IFT 6113
SURFACE RECONSTRUCTION

tiny.cc/ift6113

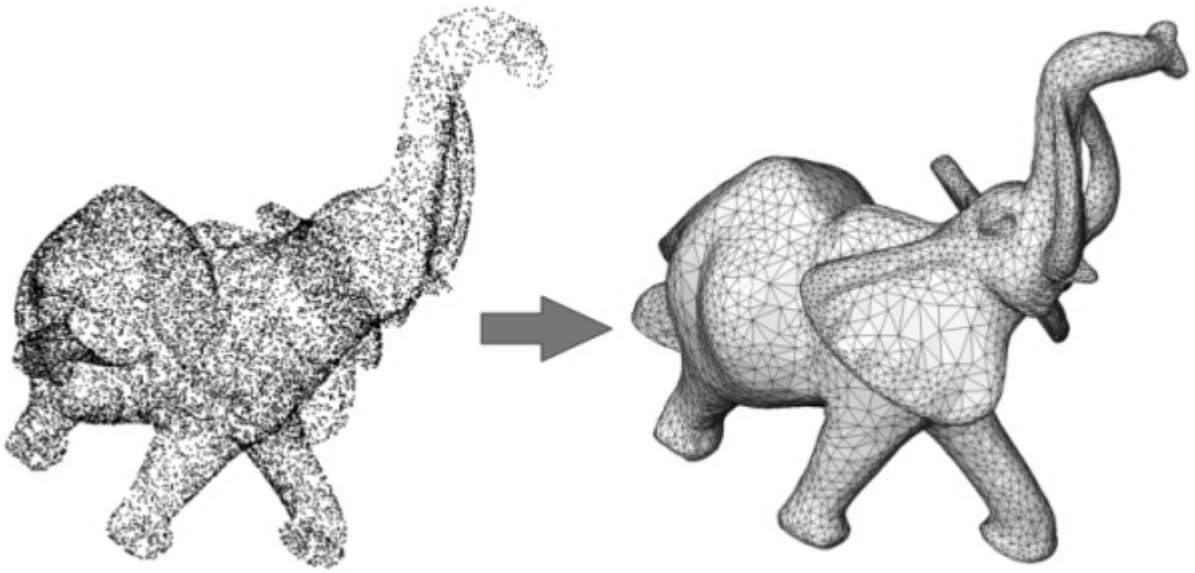


Image from https://doc.cgal.org/latest/Poisson_surface_reconstruction_3/index.html

Mikhail Bessmeltsev

Formalities

- A3 due tomorrow
- Paper presentations next week!
  - Sign up for presentation order

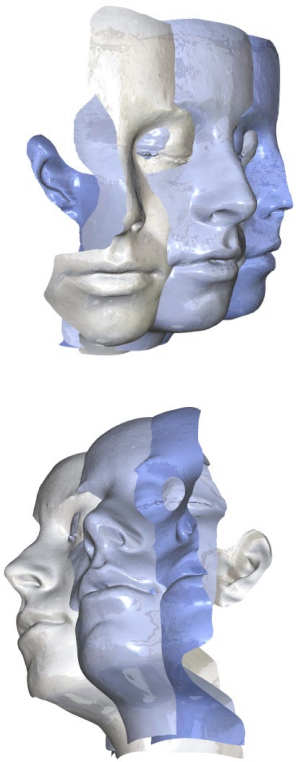
Formalities

- A3 due ~~tomorrow~~ Monday night (16th)
- Paper presentations next week!
  - Sign up for presentation order

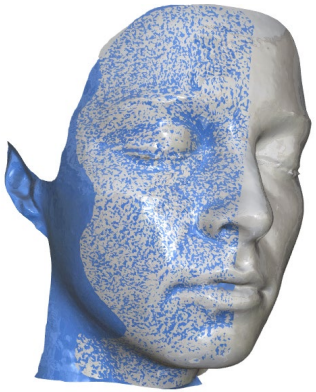
Some slides from Alla Sheffer,
Justin Solomon, and Hao Li

3D Reconstruction Pipeline

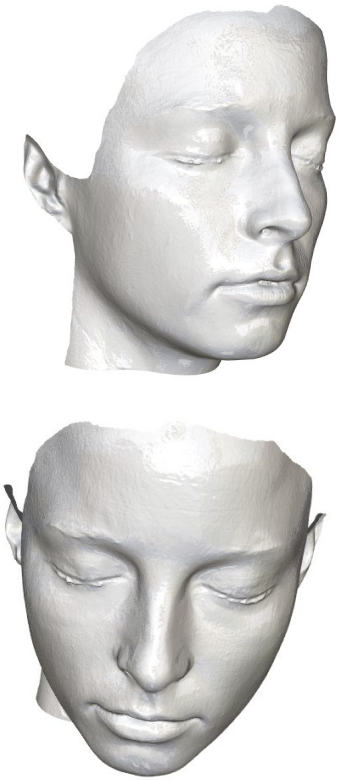
acquisition



initial alignment



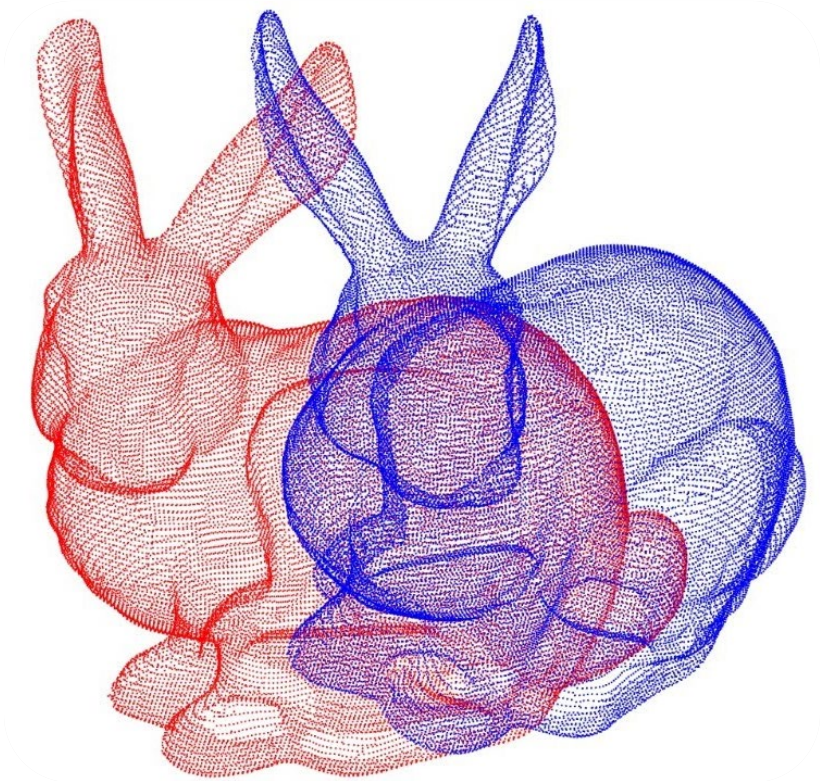
registration



merging

Data provided by Paramount Pictures and Aguru Images

Two components



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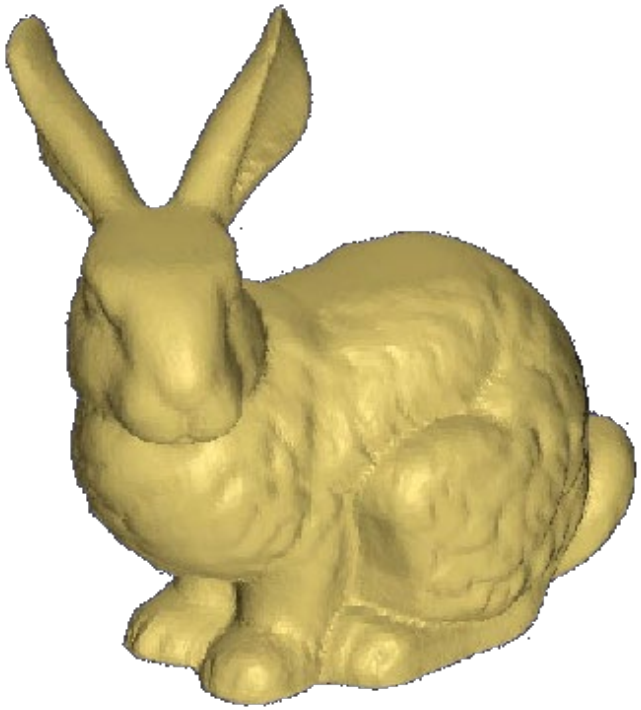
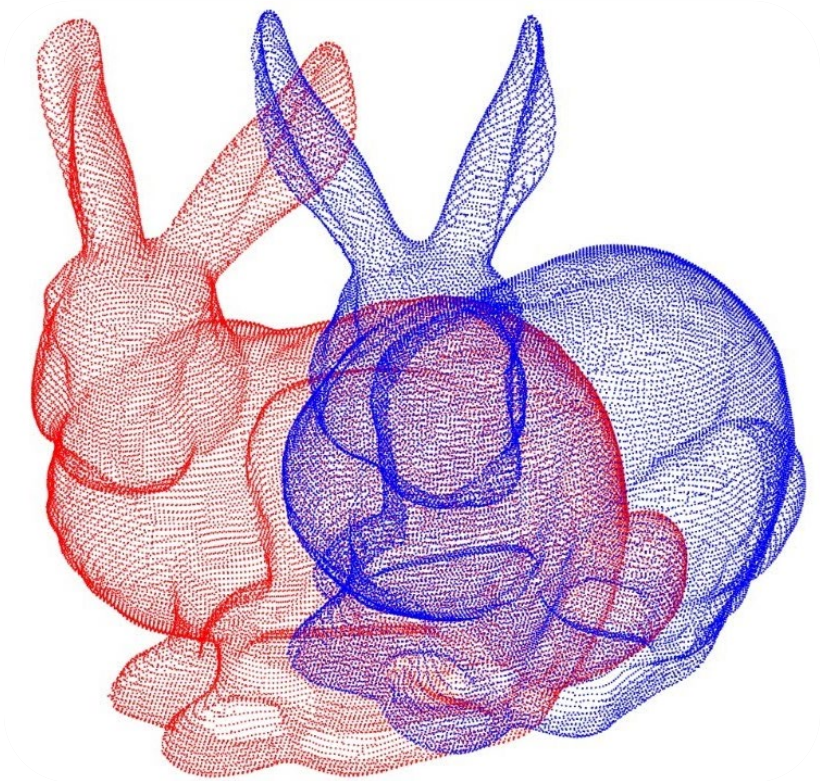
Registration



