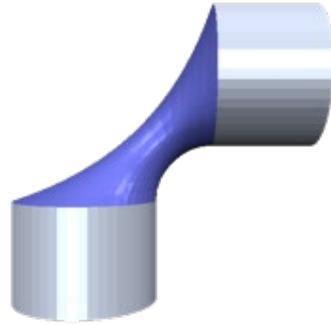


$$\Delta^2 d = 0$$

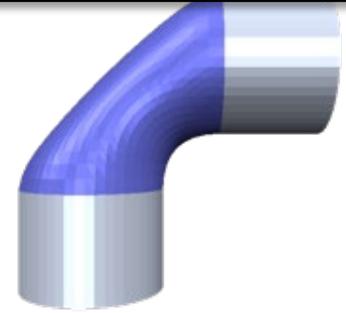
(Bilaplacian)

Deformation Energy

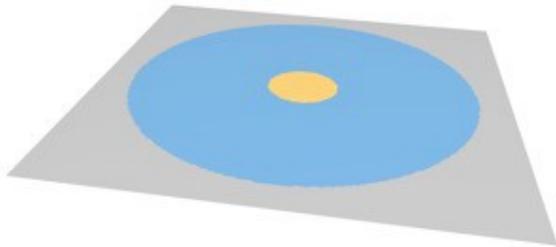
Higher order =>
more boundary
conditions



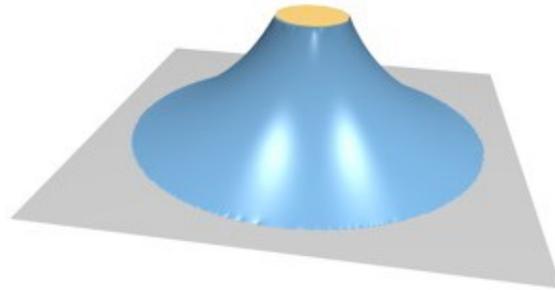
$$\Delta p = 0$$



$$\Delta^2 p = 0$$

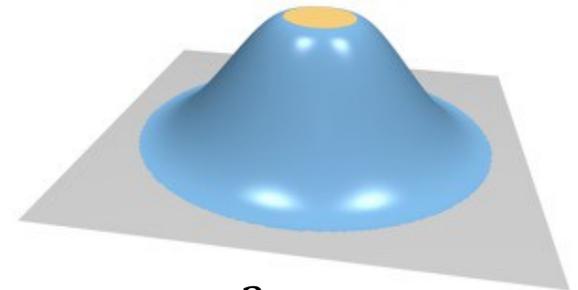


Initial state



$$\Delta d = 0$$

(Membrane)

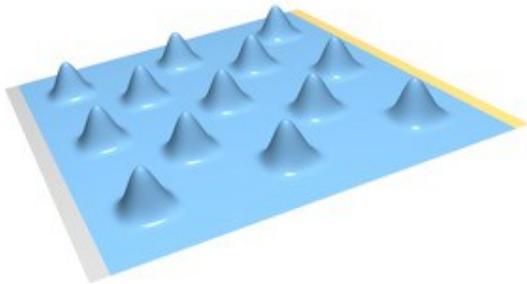


$$\Delta^2 d = 0$$

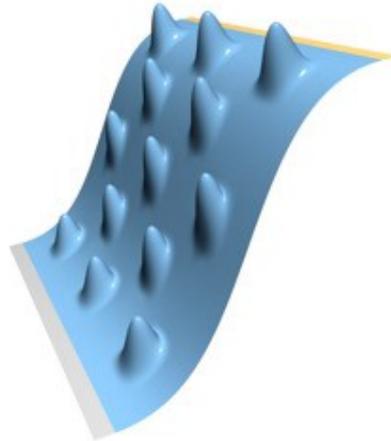
(Bilaplacian)

Solved?

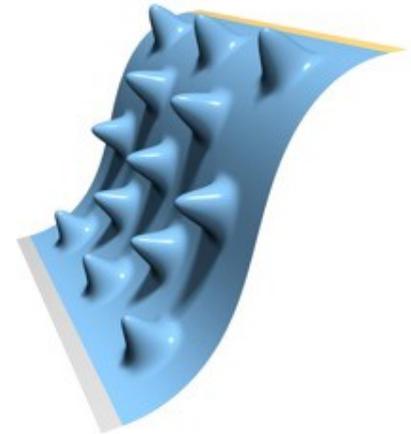
- Very fast
 - One linear solve!
- Physically-based
- Linearization => **lose details**



Original



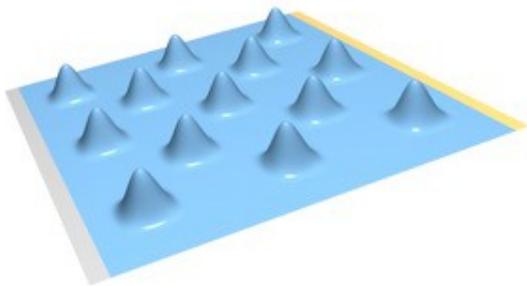
Linear
deformation



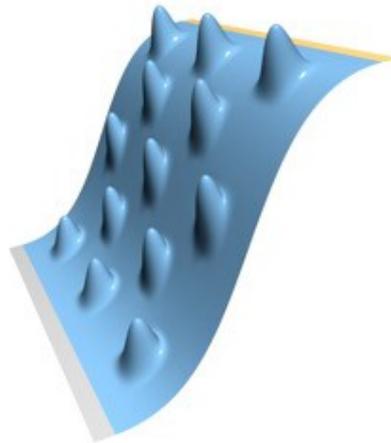
Non-linear
deformation

Issue

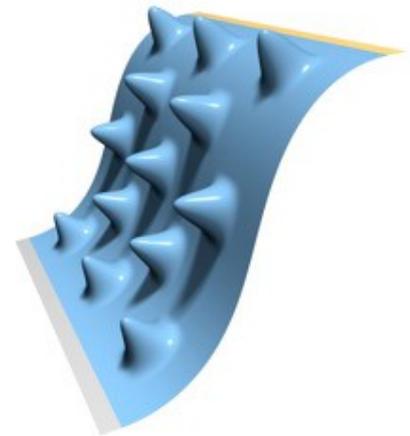
- We need to rotate details
 - Local rotation is nonlinear!
- Can we still survive with linear solves?



Original

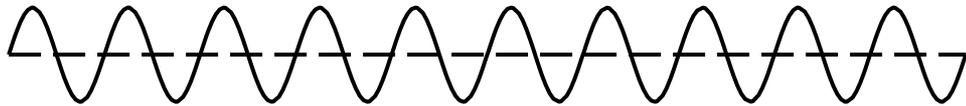


Linear
deformation



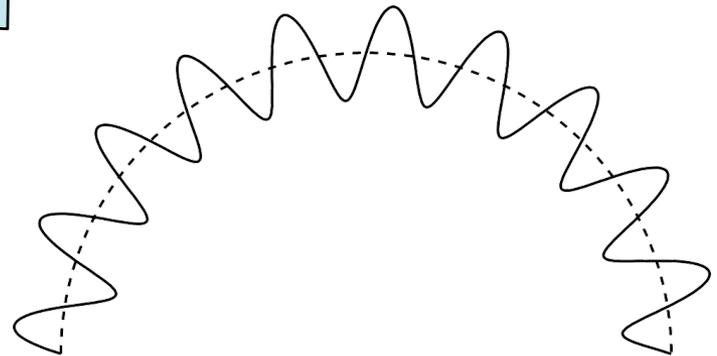
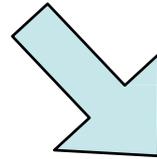
Non-linear
deformation

Multiresolution Editing



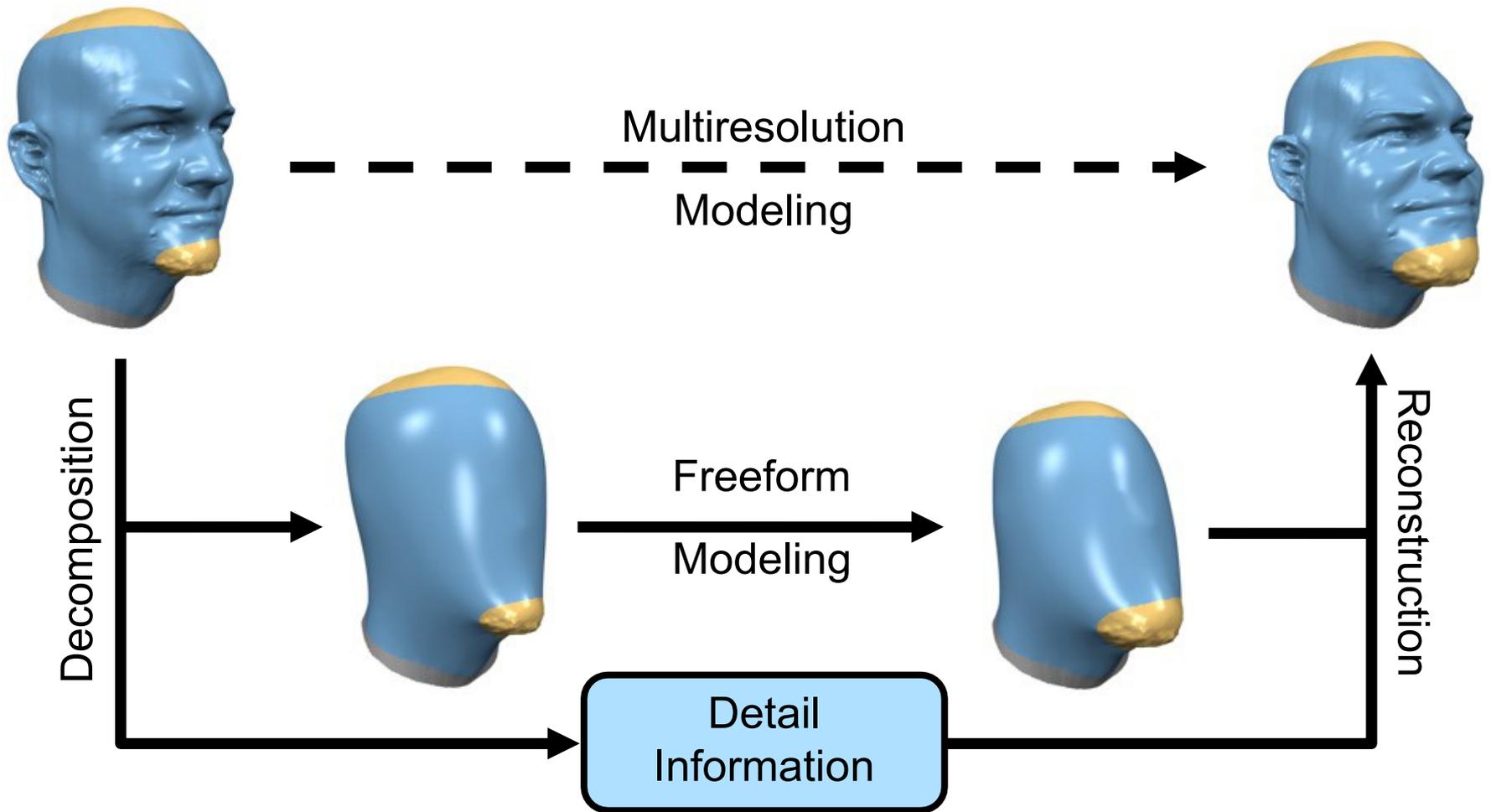
Frequency decomposition

Change low
frequencies



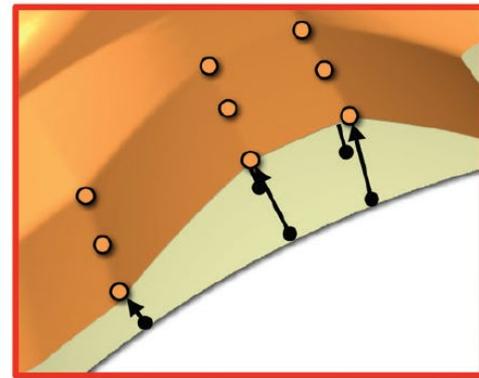
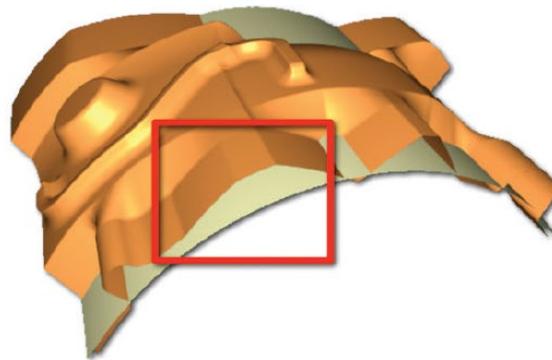
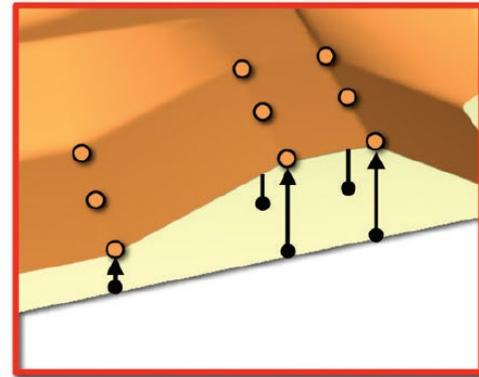
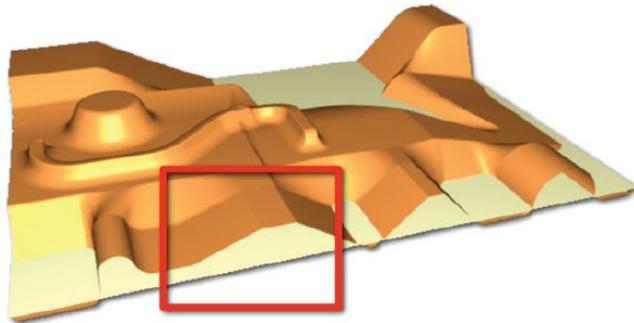
Add high frequency details,
stored in local frames

Multiresolution Editing

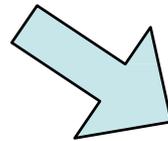
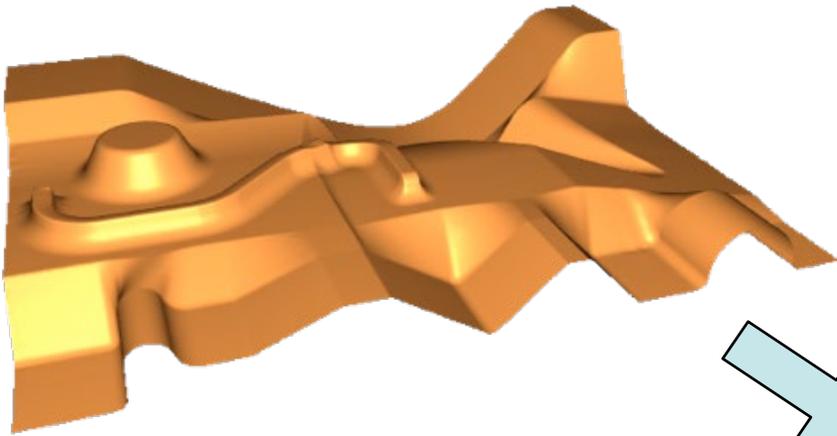


How to represent details?

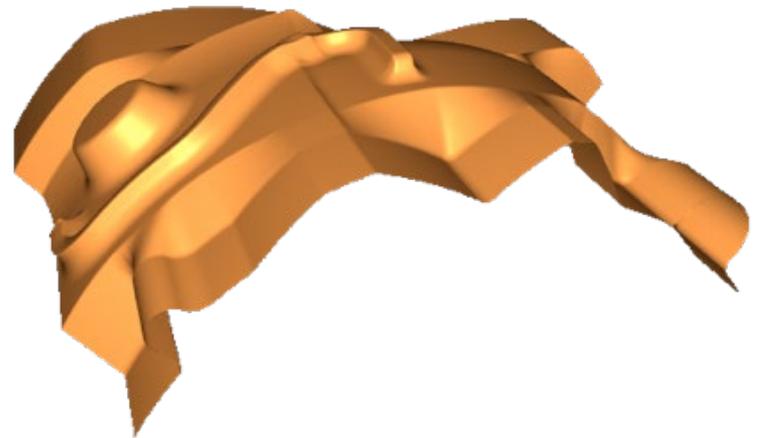
- For example, normal displacements



Result



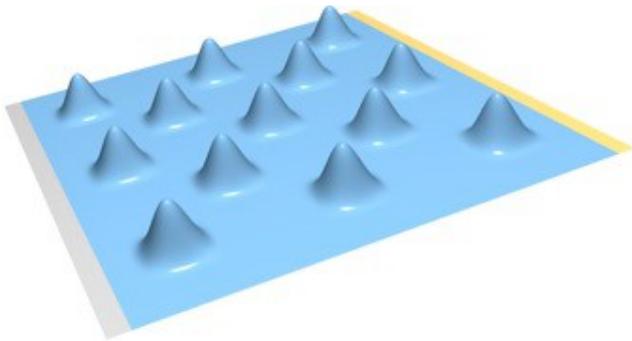
Global deformation
with intuitive detail
preservation



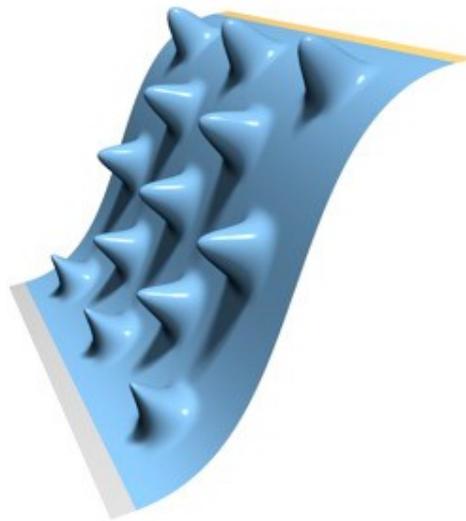
Limitations

Neighboring displacements are not coupled

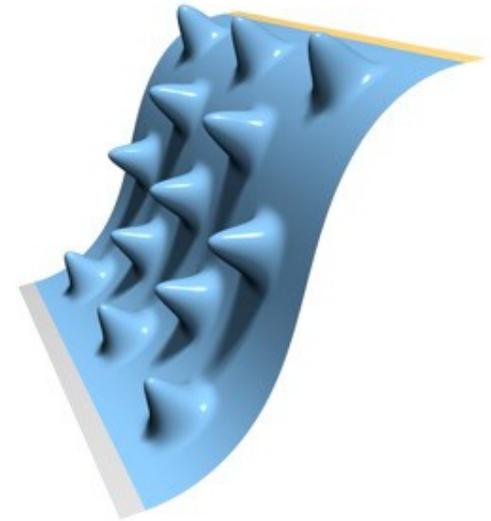
- Surface bending changes their angle
- Leads to volume changes or self-intersections



Original



Normal Displ.

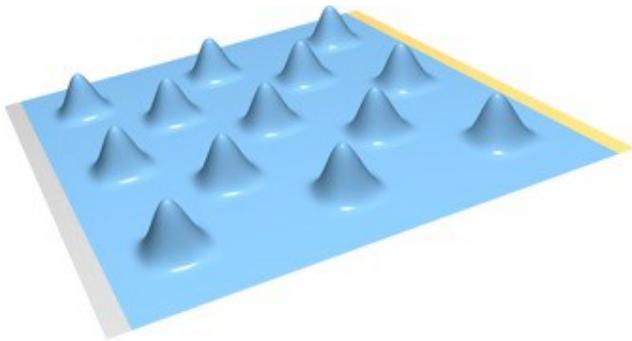


Nonlinear

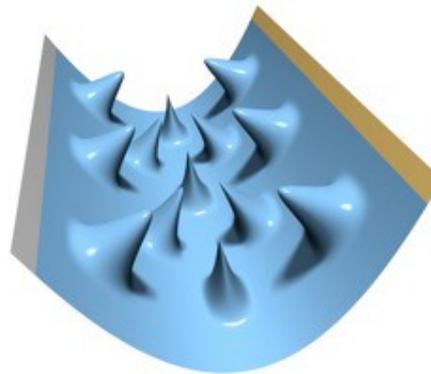
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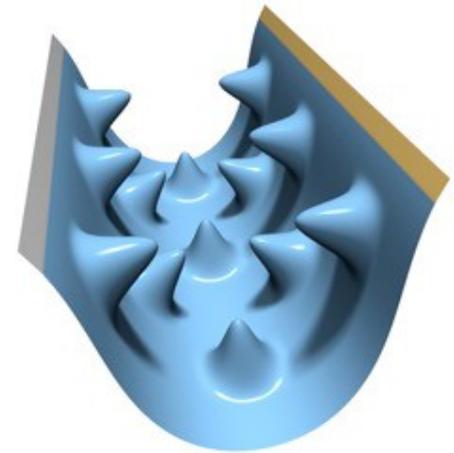
- Surface bending changes their angle
- Leads to volume changes or self-intersections



Original



Normal Displ.



Nonlinear

New coordinates?

Express shape in *differential coordinates*

Transform those,
then reconstruct the new shape

Long time ago:

Mean Value Property

$$L_{vw} = A - D = \begin{cases} 1 & \text{if } v \sim w \\ -\text{degree}(v) & \text{if } v = w \\ 0 & \text{otherwise} \end{cases}$$

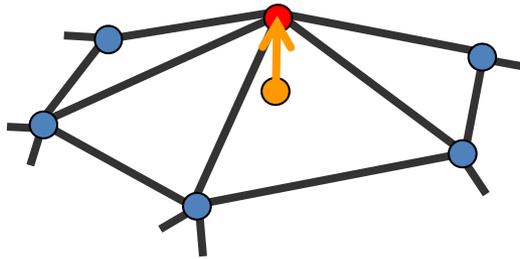
$$(Lx)_v = 0$$



Value at v is average of neighboring values

Laplacian Mesh Editing

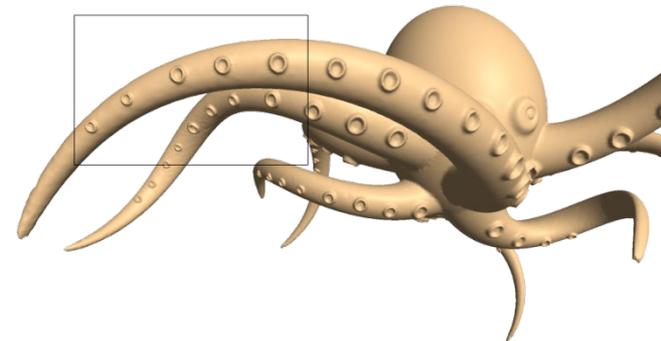
Graph Laplacian:



$$\delta_i = \mathbf{v}_i - \frac{1}{d_i} \sum_{j \in N(i)} \mathbf{v}_j$$

Local
coordinates!

$$\delta = L\mathbf{v}$$



Laplacian Mesh Editing

- Represent mesh using only δ
- Find a surface whose Laplacian coordinates are as close as possible to δ

$$\int_{\mathcal{S}} \|\Delta \mathbf{p}' - \delta'\|^2 d\mathcal{S} \rightarrow \min$$

$$\text{s.t. } p'_i = p_i, i \in \{\text{point constraints}\}$$

Laplacian Mesh Editing

Find a surface whose Laplacian coordinates are as close as possible to δ

$$\min \sum \|\delta_i - L(p'_i)\|^2 + \sum_{i \in c} \|p'_i - p_i\|^2$$

Laplacian Mesh Editing

Find a surface whose Laplacian coordinates are as close as possible to δ

$$\int_{\mathcal{S}} \|\Delta \mathbf{p}' - \delta'\|^2 d\mathcal{S} \rightarrow \min$$

$$\text{s.t. } p'_i = p_i, i \in \{\text{point constraints}\}$$

$$\text{Gateaux derivative} \Rightarrow \Delta^2 \mathbf{p}' = \Delta \delta'$$

Before:

Physically-Based

$$\int_{\Omega} k_s \left(\|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left(\|\mathbf{d}_{uu}\|^2 + 2 \|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) dudv$$

stretching bending

Gateaux derivative =>

$$-k_s \Delta d + k_b \Delta^2 d = 0$$

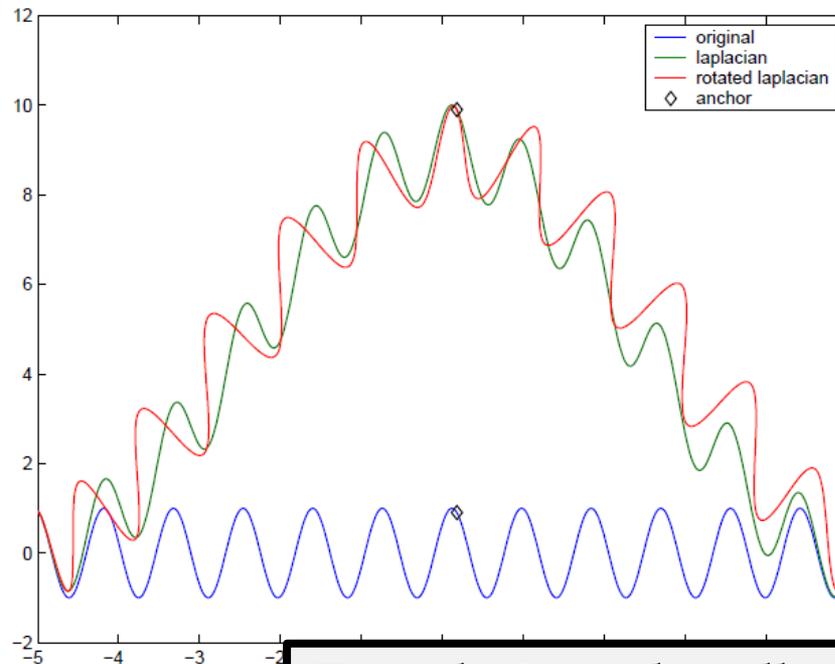
(almost) the same
equation?

Bi-Laplacian

Issue

Reconstructing from differential coordinates
makes sense only if they are
rotation and translation invariant

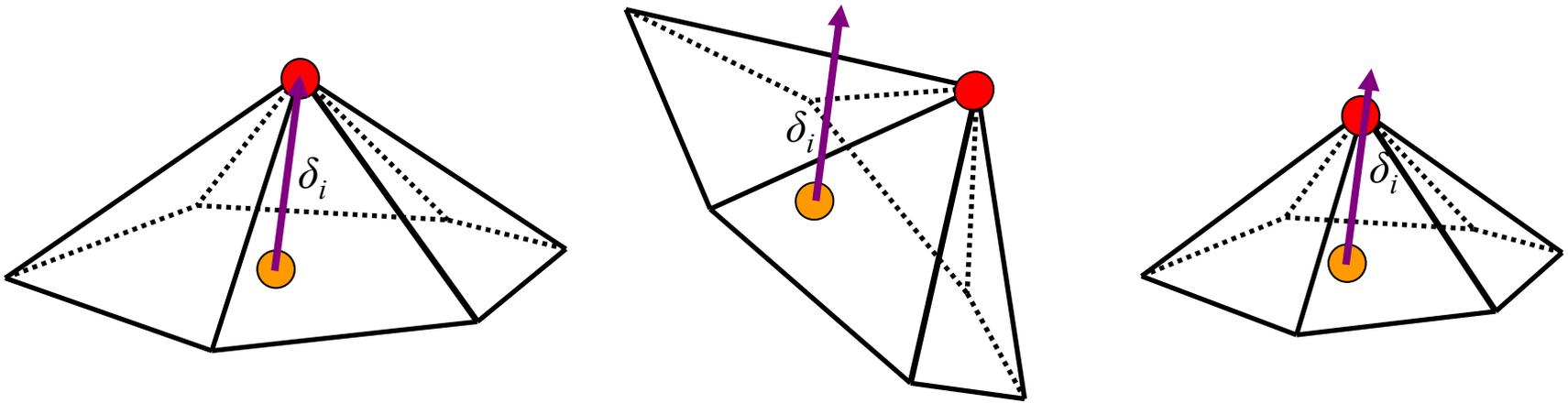
Otherwise, you get this



Translating a handle induces local rotations!

Laplacian Coordinates

- Translation invariant
- Not rotation/scale invariant



$$\delta_i = L(\mathbf{v}_i) = L(\mathbf{v}_i + \mathbf{t}); \forall \mathbf{t} \in \mathbb{R}^3$$

Solutions

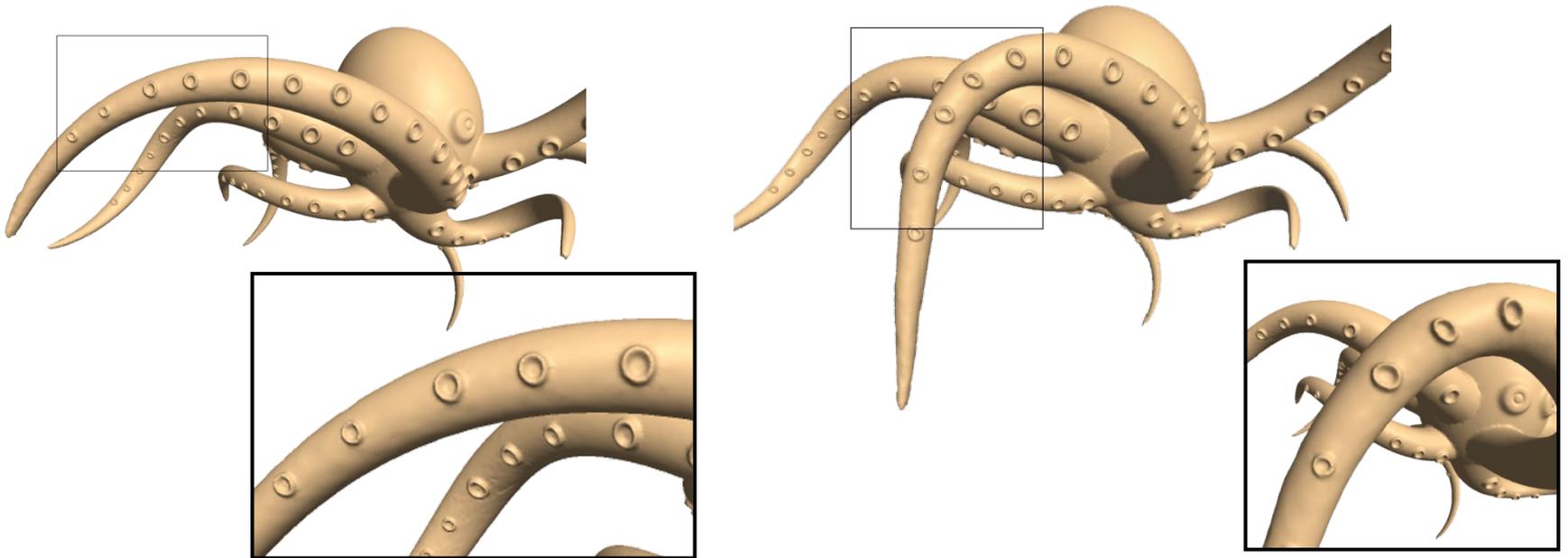
1. Transform, ignoring rotations or details
2. **while** (not converged)
 - Estimate rotations (*from normals*)
 - Rotate differential coordinates and solve

$$E(\mathbf{V}') = \sum_{i=1}^n \|R_i \delta_i - L(p'_i)\|^2 + \sum_{i \in C} \|p'_i - p_i\|^2$$

[Y. Lipman, O. Sorkine, D. Cohen-Or, D. Levin, C. Rossi and H. P. Seidel, "Differential coordinates for interactive mesh editing," *Proceedings Shape Modeling Applications*, 2004]

Solutions

1. Transform, ignoring rotations or details
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[Y. Lipman, O. Sorkine, D. Cohen-Or, D. Levin, C. Rossi and H. P. Seidel, "Differential coordinates for interactive mesh editing," *Proceedings Shape Modeling Applications*, 2004]

Rotations + **scaling** – invariant?

Add local transformations T_i as variables

$$E(\mathbf{V}') = \sum_{i=1}^n \|T_i \delta_i - L(p'_i)\|^2 + \sum_{i \in C} \|p'_i - p_i\|^2$$

[O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rossl, H.-P. Seidel, Laplacian Surface Editing, EUROGRAPHICS/Symposium on Geometry Processing, 2004]

Rotations + **scaling** – invariant?

Add local transformations T_i as variables

$$E(\mathbf{V}') = \sum_{i=1}^n \|T_i \delta_i - L(p'_i)\|^2 + \sum_{i \in \mathcal{C}} \|p'_i - p_i\|^2$$

$$\min_{T_i} \left(\|T_i \mathbf{v}_i - \mathbf{v}'_i\|^2 + \sum_{j \in \mathcal{N}_i} \|T_i \mathbf{v}_j - \mathbf{v}'_j\|^2 \right).$$

[O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Ross, H.-P. Seidel, Laplacian Surface Editing, EUROGRAPHICS/Symposium on Geometry Processing, 2004]

Rotations + **scaling** – invariant?