Meshing

Two components



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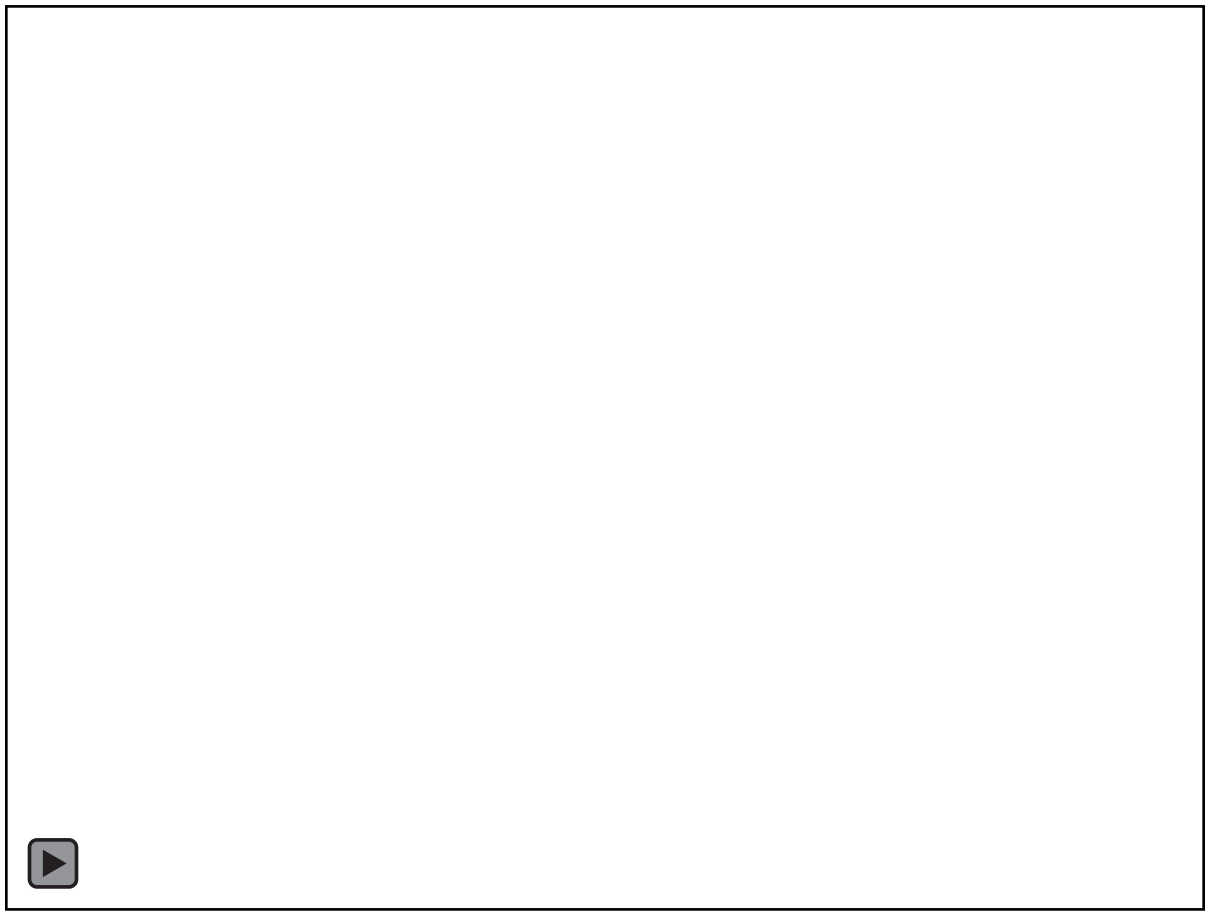
Registration

Meshing

Registration Problem



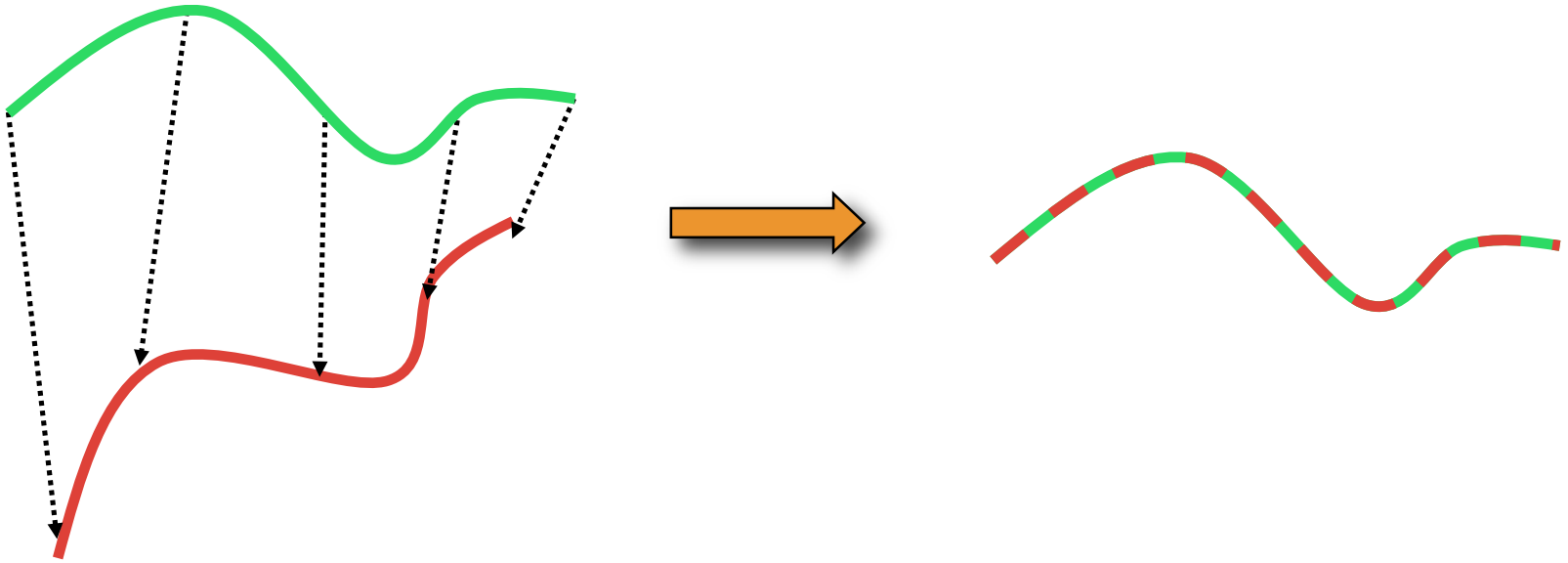
Align two overlapping objects

Rough Plan

- **ICP algorithm**
  A classic!

- **ICP variants**

Starting Point



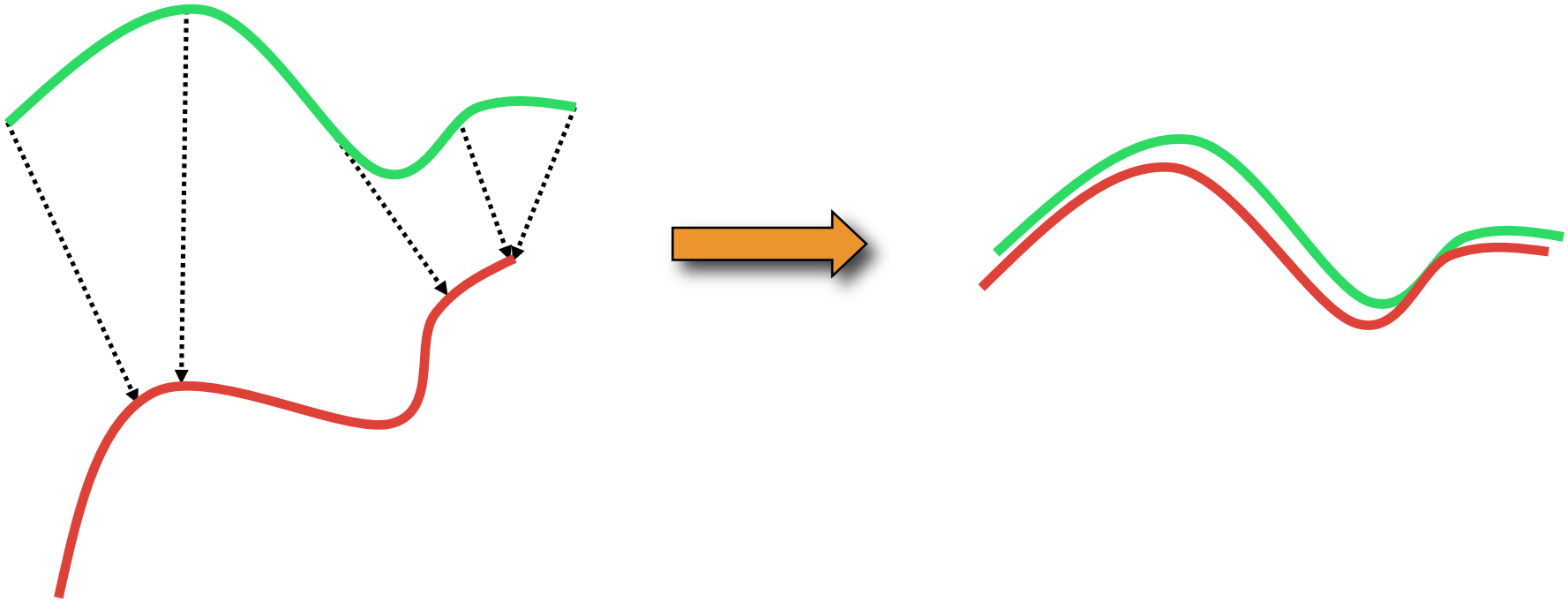
$$q_i = Rp_i + t$$

Can align given enough matches

How many correspondences determine R and t?

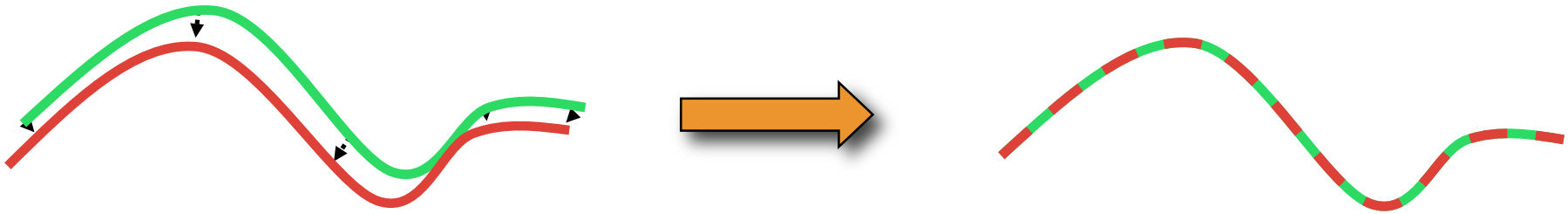
How do you get correspondences?

Rough Approximation



Closest points correspond

Try a Second Time...



Iterative Closest Point (ICP)

- **Choose e.g. 1000 random points**
- **Match each to closest point on other scan**
- **Reject pairs with distance > *k* times median**
- **Minimize**

$$E[R,t] := \sum_i \|Rp_i + t - q_i\|^2$$

- **Iterate**

"A method for registration of 3-D shapes."
Besl and McKay, PAMI 1992.

On the Board

$$\min_{t \in \mathbb{R}^3,\ R^\top R = I} \sum_i \| R p_i + t - q_i \|^2$$

Closed-form formulas!