Add local transformations T_i as variables

$$\min_{T_i} \left(\|T_i \mathbf{v}_i - \mathbf{v}'_i\|^2 + \sum_{j \in \mathcal{N}_i} \|T_i \mathbf{v}_j - \mathbf{v}'_j\|^2 \right).$$

T_i = translation + rotation + scaling

Represent (a linearization of) T_i using translation/rotation/scaling parameters

Rotations + **scaling** – invariant?

Add local transformations T_i as variables

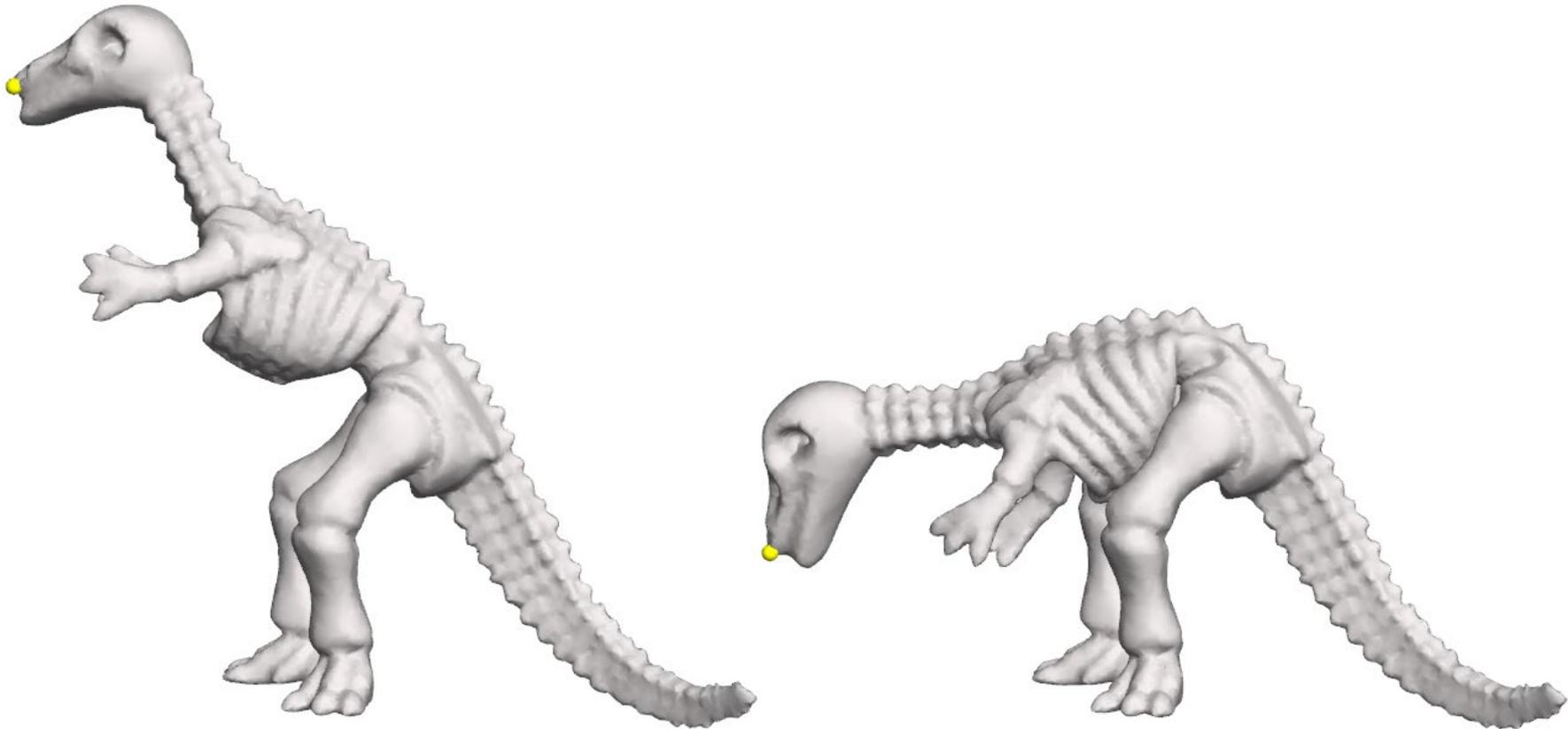
$$E(\mathbf{V}') = \sum_{i=1}^n \|T_i \delta_i - L(p'_i)\|^2 + \sum_{i \in C} \|p'_i - p_i\|^2$$

$\Rightarrow T_i$ is a linear function of V'
 \Rightarrow Quadratic optimization
 \Rightarrow Linear solve!

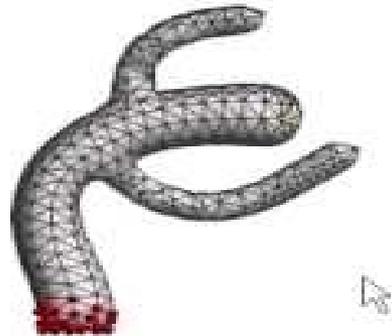


WHY?!?!

As-Rigid-As-Possible Surface Modelling

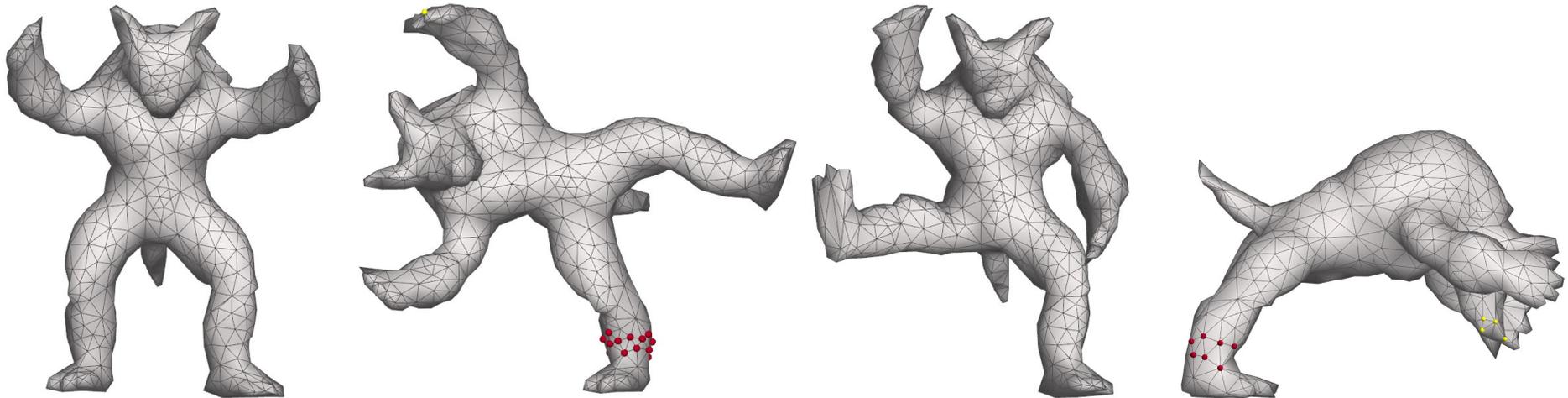


As-rigid-as-possible (ARAP)



As-rigid-as-possible (ARAP)

- “Intuitive” deformations
 - Smooth deformations at large scale
- Preserve local features
- Fast, for interactive mesh editing



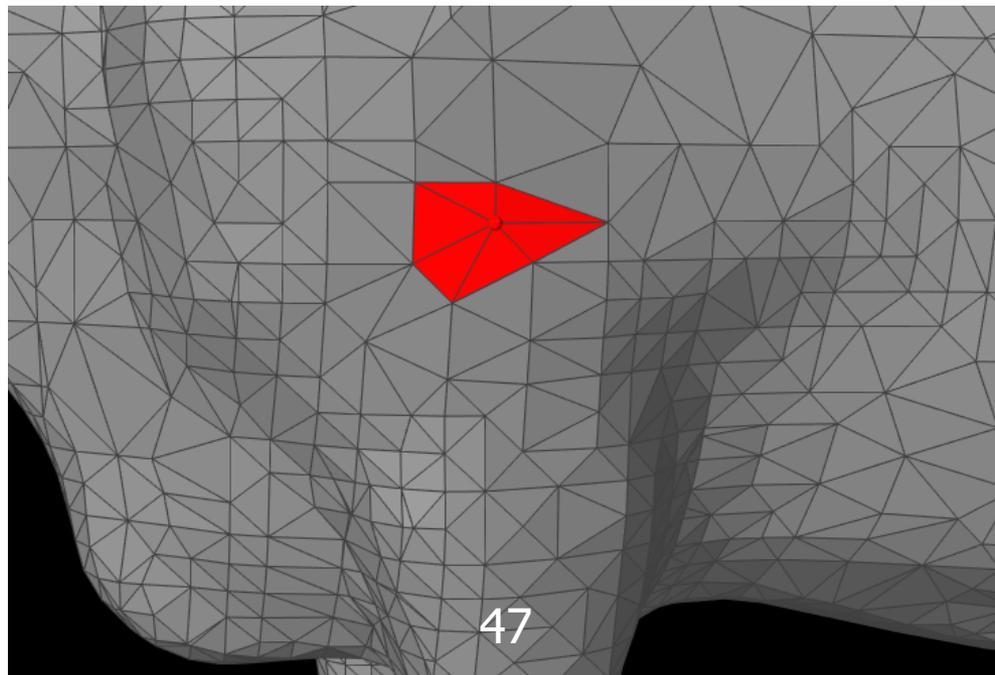
ARAP in a nutshell...

1. Break mesh into overlapping pieces
2. Try to move each piece rigidly
3. Combine all local transformations into a smooth one

Pieces

Vertex Umbrella

- Covers entire surface
- One cell per vertex
- All triangles exist in 3 cells

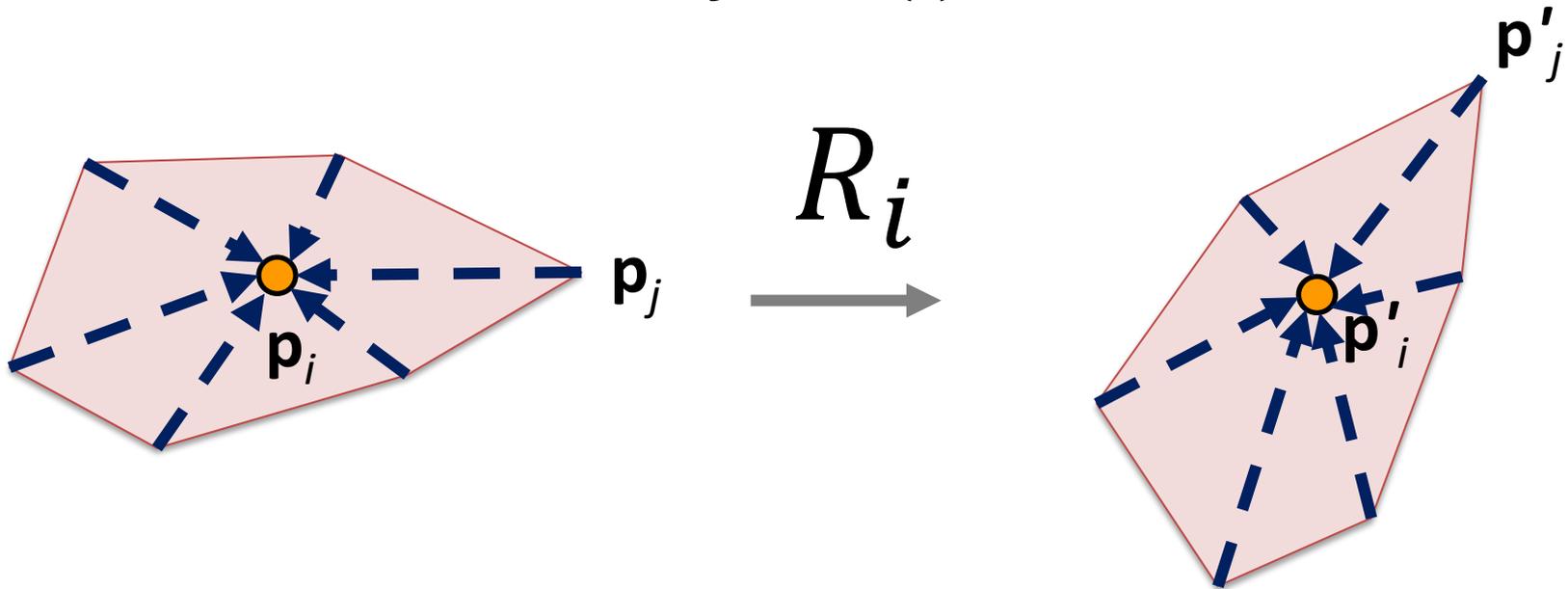


Rigid motion

If cell i moved rigidly:

$$p'_j - p'_i = R_i(p_j - p_i)$$

$$\forall j \in N(i)$$



Deviation from rigid motion

If cell i moved rigidly:

$$p'_j - p'_i = R_i(p_j - p_i)$$

$$\forall j \in N(i)$$

$$E = \sum_{j \in N(i)} \|p'_j - p'_i - R_i(p_j - p_i)\|^2$$

For the whole mesh

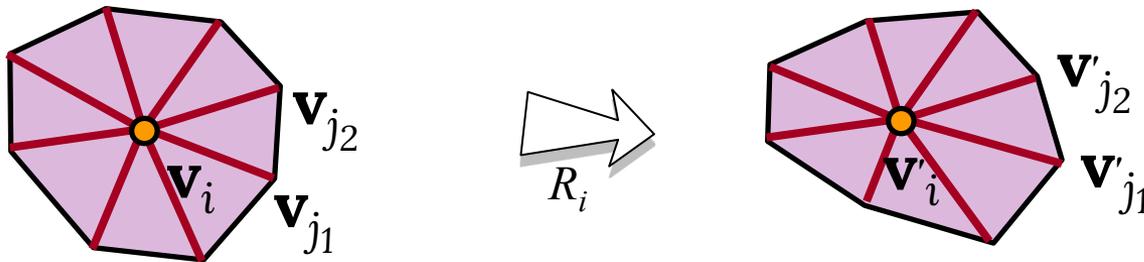
$$E = \sum_i \sum_{j \in N(i)} \|p'_j - p'_i - R_i(p_j - p_i)\|^2$$

For the whole mesh

$$E = \sum_i \sum_{j \in N(i)} w_{ij} \|p'_j - p'_i - R_i(p_j - p_i)\|^2$$

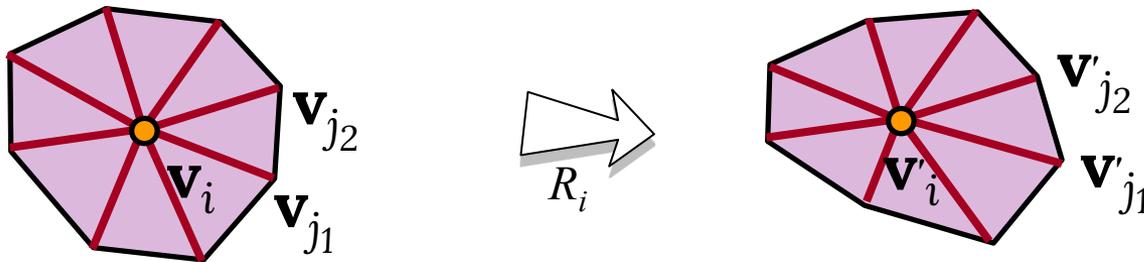
Orthogonal Procrustes problem

How to find the best rotation matrix aligning V with V' ?



Orthogonal Procrustes problem

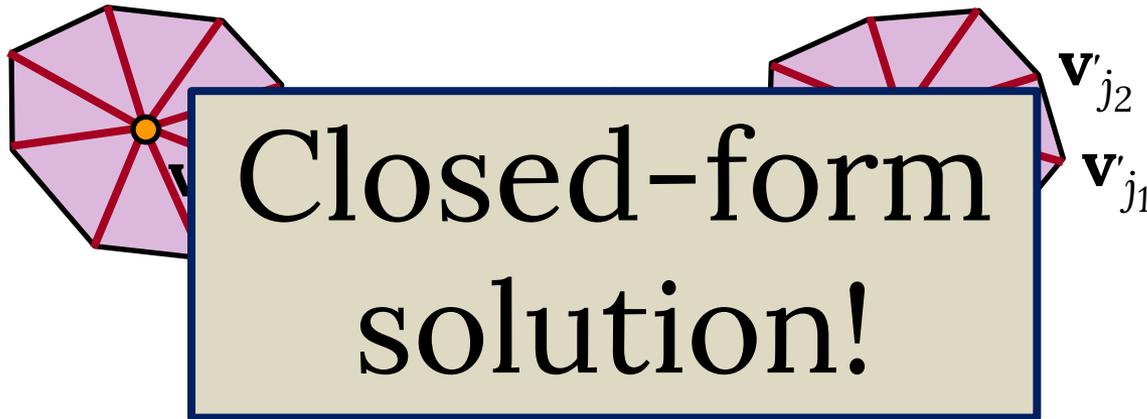
How to find the best rotation matrix aligning V with V' ?



$$\operatorname{argmin}_R \|RA - B\|_F$$
$$\text{s. t. } R^T R = I$$

???

Procrustes problem



1. Build covariance matrix $S = VV^T$
2. SVD: $S = U\Sigma W^T$
3. $R_i = UW^T$

Mesh Deformation

$$\min \sum_i \sum_{j \in N(i)} w_{ij} \|p'_j - p'_i - R_i(p_j - p_i)\|^2$$
$$\text{s.t. } p'_i = \tilde{p}_i$$

 point constraints

Caveats:

- $\{p'_i\}$ and $\{R_i\}$ are unknown
- Non-linear optimization problem

Mesh Deformation

1. Start with initial guess of $\{\mathbf{p}'_i\}$, find $\{R_i\}$
2. Given $\{R_i\}$, minimize energy to find $\{\mathbf{p}'_i\}$
3. Repeat

$$\sum_{j \in N(i)} w_{ij} (\mathbf{p}'_i - \mathbf{p}'_j) = \sum_{j \in N(i)} \frac{w_{ij}}{2} (R_i + R_j) (\mathbf{p}_i - \mathbf{p}_j)$$

$$L\mathbf{p}' = \mathbf{b}$$

Advantages

Laplacian

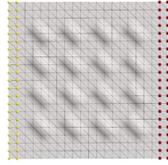
- Depends only on original mesh
- Only needs to be factored once!

Rotations can be computed in parallel

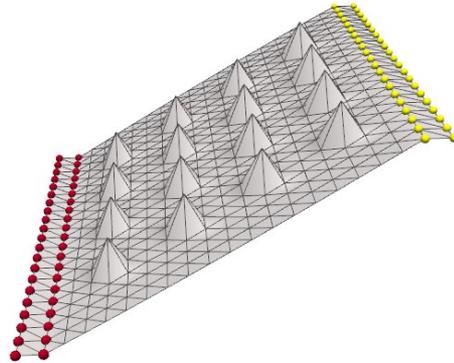
- Each iteration reduces energy
 - Updating rotations guaranteed to reduce cell-error
 - Updating positions guaranteed to reduce global error

Guaranteed Convergence

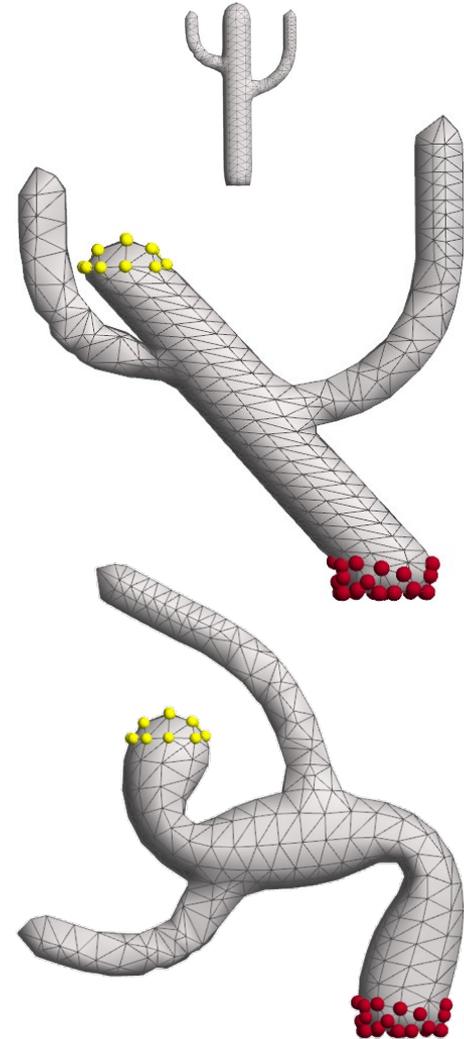
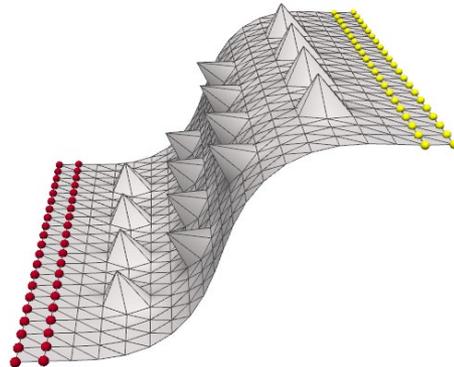
Results (vs Poisson)



Poisson:



ARAP:



Deformation models

Direct

$$v' = \left(\sum w_j T_j \right) v$$

- Linear Blend Skinning
- Dual Quaternion Skinning
- ...

Variational

$$v' = \operatorname{argmin}_x E(x)$$

- Multiresolution editing
- As-Rigid-As-Possible
- Laplacian Mesh Editing
- ...

Deformation models

Direct

$$v' = \left(\sum w_j T_j \right) v$$

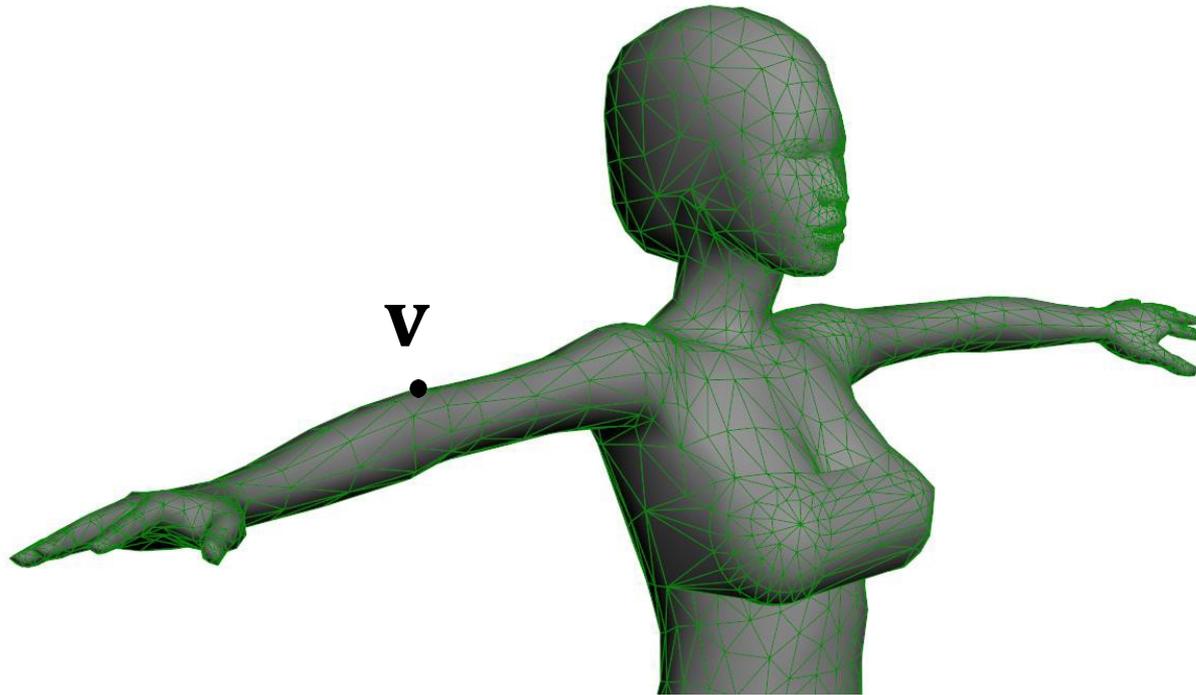
- Linear Blend Skinning
- Dual Quaternion Skinning
- ...

Variational

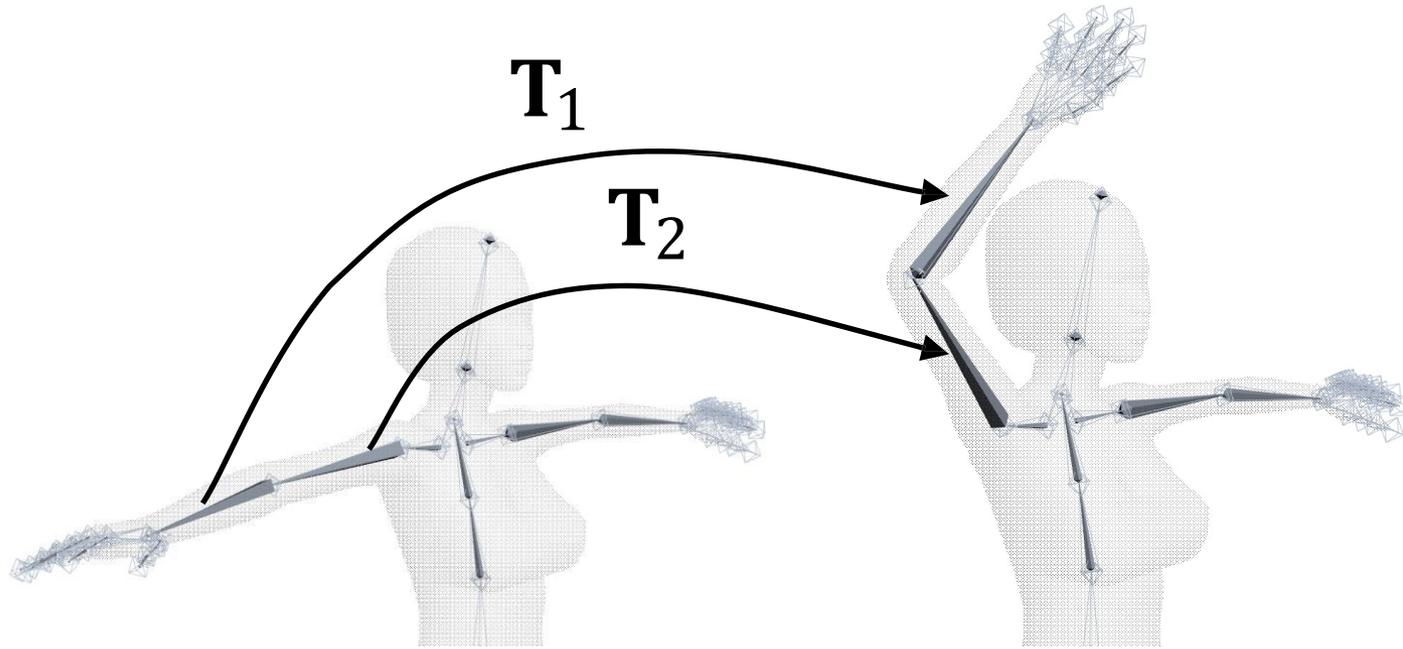
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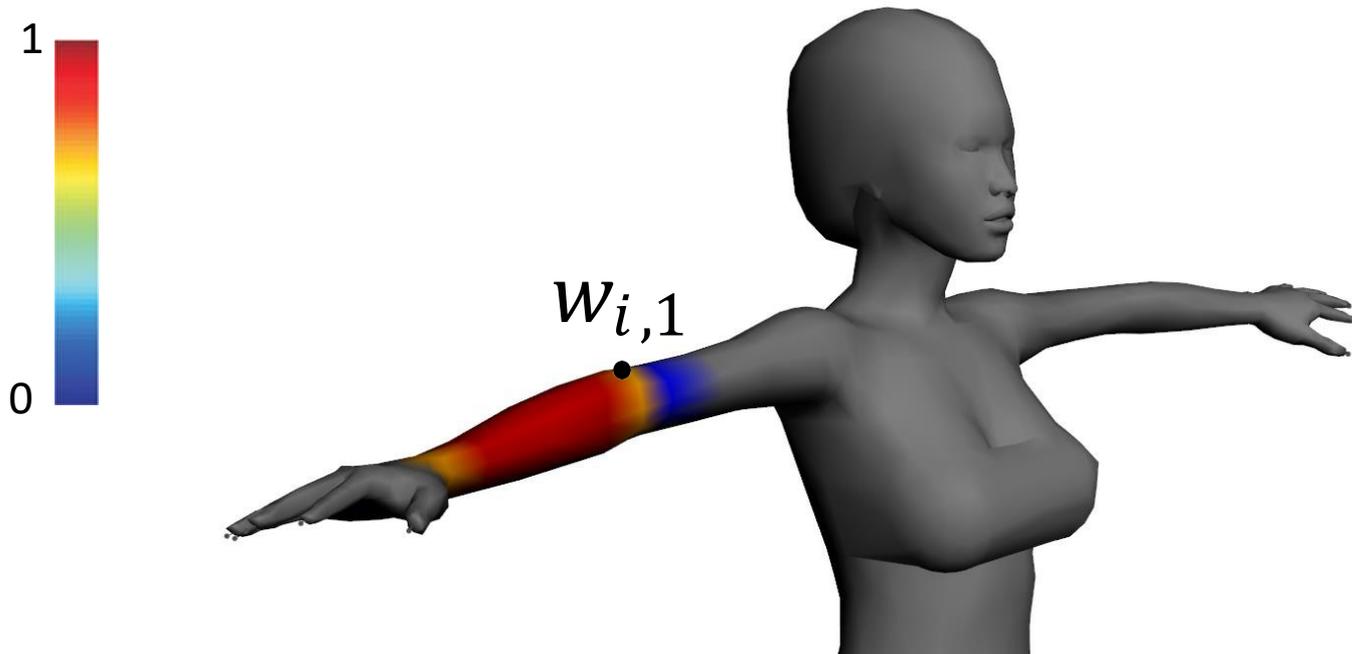
1) Rest pose