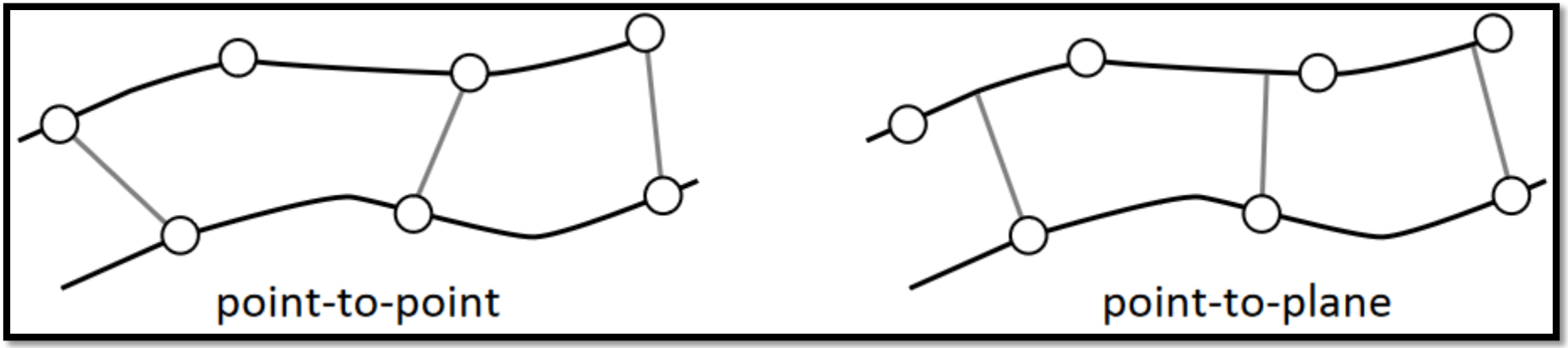
Many (!) Variants of ICP

- **Source points from one or both meshes**
- **Matching to points in the other mesh**
- **Weighting correspondences**
- **Rejecting outlier point pairs**
- **Alternative error metrics**

See [Rusinkiewicz & Levoy, 3DIM 2001]

Point-to-Plane Error Metric

Flat parts can slide along each other



$$E[R,t] := \sum_i ((Rp_i + t - q_i)^\top n_i)^2$$

$$\approx \sum_i [(p_i - q_i)^\top n_i + r^\top (p_i \times n_i) + t^\top n_i)^2 \text{ after linearizing}$$

where $r := (r_x, r_y, r_z)$

Least-squares!

"Object modelling by registration of multiple range images"
Chen and Medioni, Image and Vision Computing 10.3 (1992); image courtesy N. Mitra

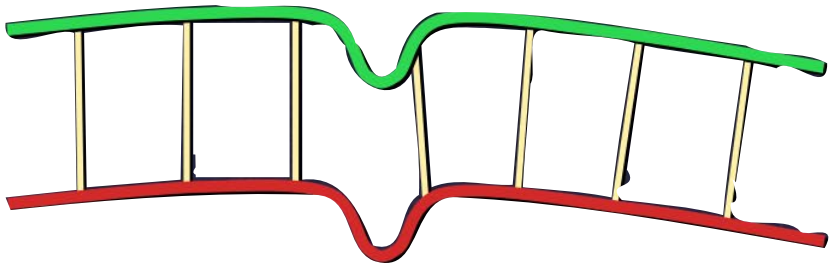
Closest Compatible Point

Can improve matching effectiveness by restricting match to compatible points

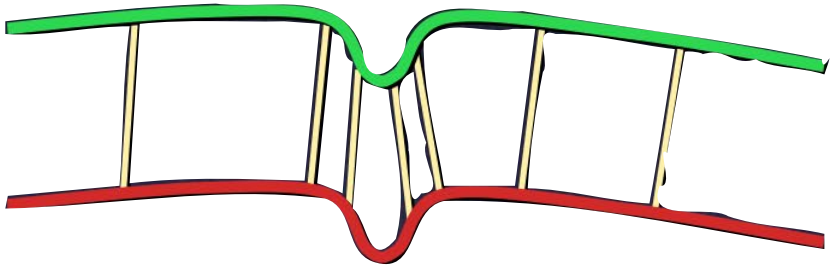
- Compatibility of colors [Godin et al. 94]
- Compatibility of normals [Pulli 99]
- Other possibilities:
  curvatures, higher-order derivatives, and other local features

Slide courtesy N. Mitra

Choose Points to Improve Stability



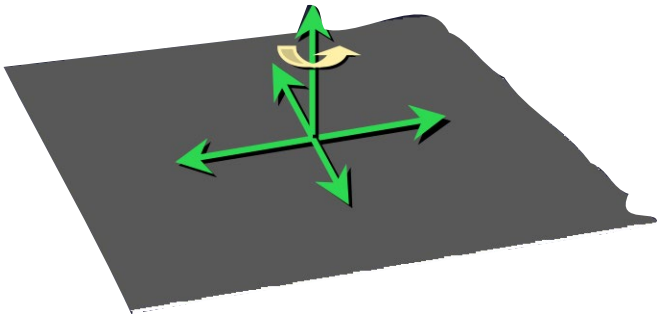
Uniform Sampling



Stable Sampling

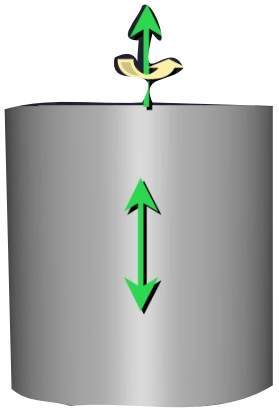
Sample discriminative points

Local Covariance

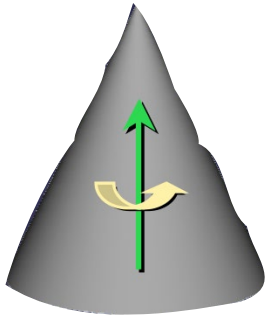


3 small eigenvalues
2 translation
1 rotation

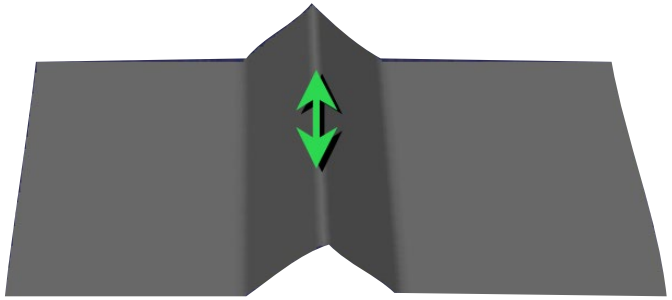
3 small eigenvalues
3 rotation



2 small eigenvalues
1 translation
1 rotation



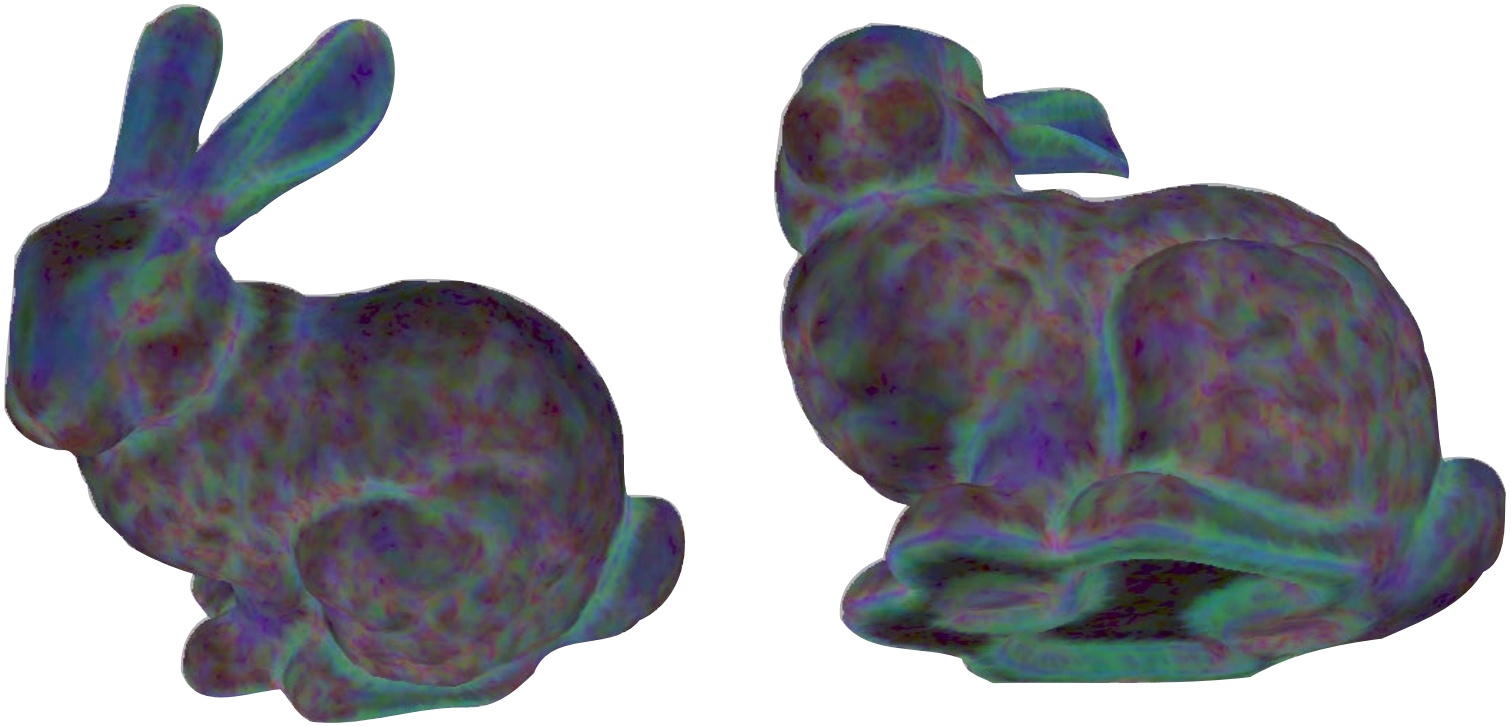
1 small eigenvalue
1 rotation



1 small eigenvalue
1 translation

[Gelfand et al. 2004]

Stability Analysis



Key:

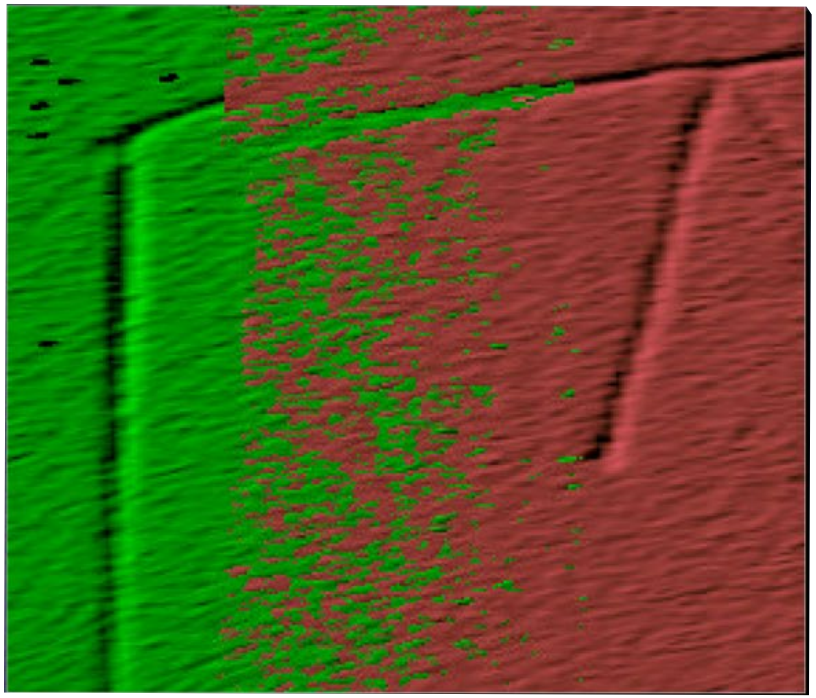
3 DOFs stable

5 DOFs stable

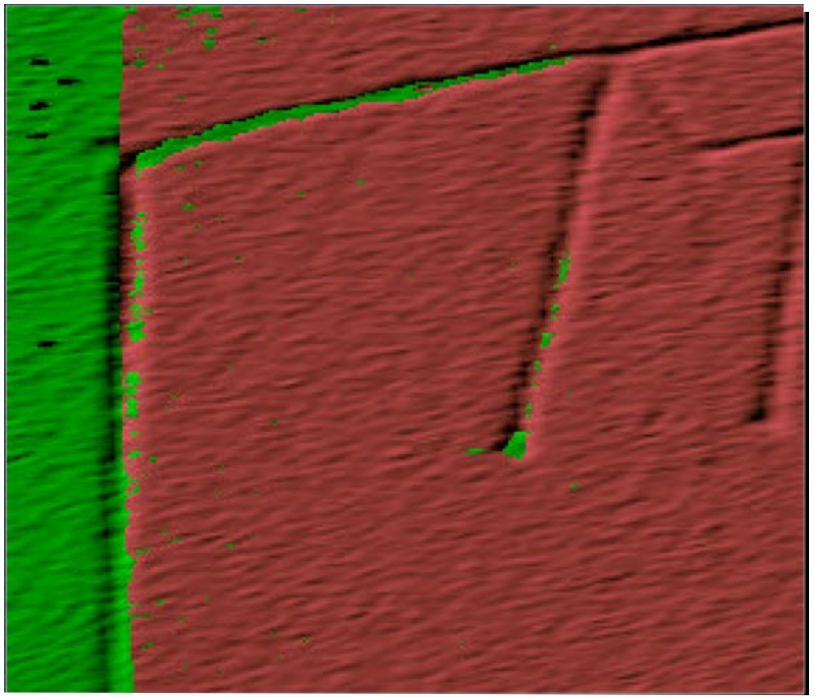
4 DOFs stable

6 DOFs stable

Alternative: Uniform Normals

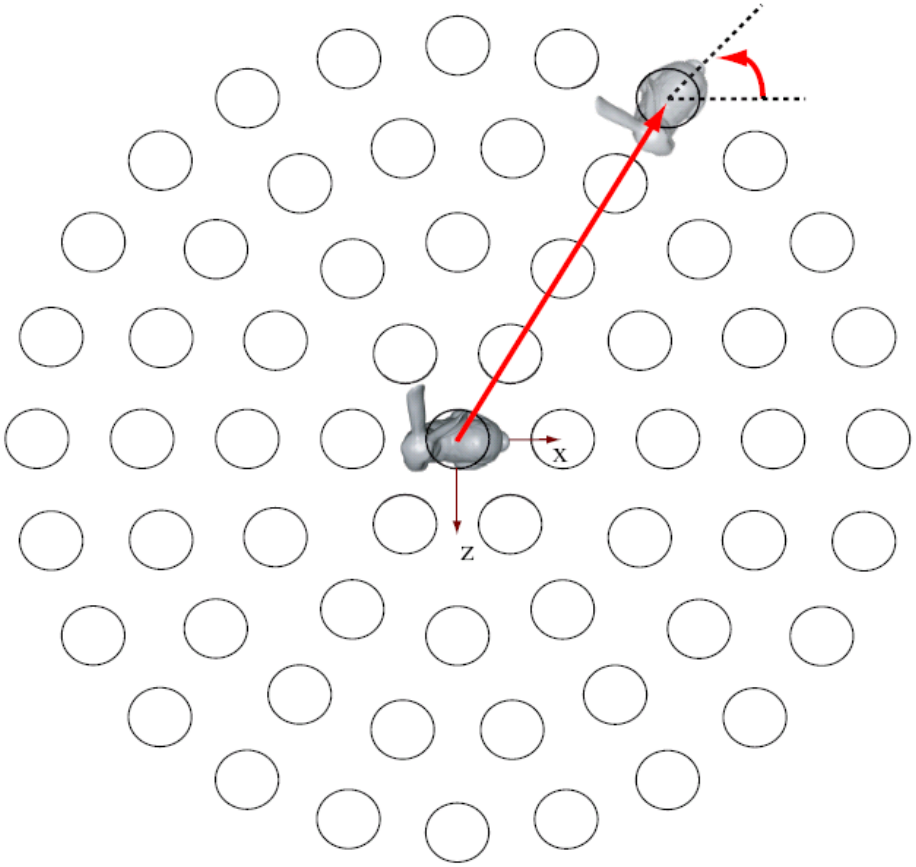


Random Sampling

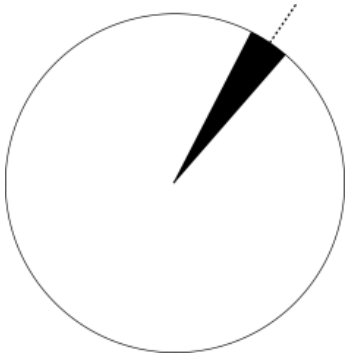


Normal-space Sampling

Convergence Funnel Visualization



Translation in xz plane
Rotation about y

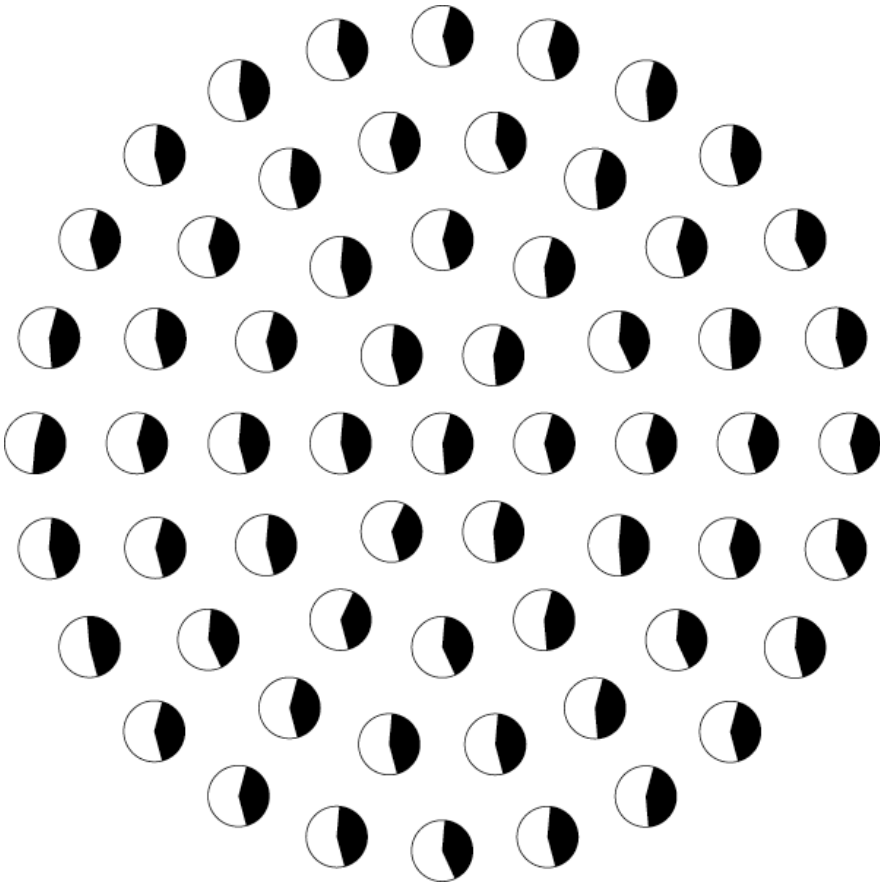


Converges

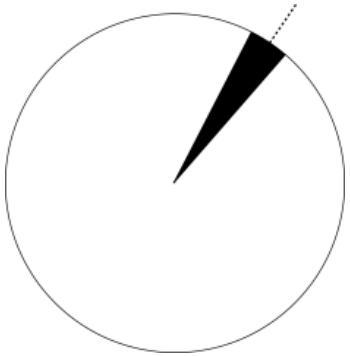
Does not converge

Slide courtesy N. Mitra

Distance Field Method



Translation in xz plane
Rotation about y

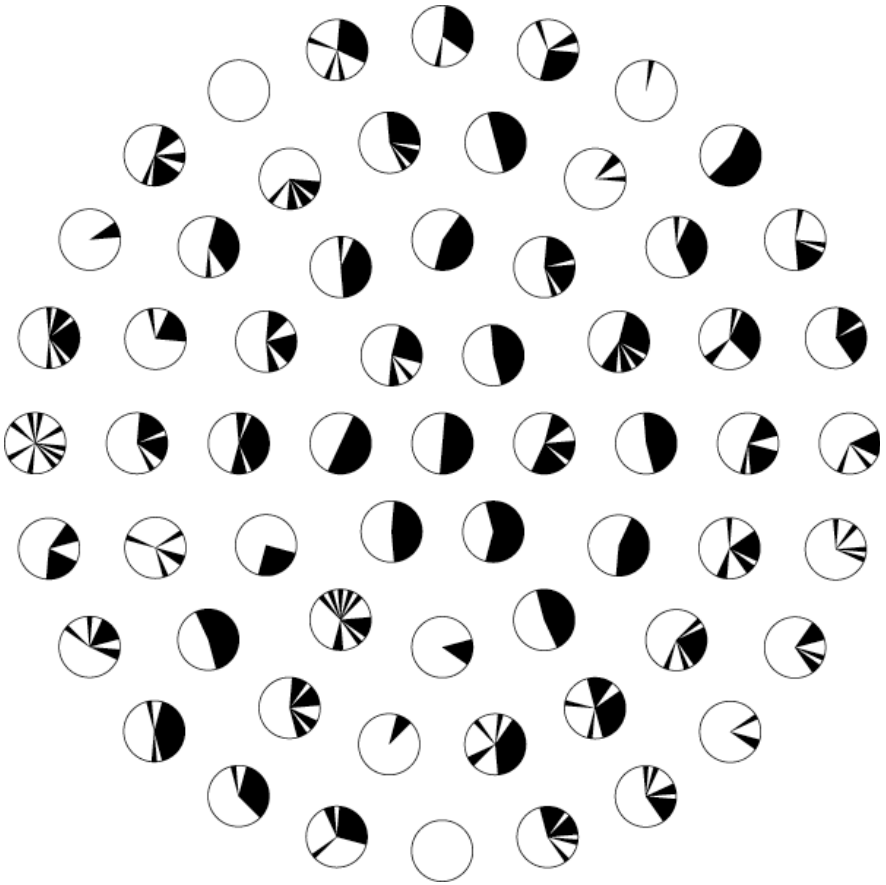


Converges

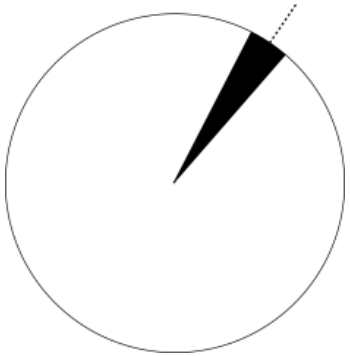