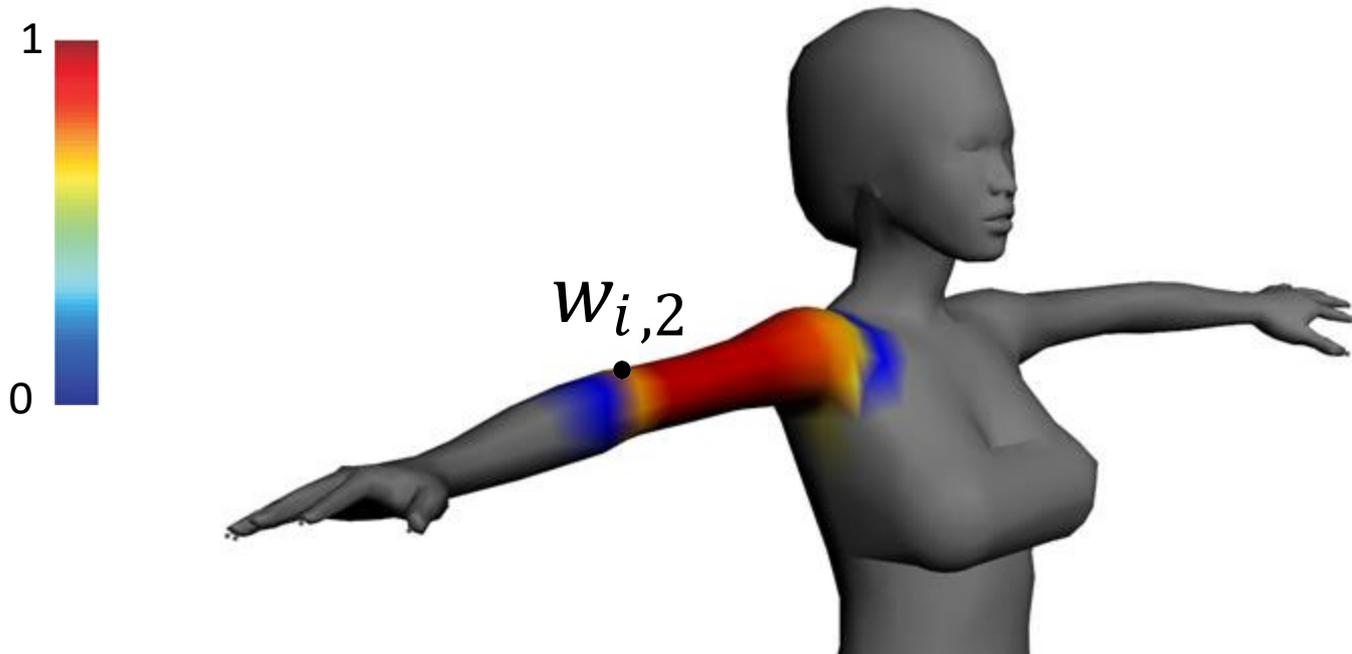
2) Skinning transformations



3) Skinning weights



3) Skinning weights



Linear blend skinning (LBS)

$$v' = \left(\sum w_j T_j \right) v$$

LBS is used widely in the industry

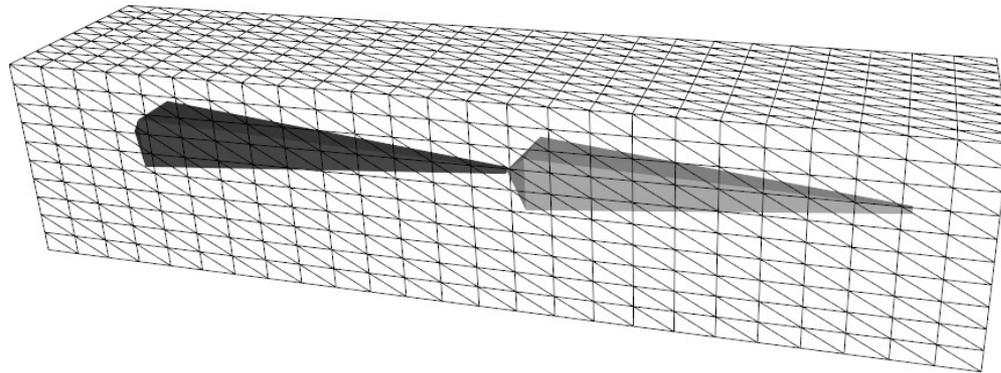


Halo 3



Bolt

LBS: candy-wrapper artifact



LBS: candy-wrapper artifact

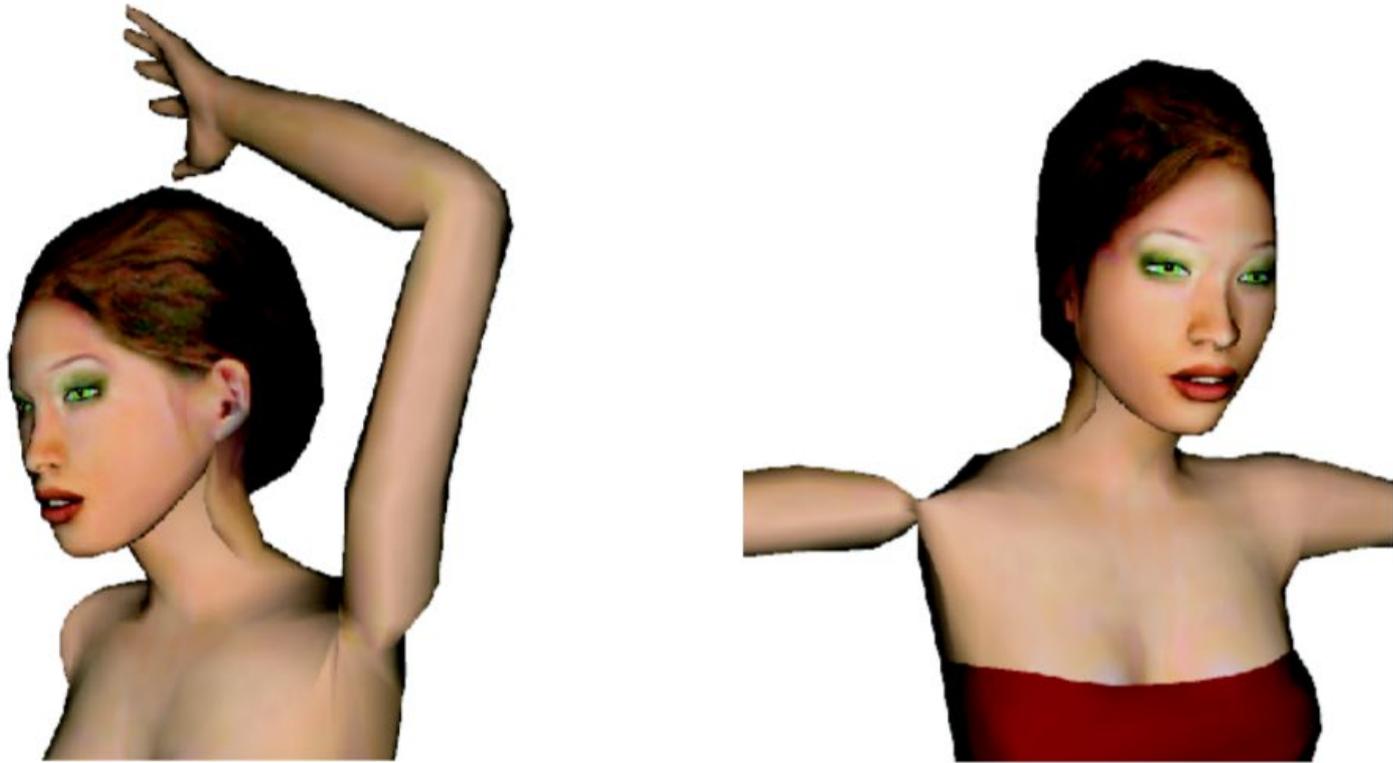


Figure 2: Typical “candy-wrapper” artifacts of linear blend skinning.

What went wrong?

$$v' = \left(\sum w_j T_j \right) v$$

What went wrong?

$$v' = \left(\sum w_j T_j \right) v$$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What went wrong?

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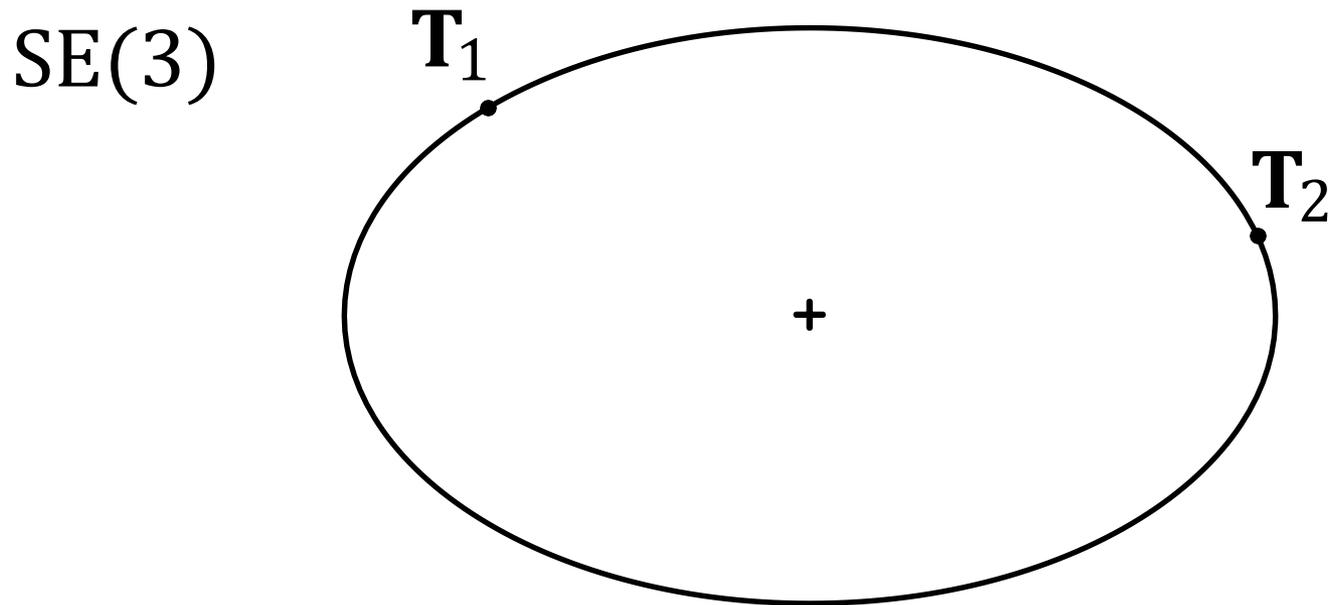
$$R_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Why can't we just sum up rotation matrices?

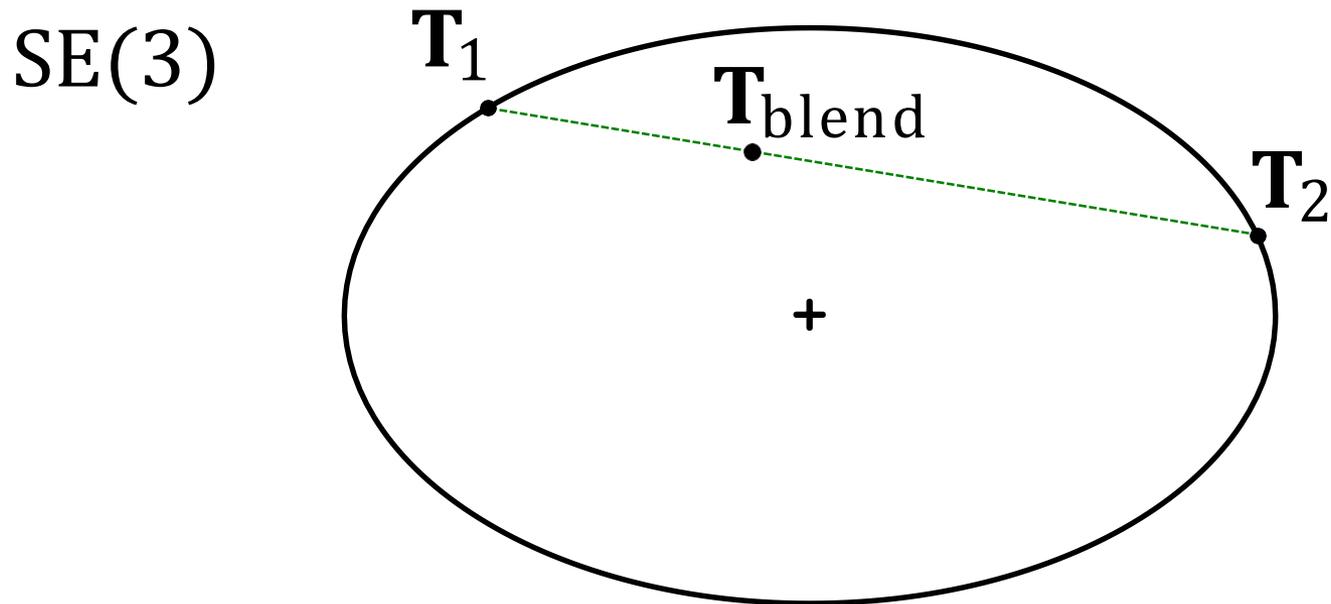
Geometry of linear blending

$SE(3)$

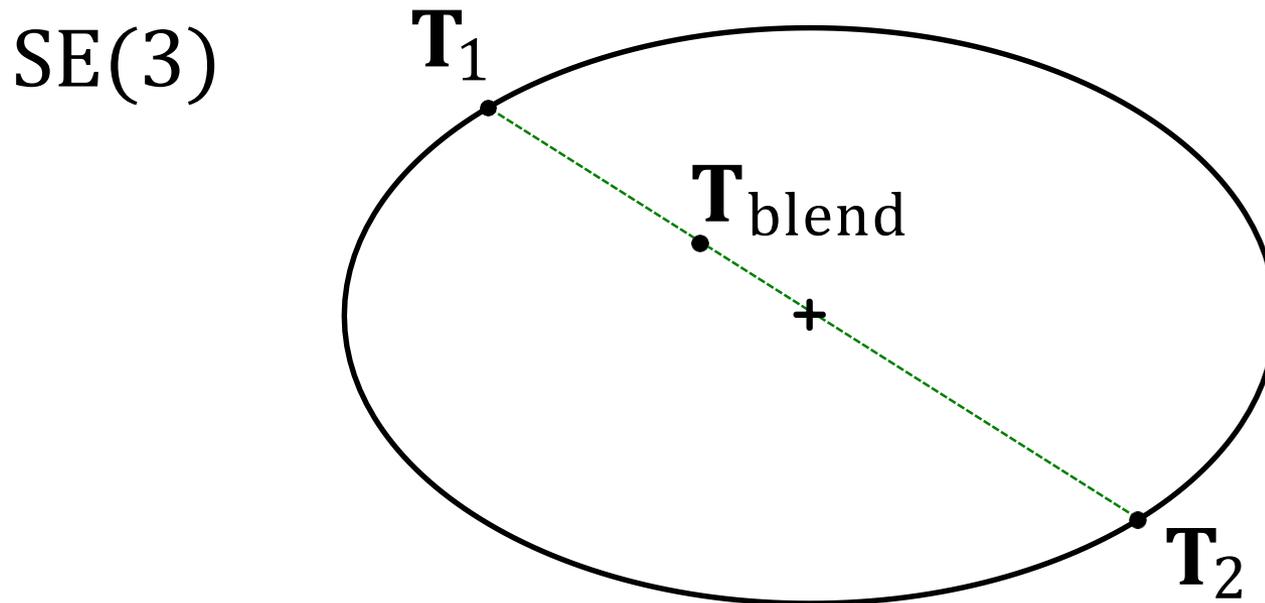
Geometry of linear blending



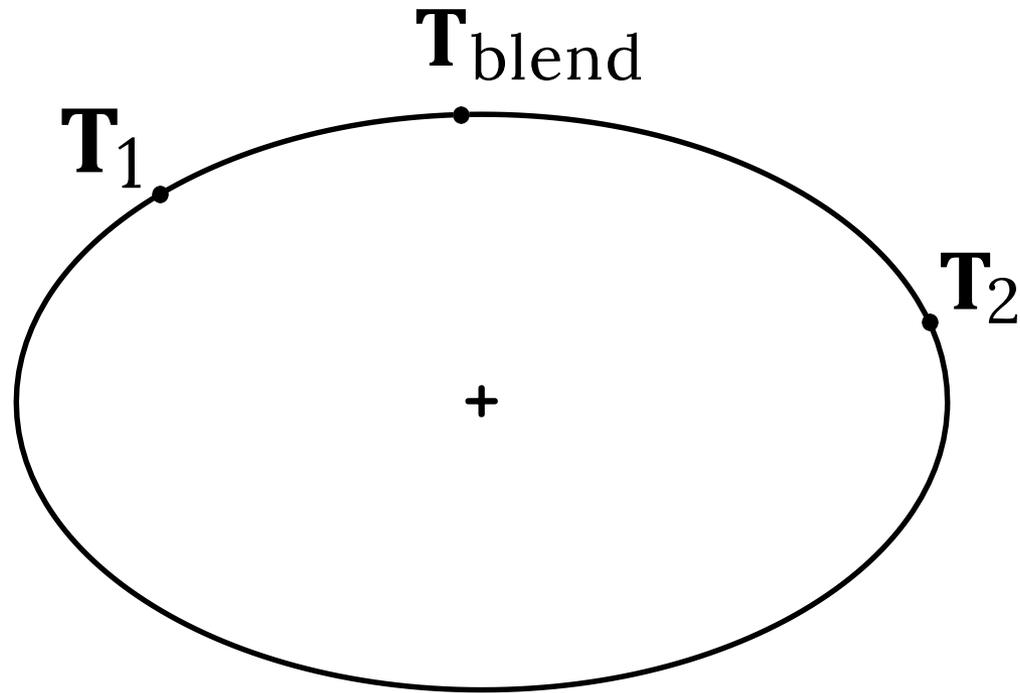
Geometry of linear blending

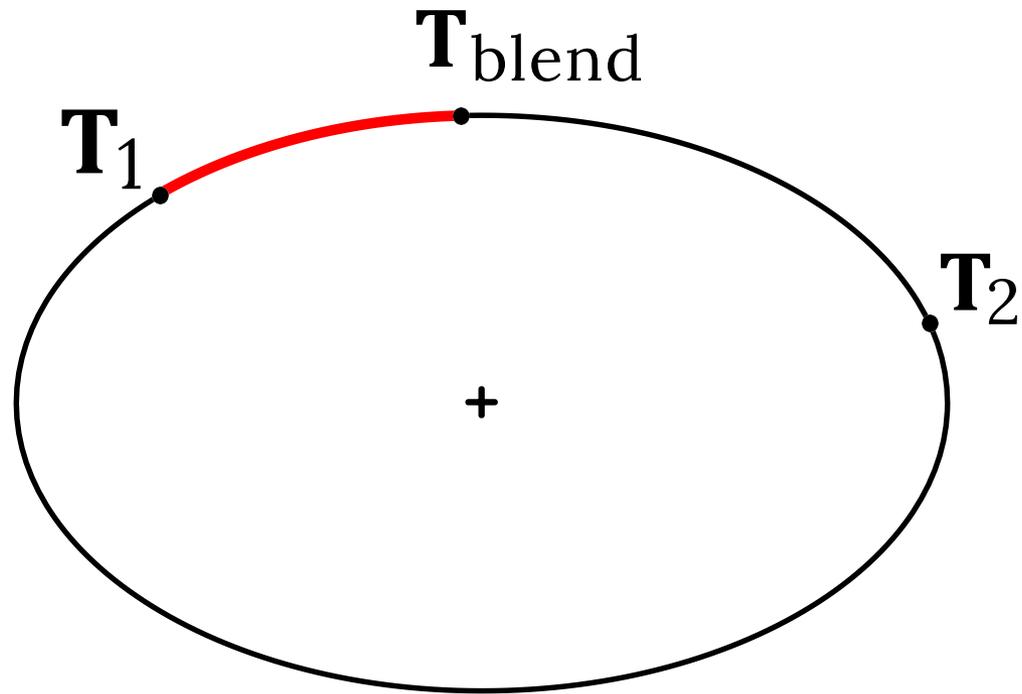


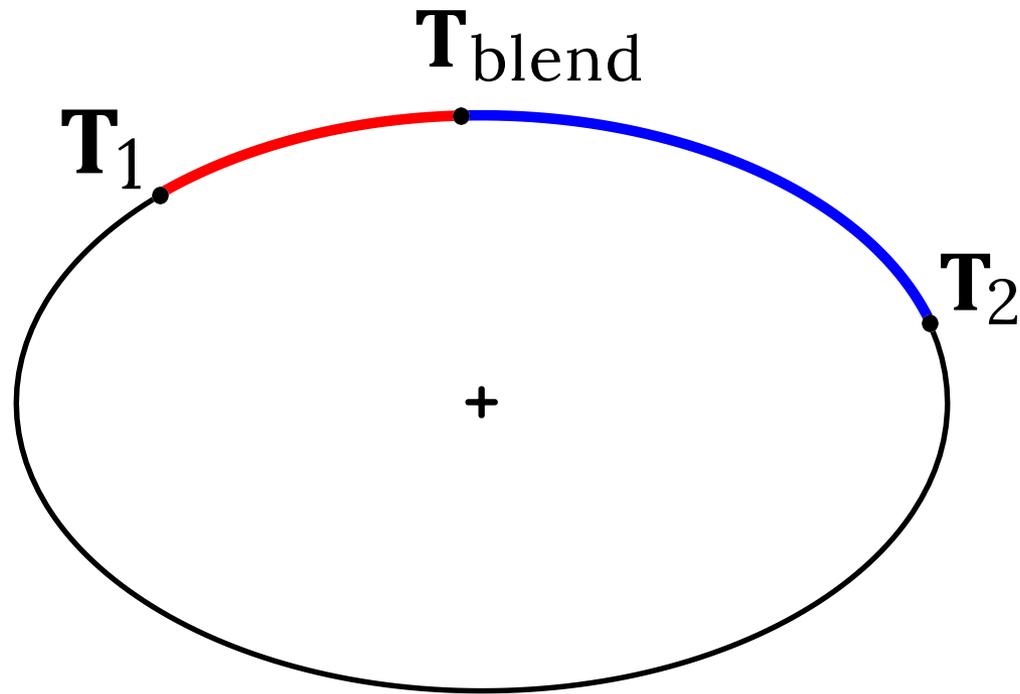
Geometry of linear blending



Intrinsic blending







Intrinsic blending using Lie algebras

[Buss and Fillmore 2001, Alexa 2002, Govindu 2004, Rossignac and Vinacua 2011]

$$\operatorname{argmin}_X \sum_X w_j d(X, T_j)$$

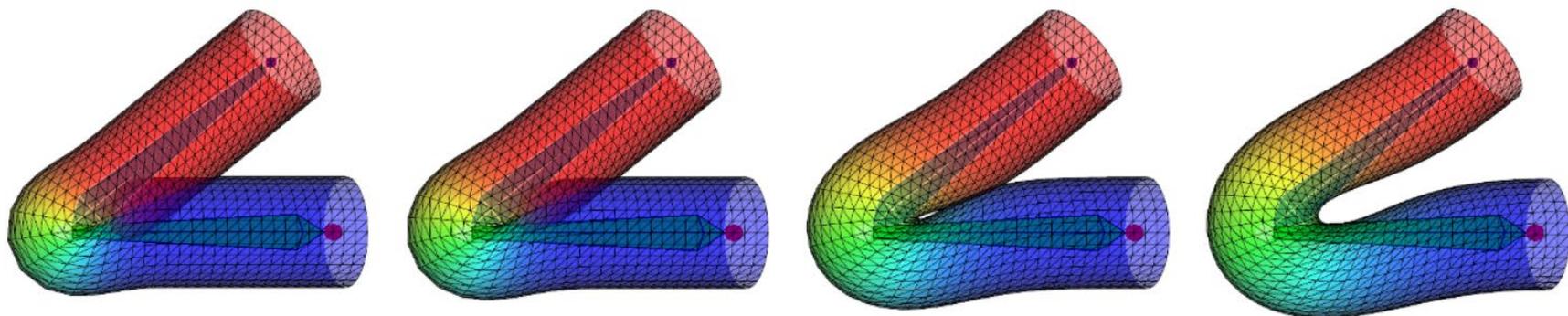
$$d(\mathbf{X}, \mathbf{Y}) = \|\log(\mathbf{Y}\mathbf{X}^{-1})\|^2$$

Karcher / Frechet mean

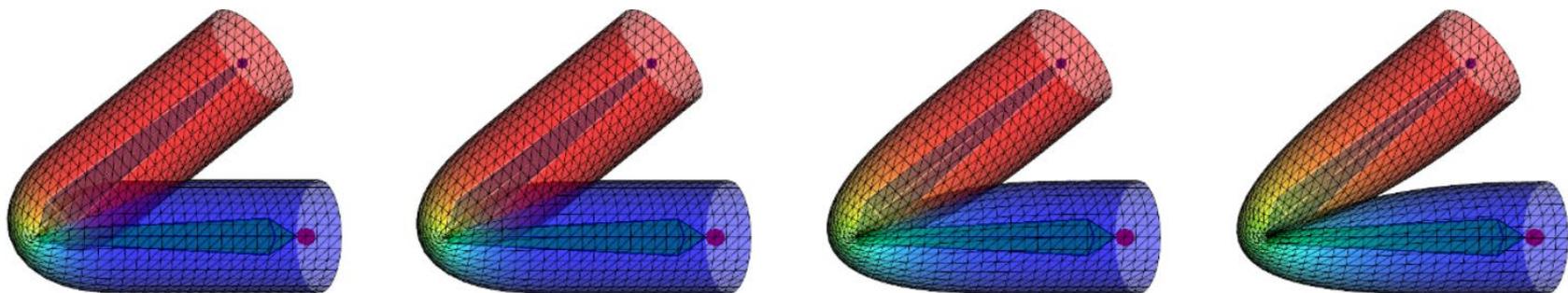
79

Dual Quaternion Skinning

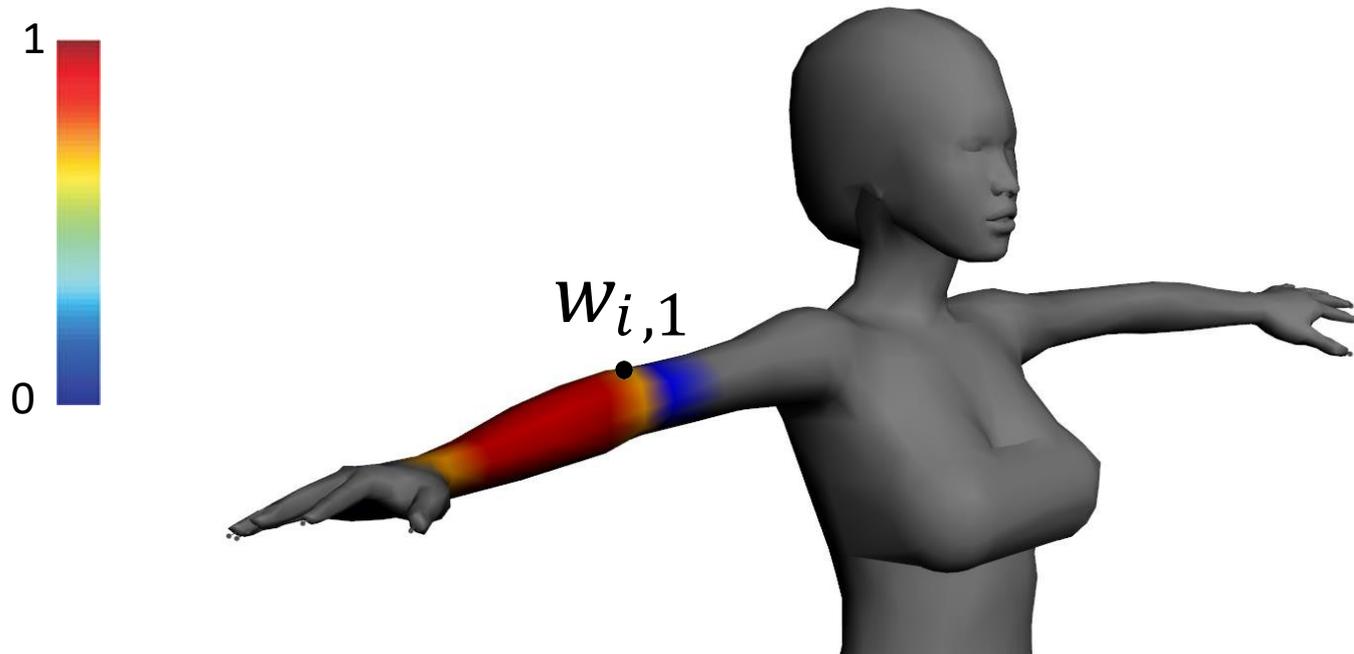
Dual
Quats



Linear
Blend



Where do the weights come from?



Manual?



Automatic skinning
weight computation

Weights should obtain a few basic qualities



Inverse Euclidean distance weights are too crude,
show obvious artifacts

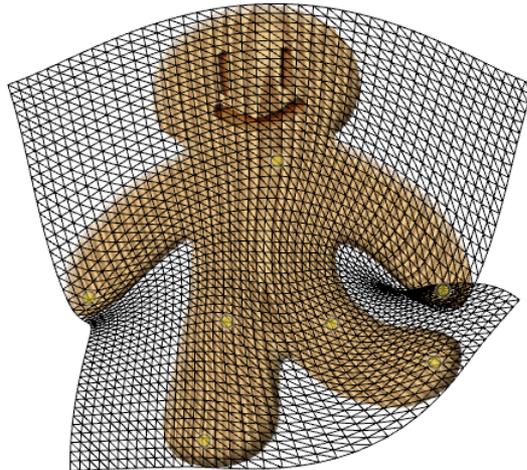


$$w_j(\mathbf{v}) = \frac{1}{\|\mathbf{c}_i - \mathbf{v}\|^2}$$

weights optimized *inside* shape

[Shepard 1968],
[Schaefer et al. 2006], etc.

Inverse Euclidean distance weights are too crude

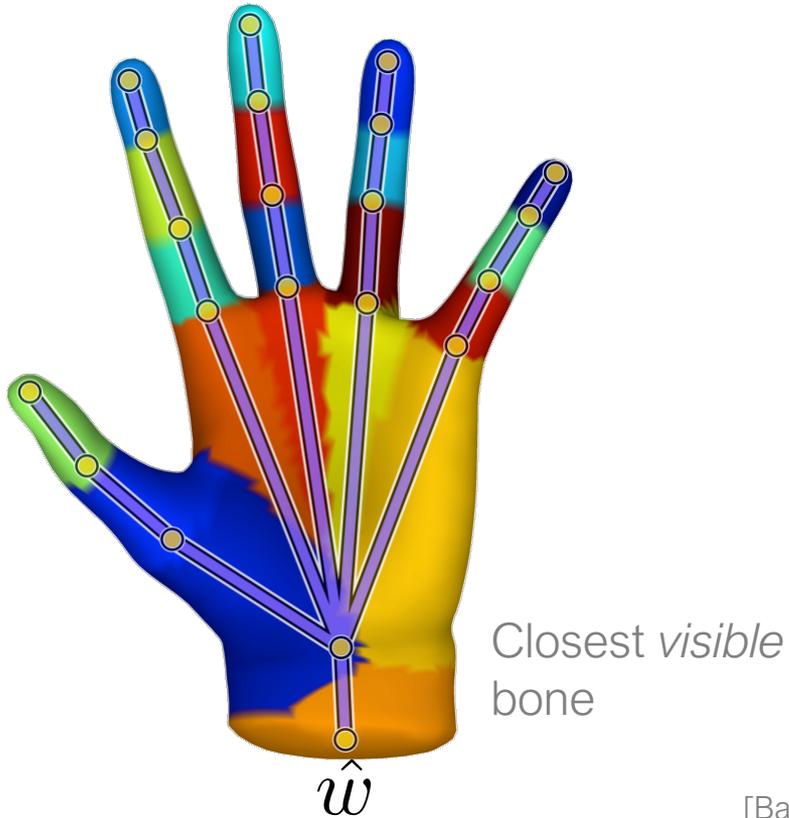


$$w_j(\mathbf{v}) = \frac{1}{\|\mathbf{c}_i - \mathbf{v}\|^2}$$

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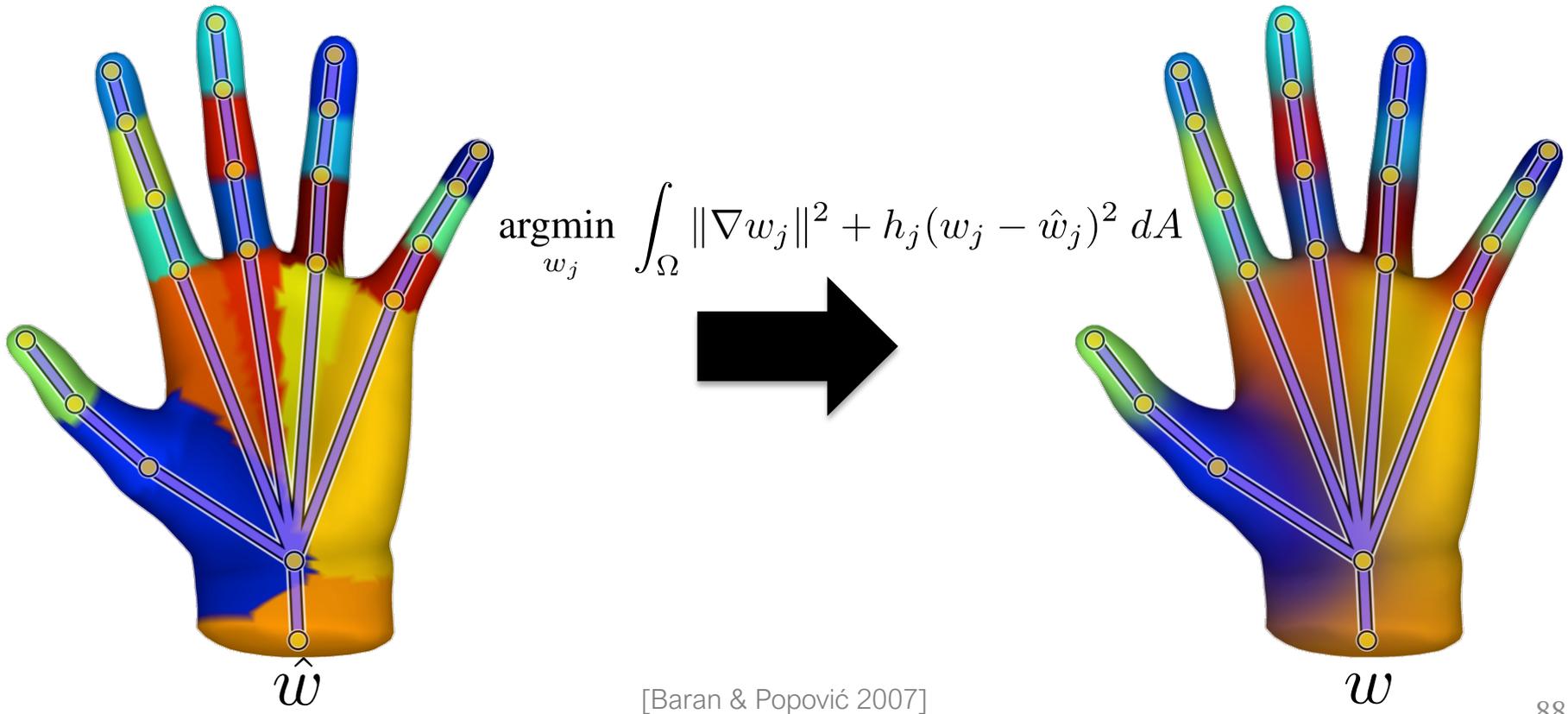
[Shepard 1968],
[Schaefer et al. 2006], etc.