Does not converge

Slide courtesy N. Mitra

Point-to-Plane



Translation in xz plane
Rotation about y

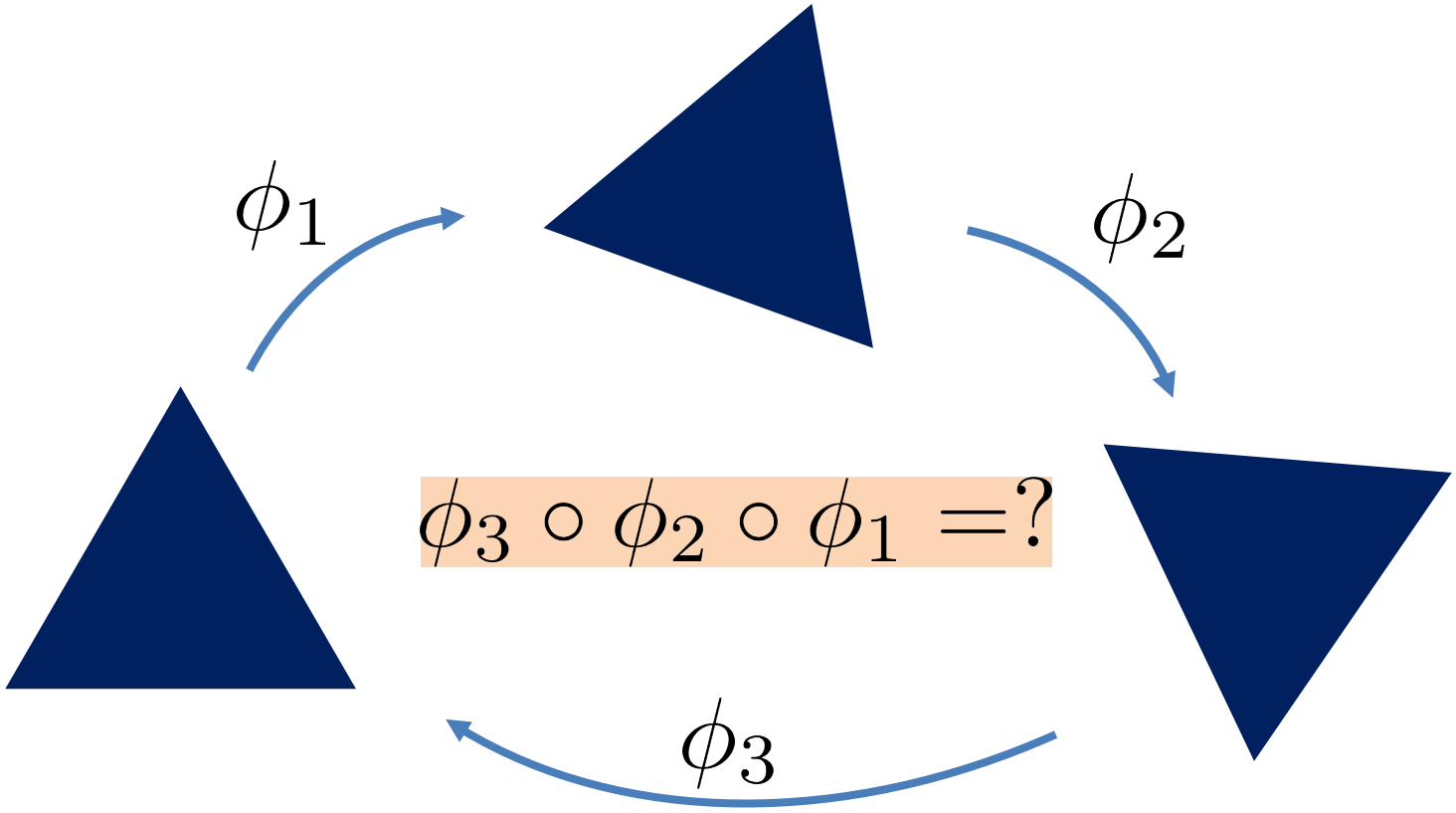


Converges

Does not converge

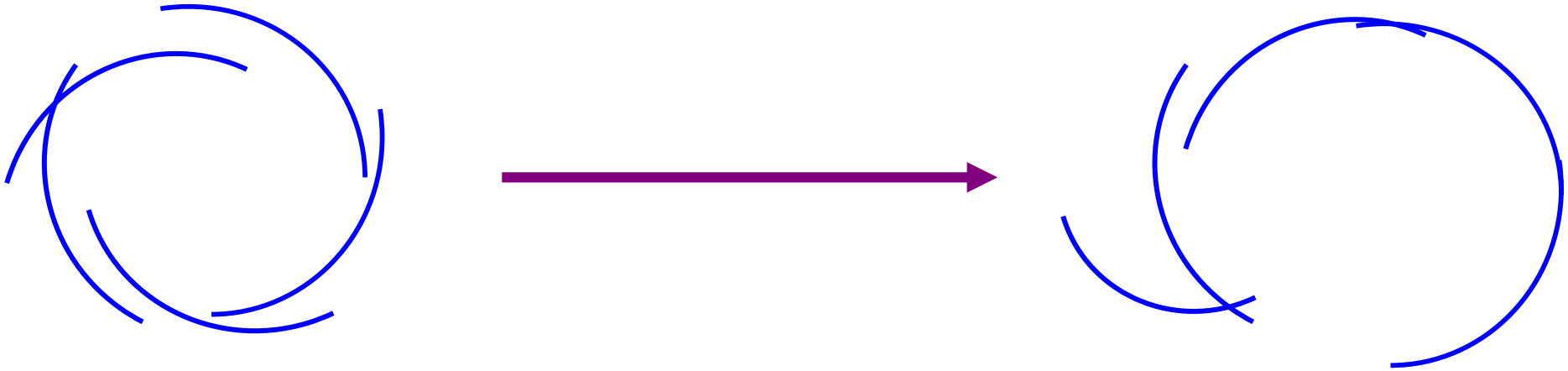
Slide courtesy N. Mitra

Issue: ICP Three Times



Usually have $\geq$ 2 scans

Improve Sequential Alignment?

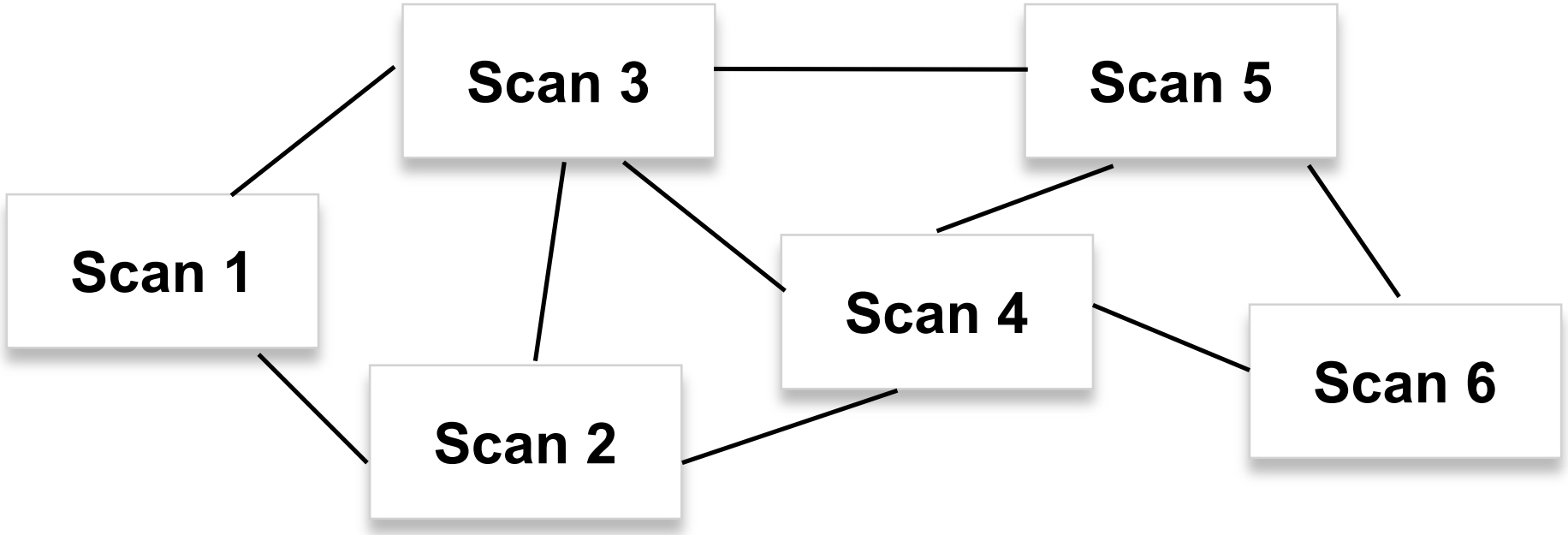


Prevent “drift”

Simple Methods

- **Align everything to anchor scan**
  Which to choose? Dependence on anchor?
- **Align to union of previous scans**
  Order dependence? Speed?
- **Simultaneously align everything using ICP**
  Local optima? Computational expense?

Graph Approach



Align similar scans, then assemble

Lu and Milios

- **Pairwise phase**

Compute pairwise ICP on graph

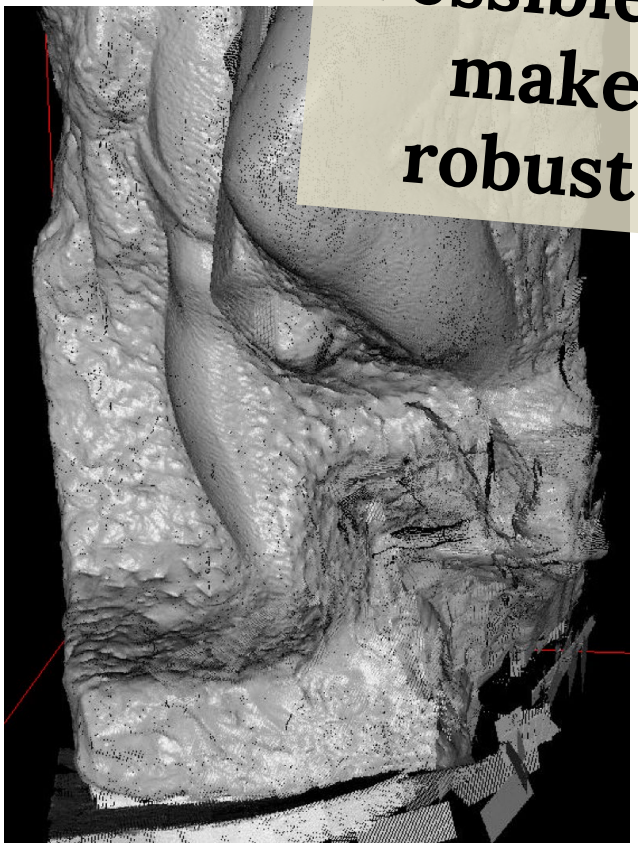
- **Global alignment**

Least-squares rotation/translation

Linearize for global alignment

Failed ICP in Global Registration

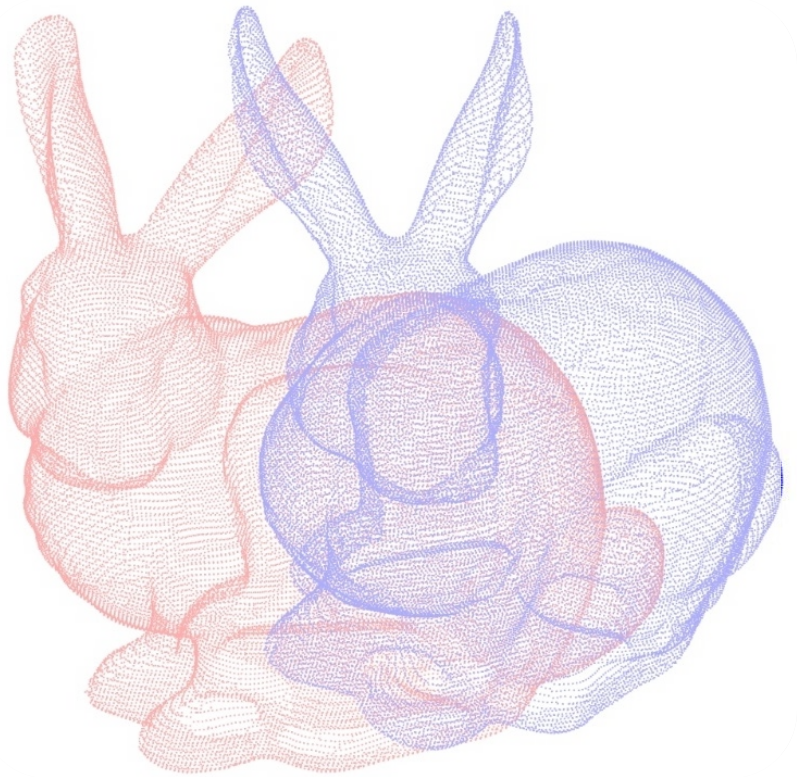
Correct global registration



Global registration including bad ICP

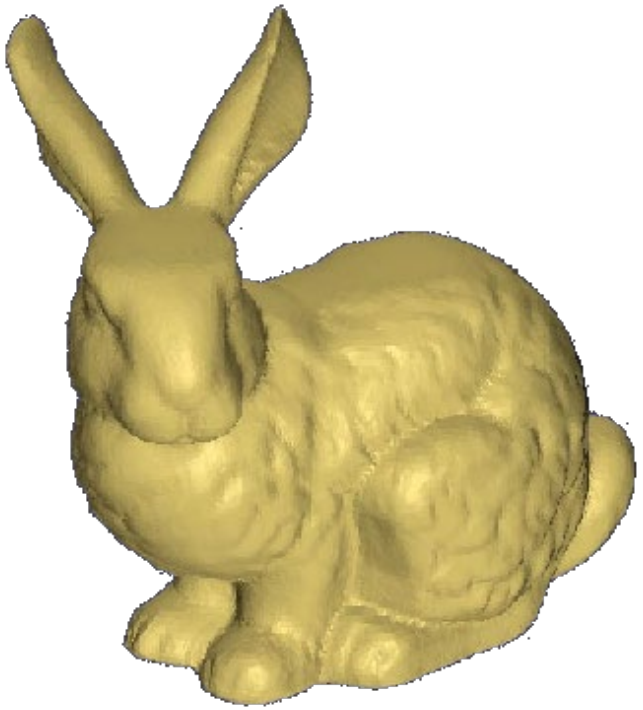
Possible to make robust?

Two components



https://i.ytimg.com/vi/uzOCS_gdZuM/maxresdefault.jpg

Registration



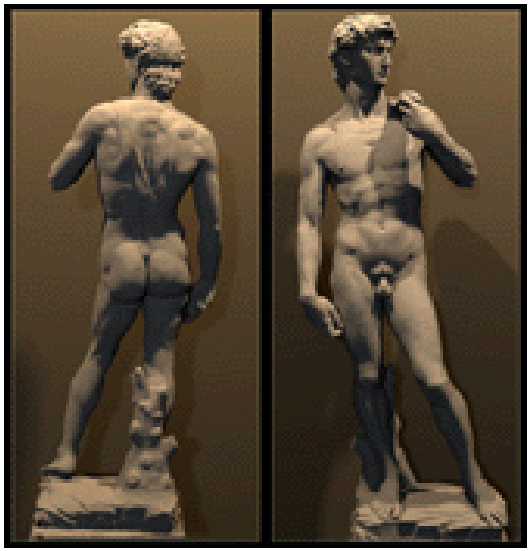
Meshing

Triangulating Point Clouds

Connect neighboring points into triangles

Point cloud data

Reconstructed surface



Triangulating Point Clouds

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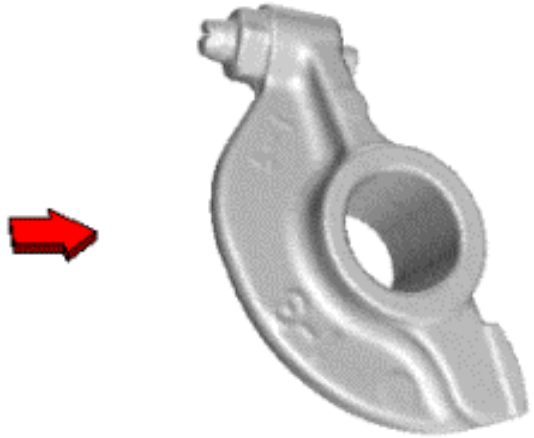
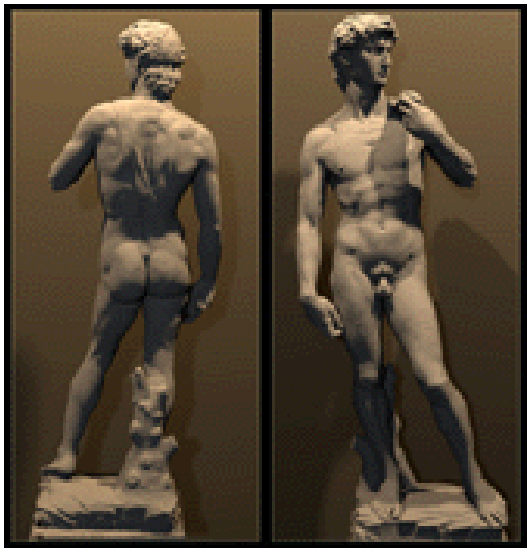


Reconstructed surface



Who are the neighbors?

⇔ What's the connectivity/topology

Methods

- Explicit, *or reconstruction circa 1998*
  - Zippering
  - Delaunay/Voronoi-based
- Implicit
  - Signed distance function
  - Poisson
- Data-driven

Methods

- Explicit, *or reconstruction circa 1998*
  - **Zippering**
  - Delaunay/Voronoi-based
- Implicit
  - Signed distance function
  - Poisson
- Data-driven

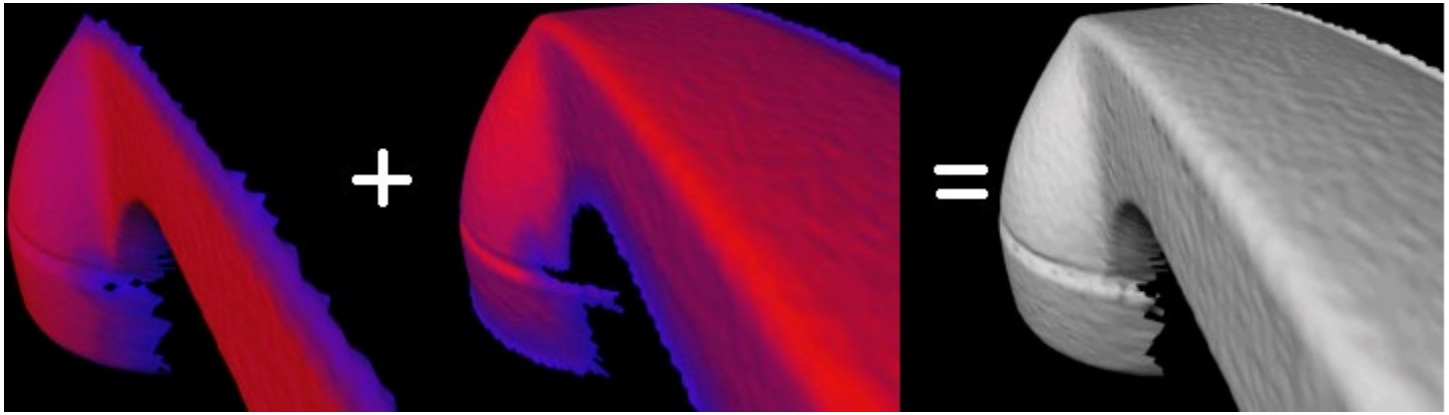
Basic Reconstruction: Zippering

Single scan → mesh

- regular lattice of points in X and Y with changing depth (Z) = height map

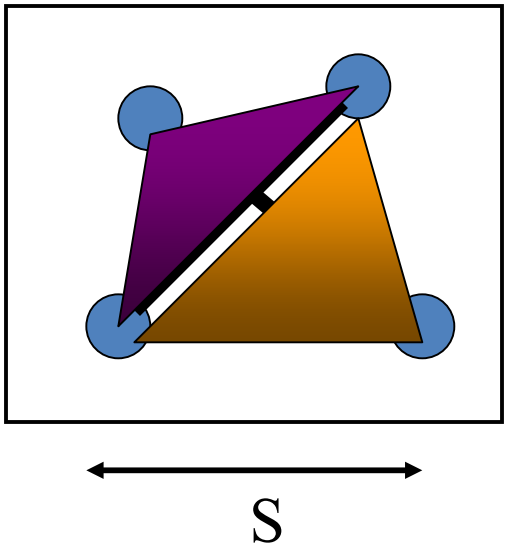
Register

Merge meshes



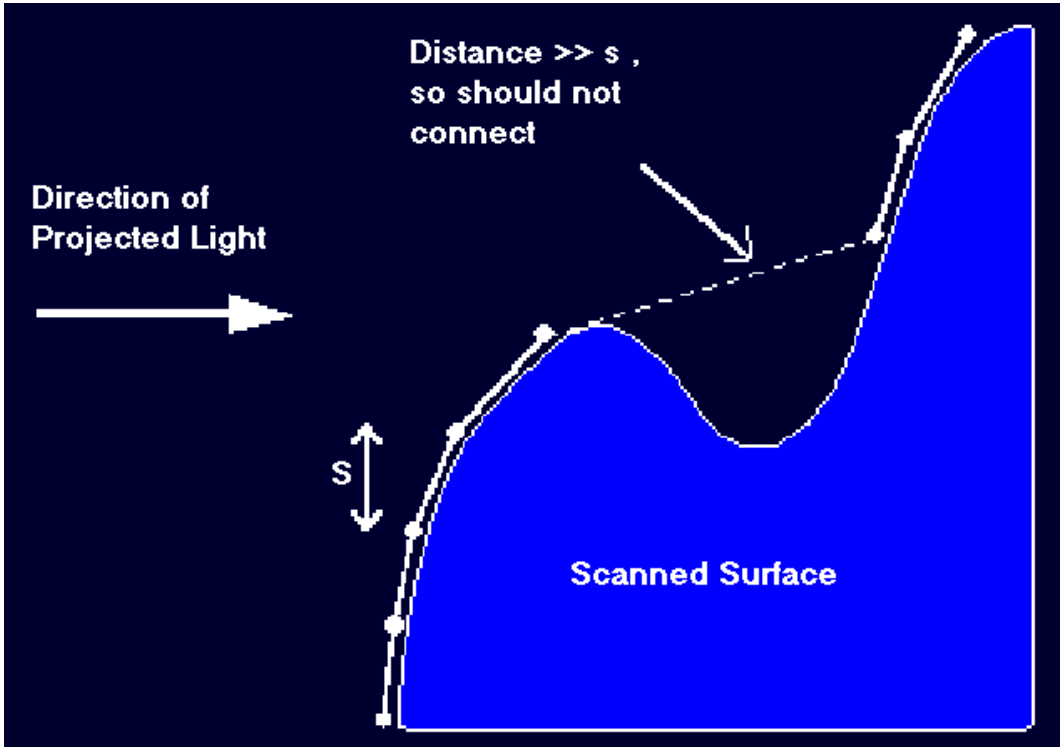
One scan → mesh

- Find quadruples of lattice points
- Form triangles
  - Find shortest diagonal
  - Form two triangles (test depth)