Discontinuous projection onto surface can be smoothed out

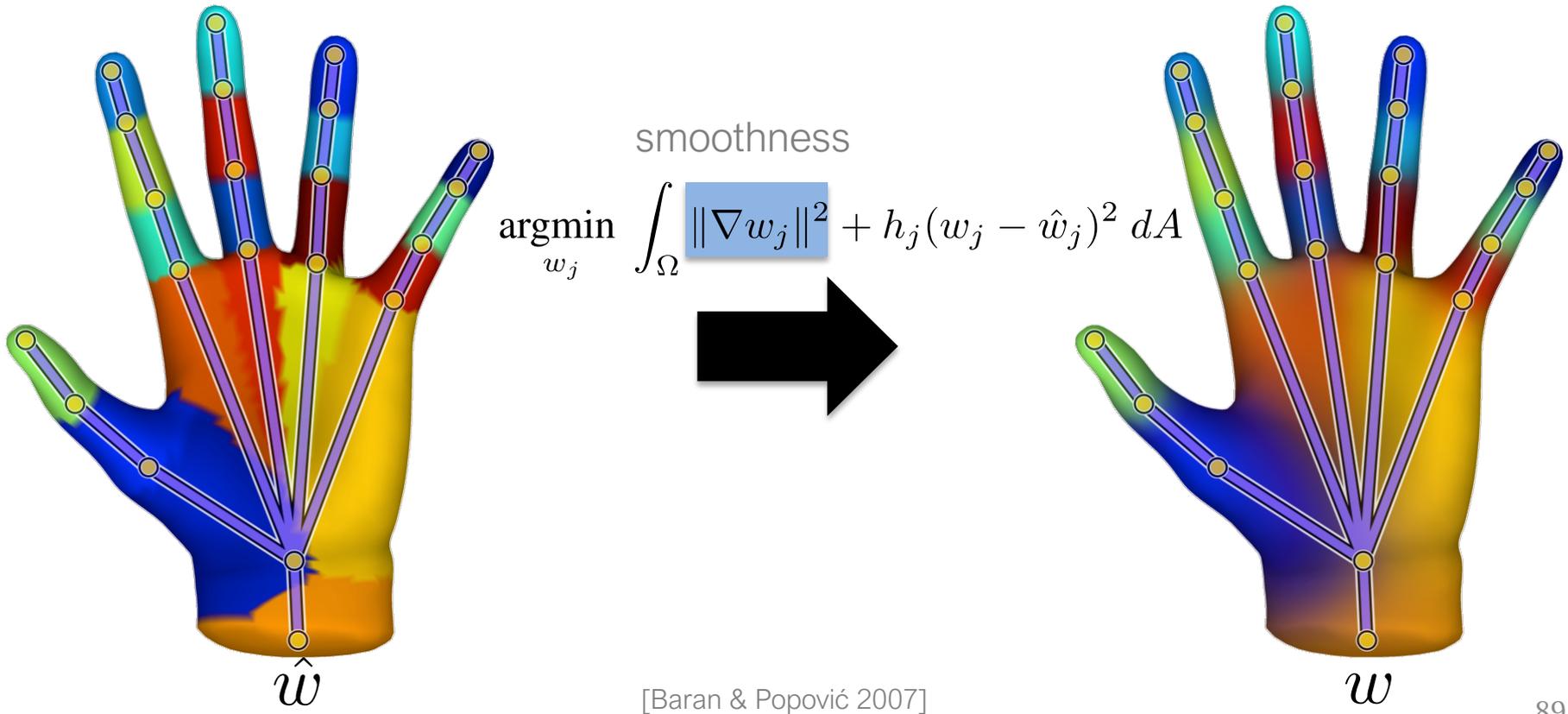


[Baran & Popović 2007]

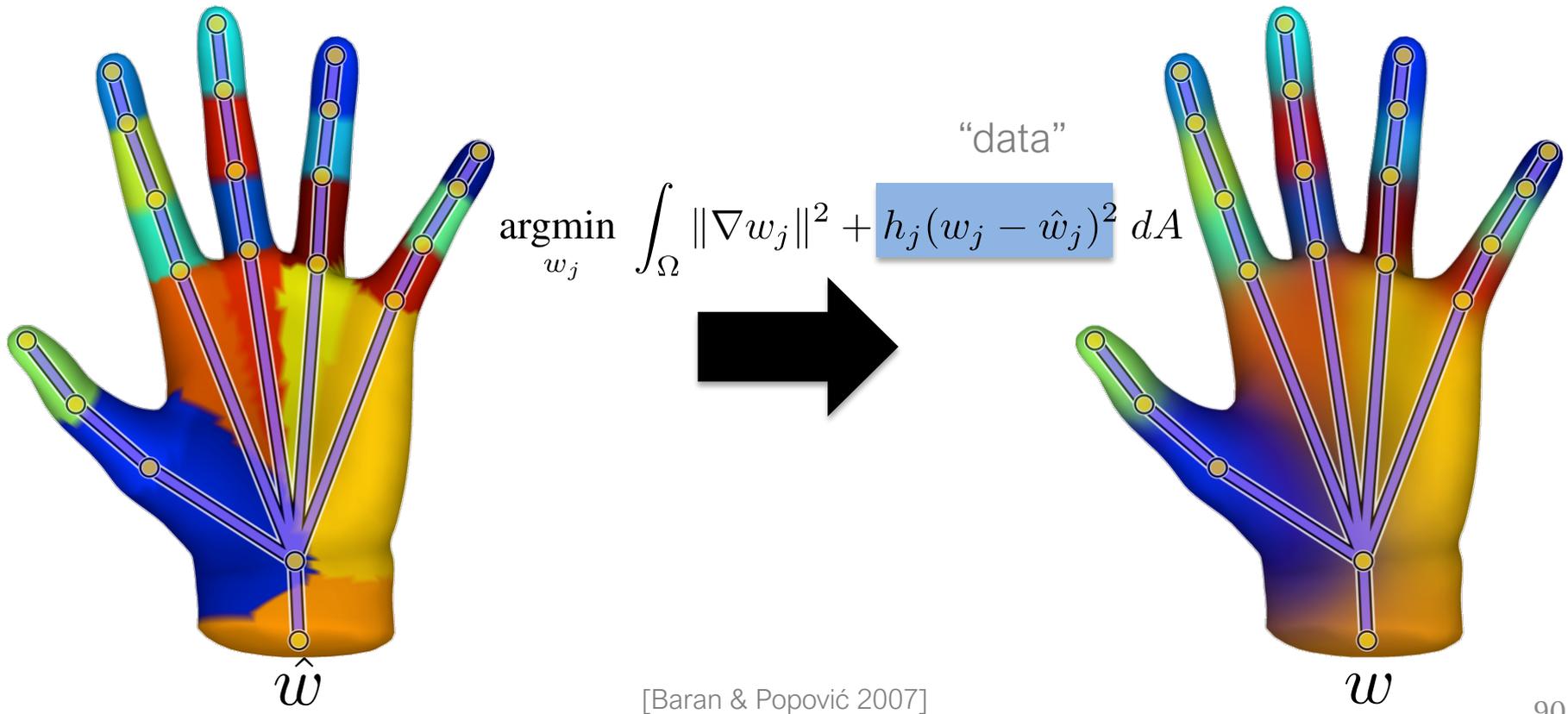
Discontinuous projection onto surface
can be smoothed out



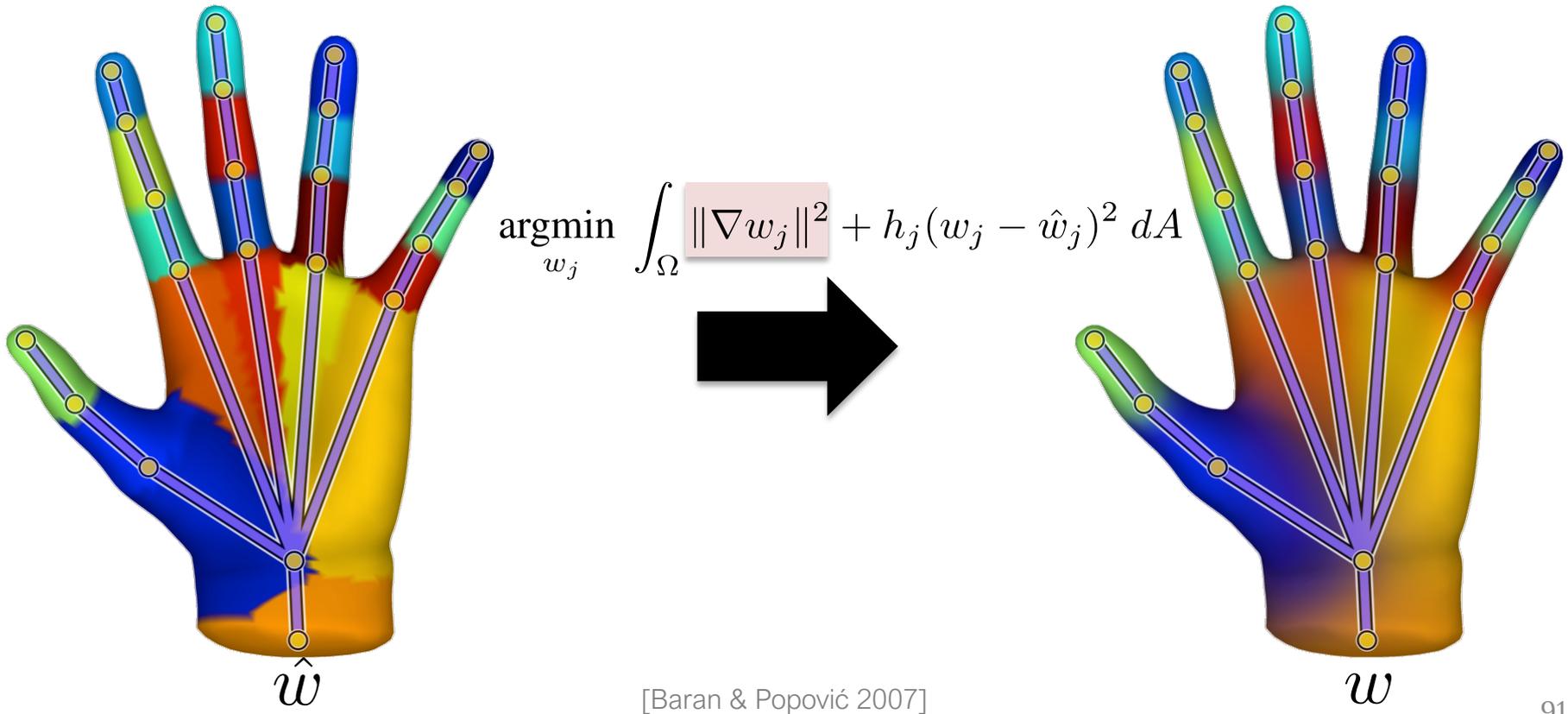
Discontinuous projection onto surface can be smoothed out



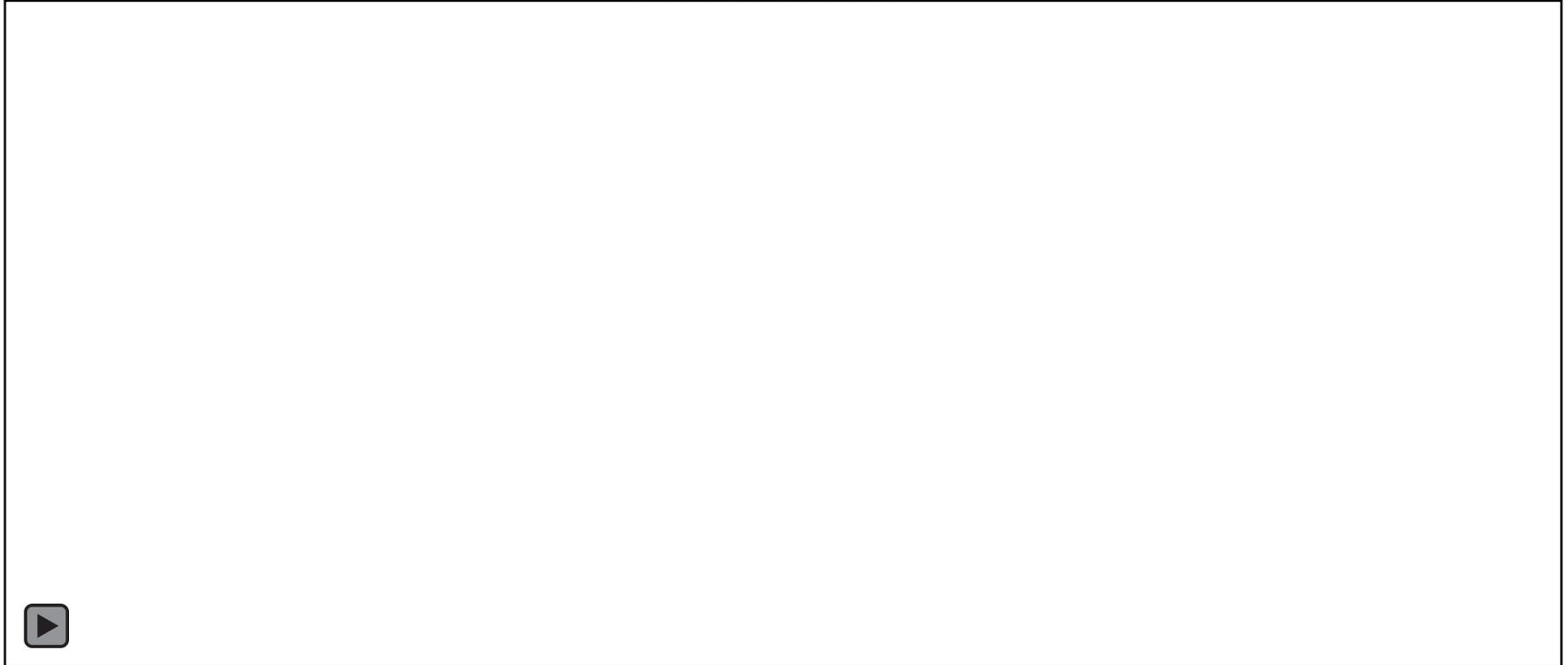
Discontinuous projection onto surface can be smoothed out



Discontinuous projection onto surface can be smoothed out



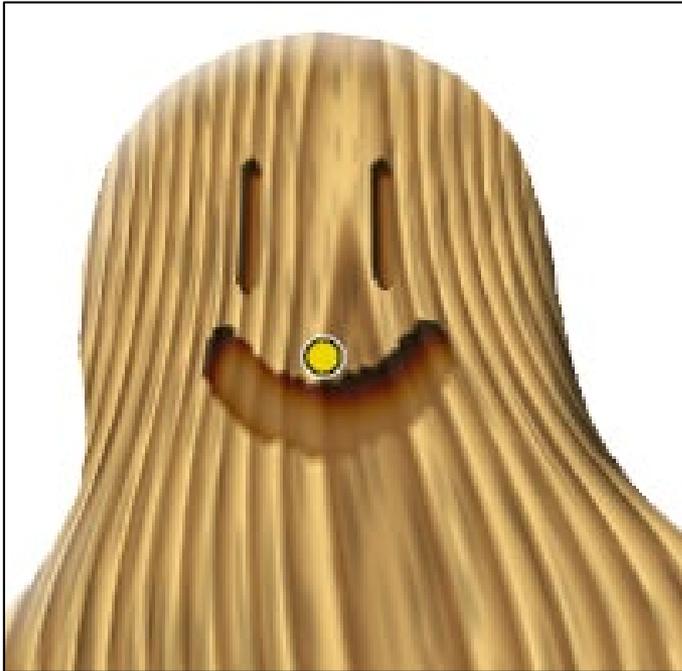
Gradient energy weights not smooth at handles