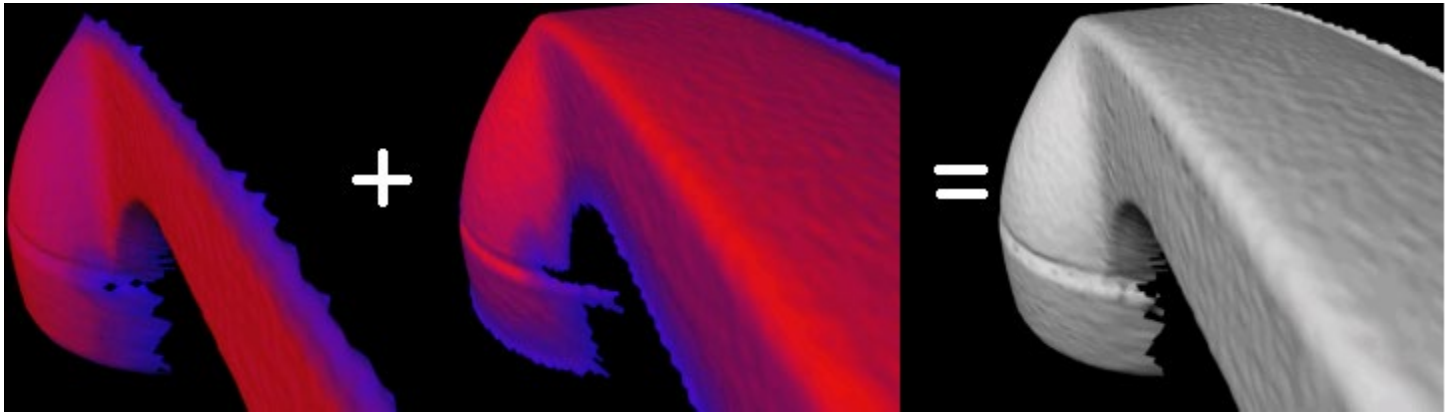
One scan → mesh

Avoid connecting depth discontinuities



Basic Reconstruction: Zippering

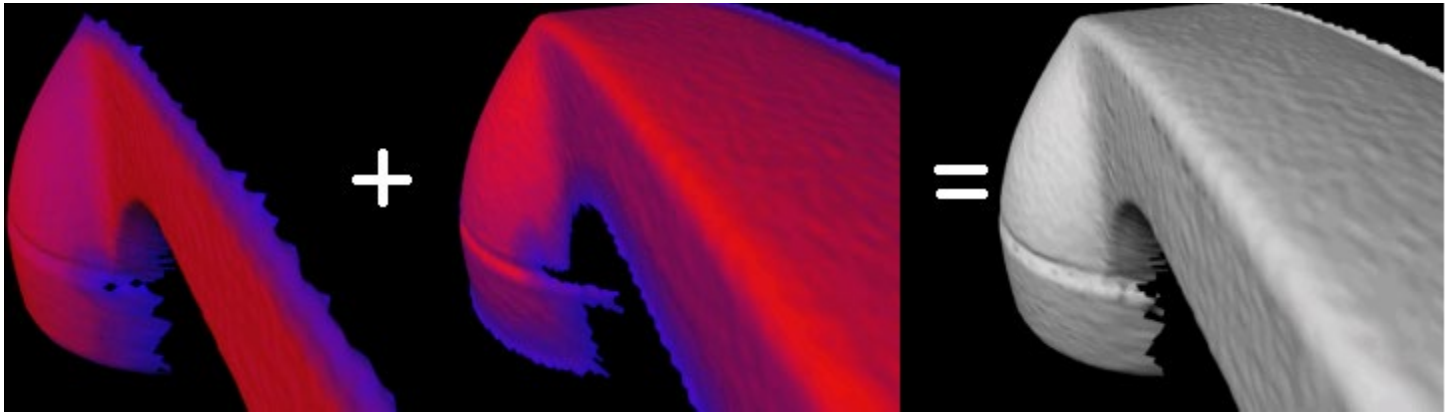
- ✔ Single scan → mesh
  Register
  Merge meshes



Basic Reconstruction: Zippering

- ✔ Single scan → mesh
- ✔ Register

  Merge meshes

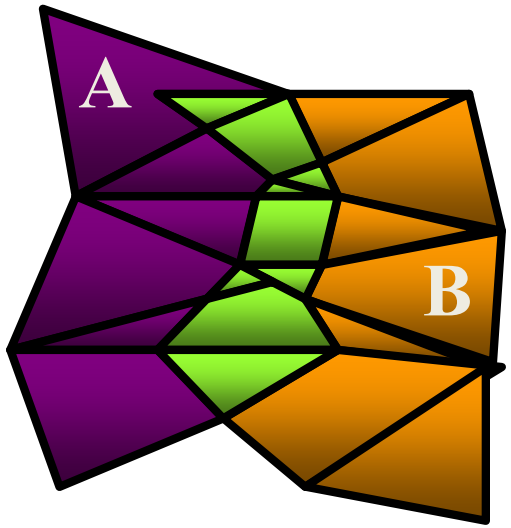


Merging

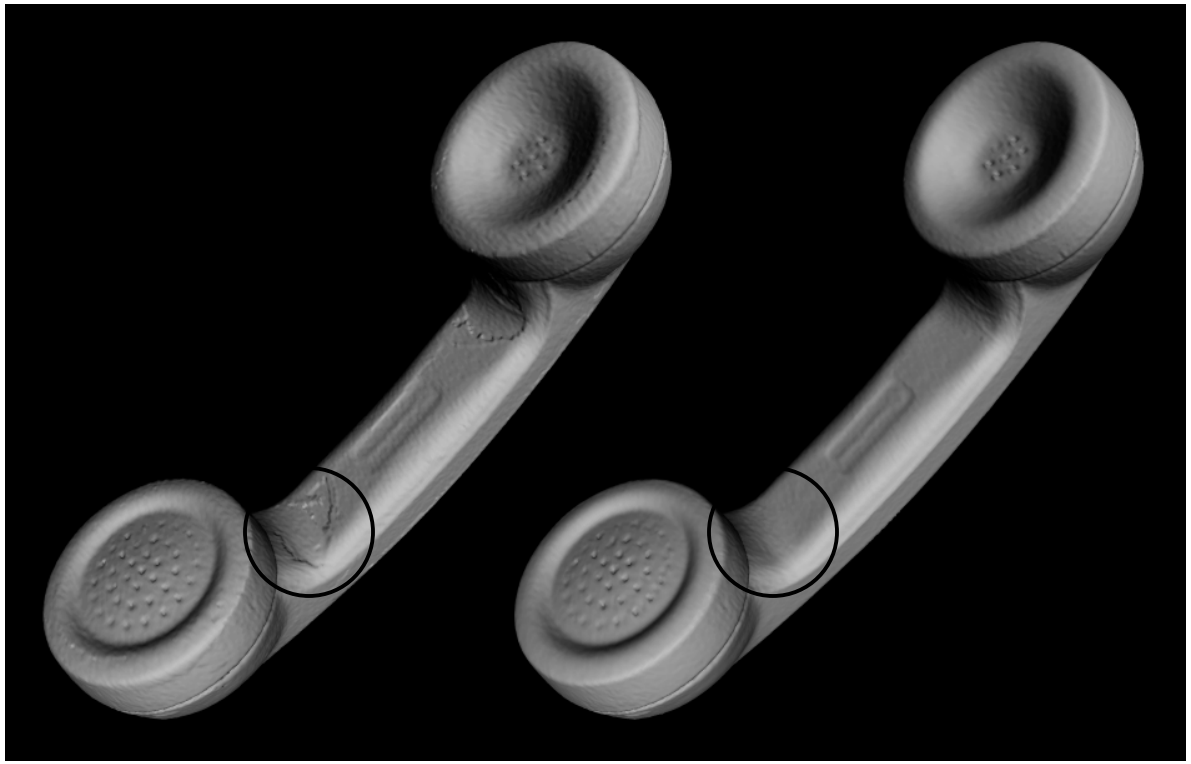
2 overlapping meshes

Zippering

- Remove overlapping portion of the mesh
  - Use for *consensus geometry*
- Clip one mesh against another
- Remove triangles introduced during clipping



Post-processing



Zippering results

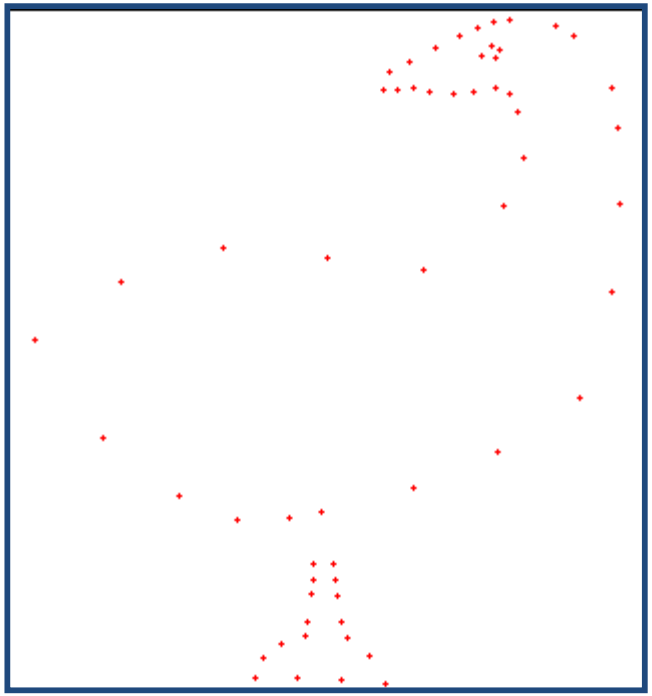
‘Consensus geometry’

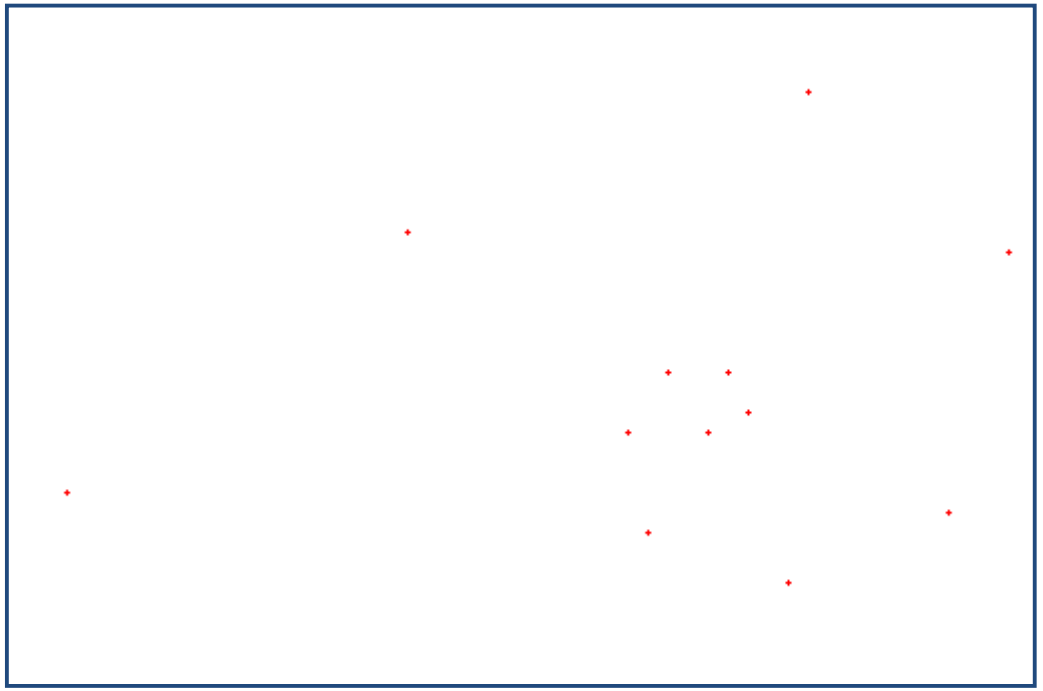
Move vertices to their average positions over all scans

Methods

- Explicit, *or reconstruction circa 1998*
  - Zippering
  - **Delaunay/Voronoi-based**
- Implicit
  - Signed distance function
  - Poisson
- Data-driven