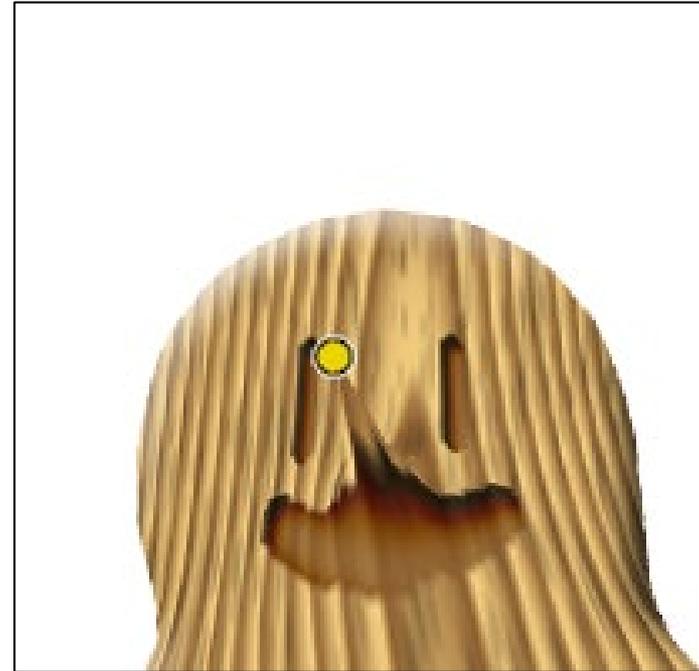
$$\operatorname{argmin}_{w_j} \int_{\Omega} (\Delta w_j)^2 dA$$

$$\operatorname{argmin}_{w_j} \int_{\Omega} \|\nabla w_j\|^2 dA$$

Gradient energy weights not smooth at handles

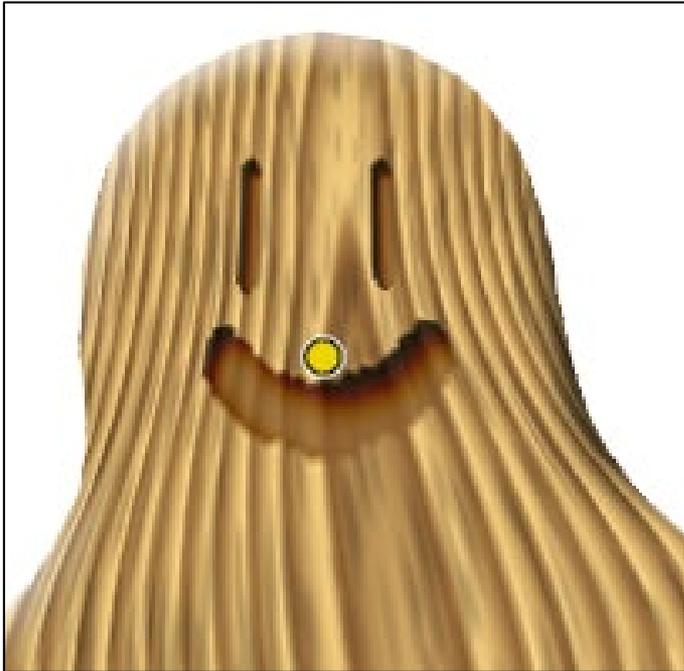


$$\operatorname{argmin}_{w_j} \int_{\Omega} (\Delta w_j)^2 dA$$

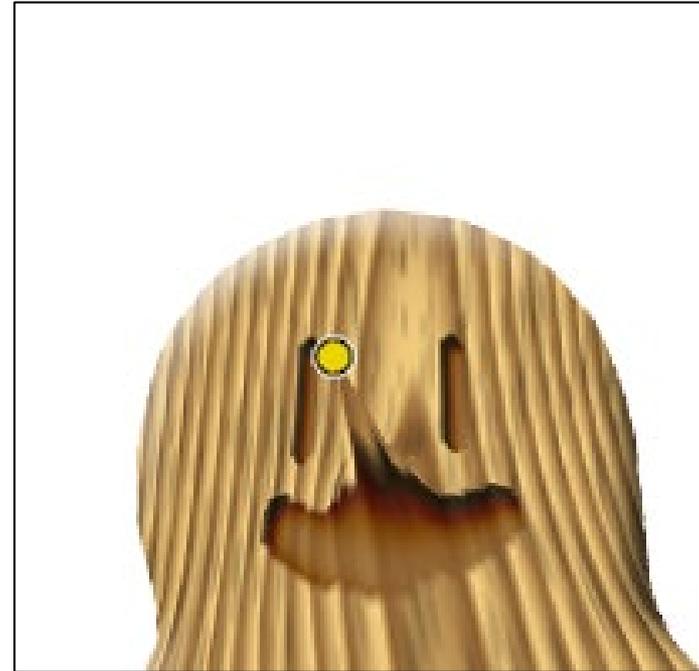


$$\operatorname{argmin}_{w_j} \int_{\Omega} \|\nabla w_j\|^2 dA$$

Gradient energy weights not smooth at handles



$$\Delta^2 w_j = 0$$



$$\Delta w_j = 0$$

Point constraints for Laplace equation



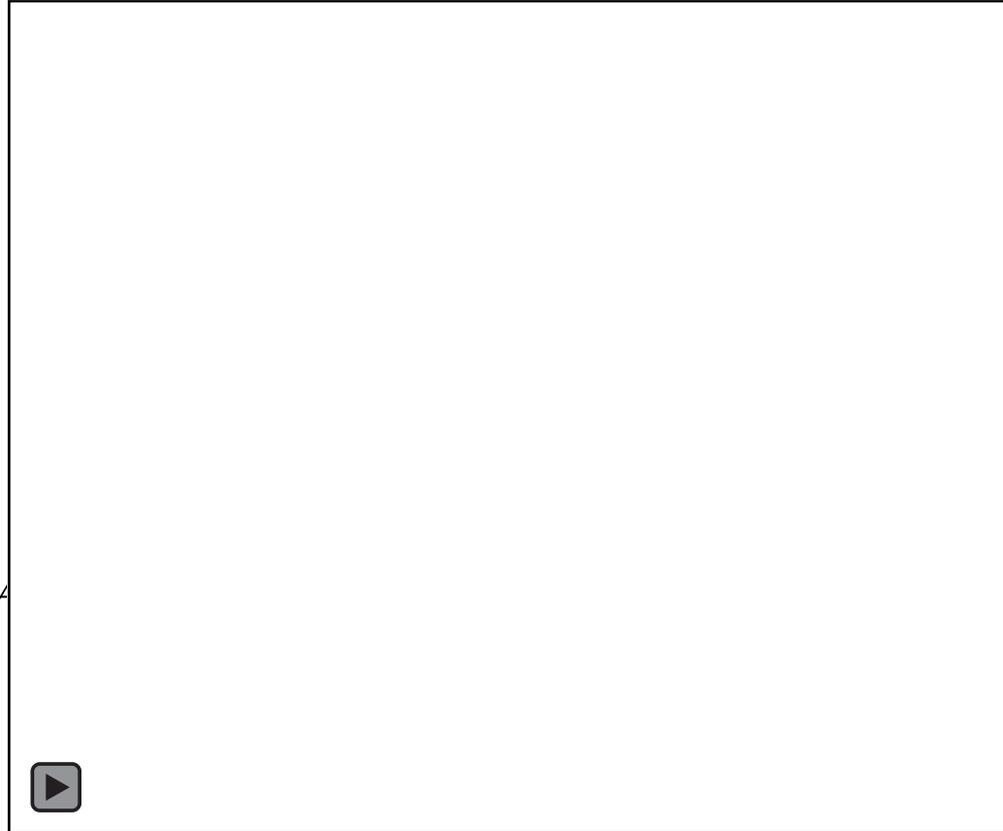
<https://www.facebook.com/521399544544480/photos/a.523048724379562/800968259920939/?type=1&theater> , Facebook group "Circus tents and circus equipment"

Non-negative, local weights are mandatory

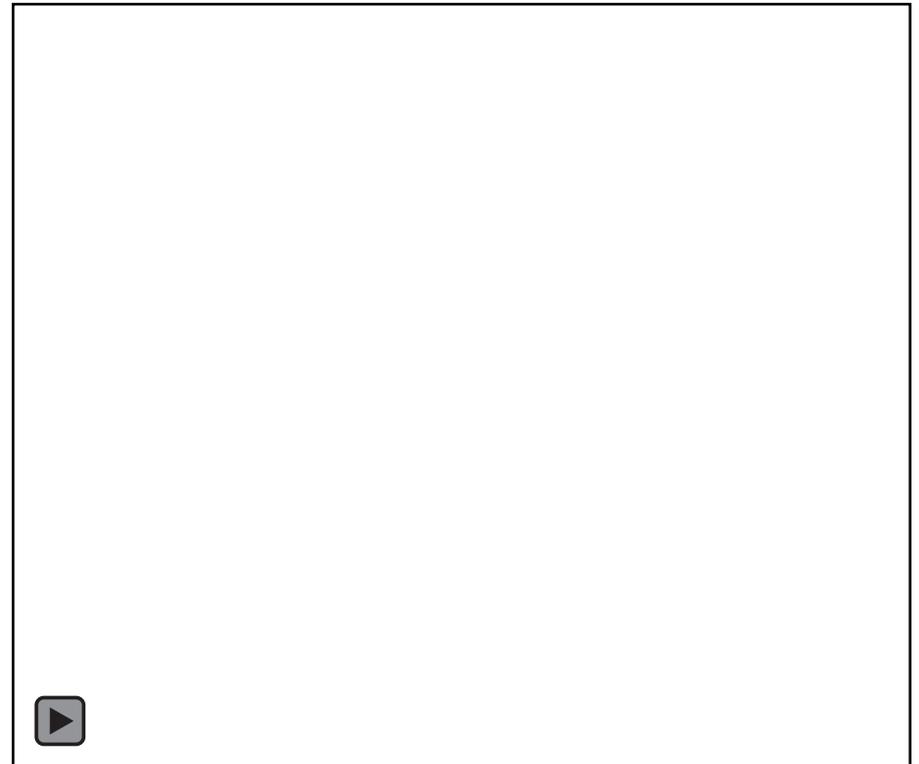
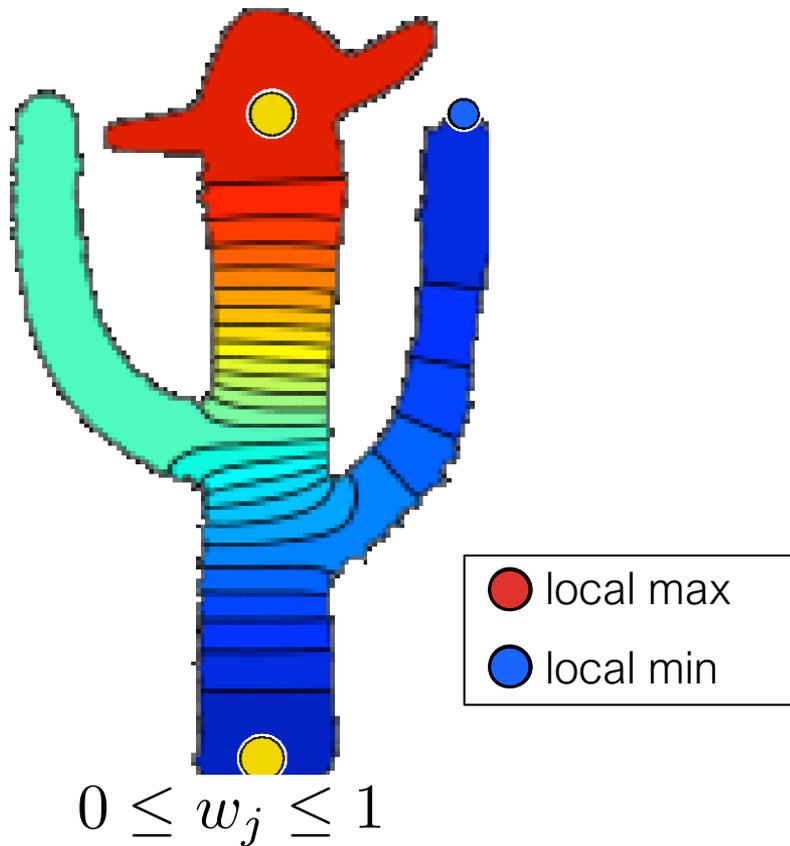
$$0 \leq w_j \leq 1$$

$$\operatorname{argmin}_{w_j} \int_{\Omega} (\Delta w_j)^2 dA$$

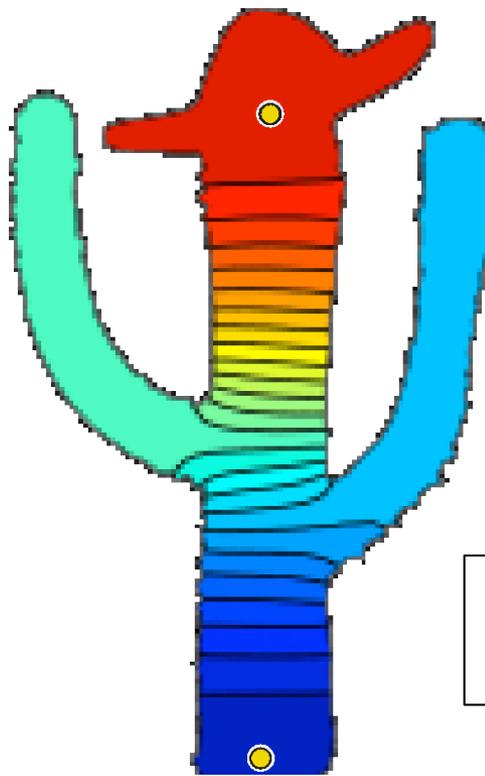
[Botsch & Kobbelt 2004]



Spurious extrema cause distracting artifacts



Must explicitly prohibit spurious extrema



- local max
- local min

w_j is “monotonic”



Previous methods fail in one way or another

	Euclidean	Δw_j	$\Delta^2 w_j$
smooth	✓	–	✓
non-negative	✓	✓	–
shape-aware	–	✓	✓
local	–/✓	–	–
monotonic	–	✓	–
arbitrary handles	–	✓	✓
	[Shepard 1968, Sibson 1980, Schaefer et al. 2006]	[Baran & Popovic 2007, Joshi et al. 2007]	[Botsch & Kobbelt 2004, Sorkine et al. 2004, Finch et al. 2011]

Constrained optimization ensures satisfaction of all properties

$$\operatorname{argmin}_{w_j, j=1, \dots, m} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 dV$$

- + shape-aware
- + smoothness

[Botsch & Kobbelt 2004, Sorkine et al. 2004, Joshi & Carr 2008, Jacobson et al. 2010, Finch et al. 2011, Andrews et al. 2011]

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$$w_j(\mathbf{v}) = \begin{cases} 1 & \mathbf{v} \in h_j, \\ 0 & \mathbf{v} \in h_k \\ \text{linear on cage facets} \end{cases}$$

- + shape-aware
- + smoothness
- + arbitrary handles

[Botsch & Kobbelt 2004, Sorkine et al. 2004, Joshi & Carr 2008, Jacobson et al. 2010, Finch et al. 2011, Andrews et al. 2011]

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$$0 \leq w_j \leq 1,$$

$$\sum_{j=1}^m w_j = 1$$

- + shape-aware
- + smoothness
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[Jacobson et al. 2011]

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$$\operatorname{argmin}_{w_j, j=1, \dots, m} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 dV$$

$$\|w\|_1 = 1$$

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[Rustamov 2011]

Constrained optimization ensures satisfaction of all properties

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[Rustamov 2011]

Constrained optimization ensures satisfaction of all properties

$$\operatorname{argmin}_{w_j, j=1, \dots, m} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 dV$$

$$\nabla w_j \cdot \nabla u_j > 0$$

- + shape-aware
- + smoothness
- + arbitrary handles
- + non-negativity
- + locality
- + monotonicity

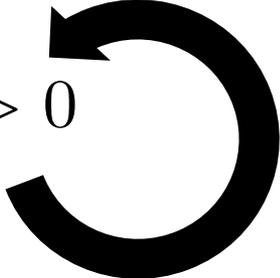
[Weinkauff et al. 2011, Jacobson et al. 2012, Günther et al. 2014]

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