2D: connect the dots

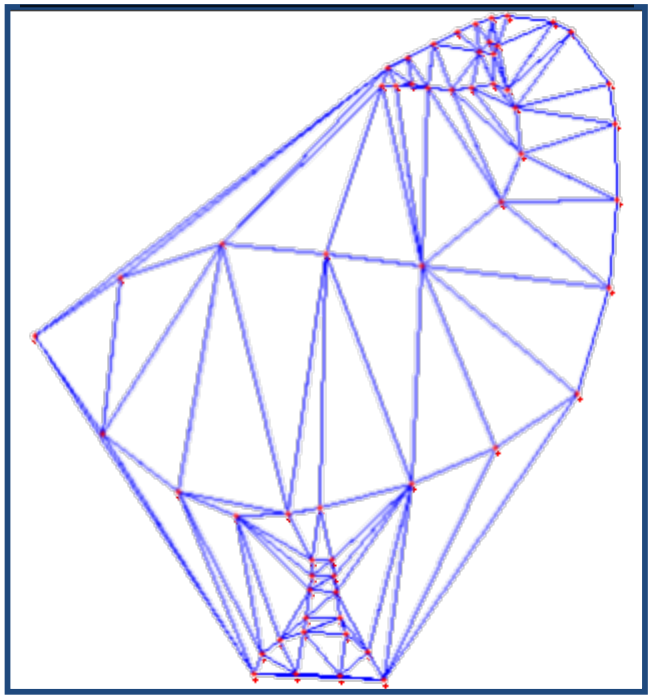


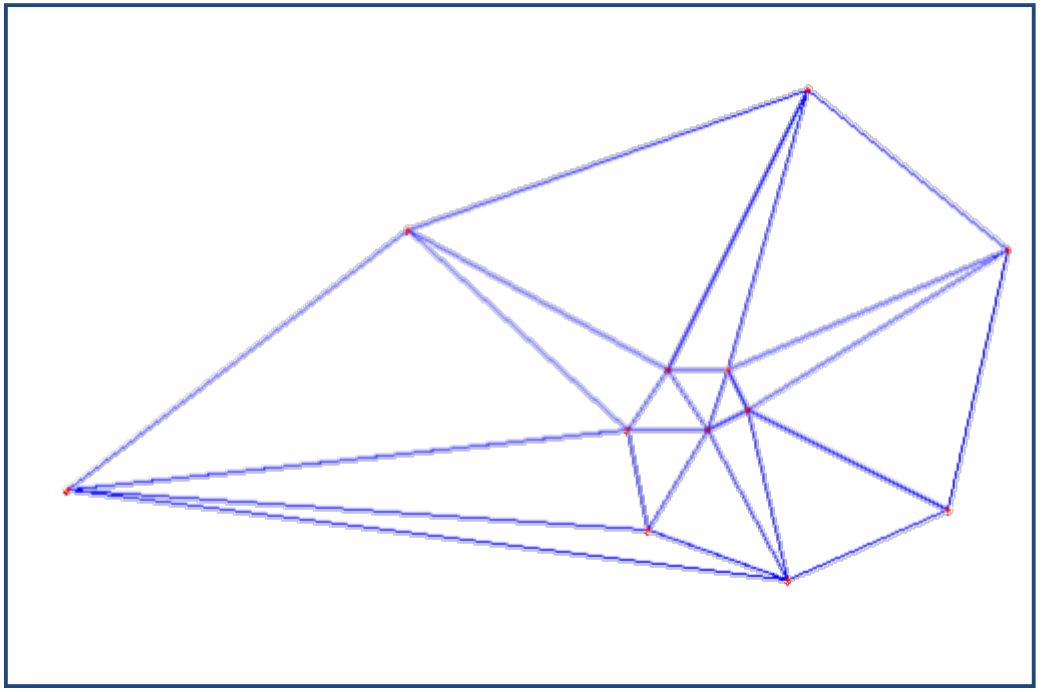


Connectivity?

Edges should be far from other points

2D: connect the dots

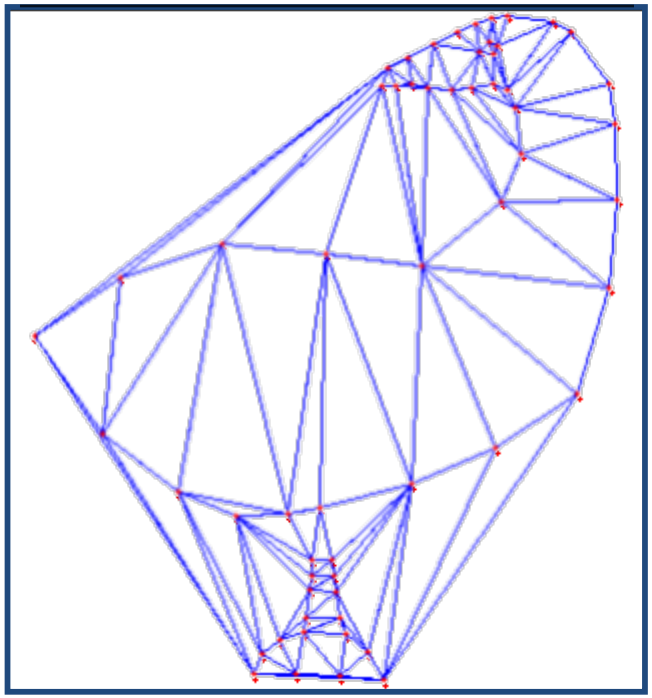


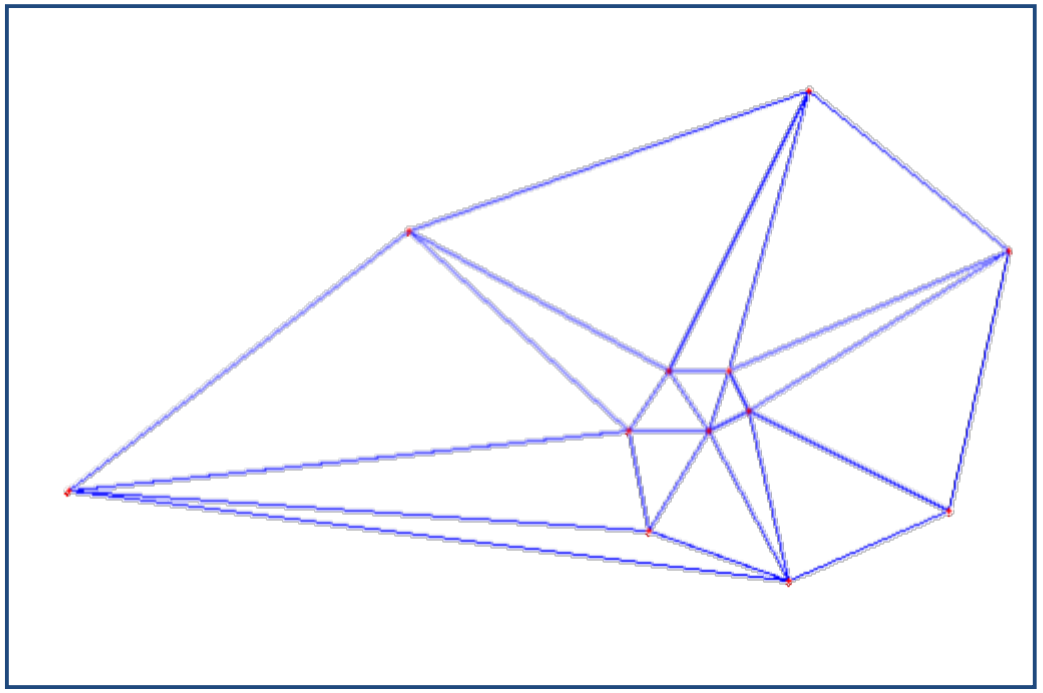


Delaunay Triangulation

Edge e is Delaunay $\Leftrightarrow$ some circumcircle of e contains no other sample points

2D: connect the dots

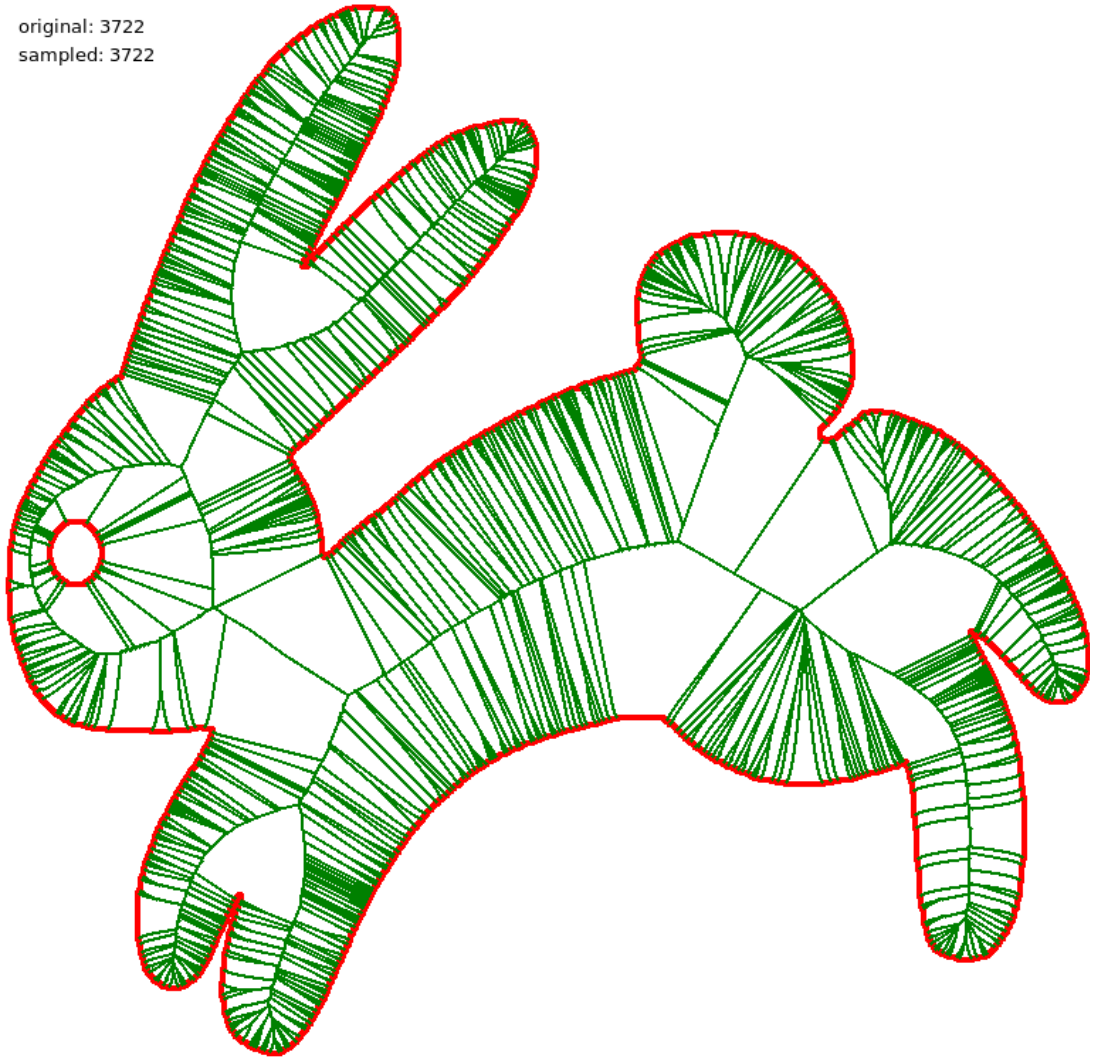




Which edges to pick?

Recall:

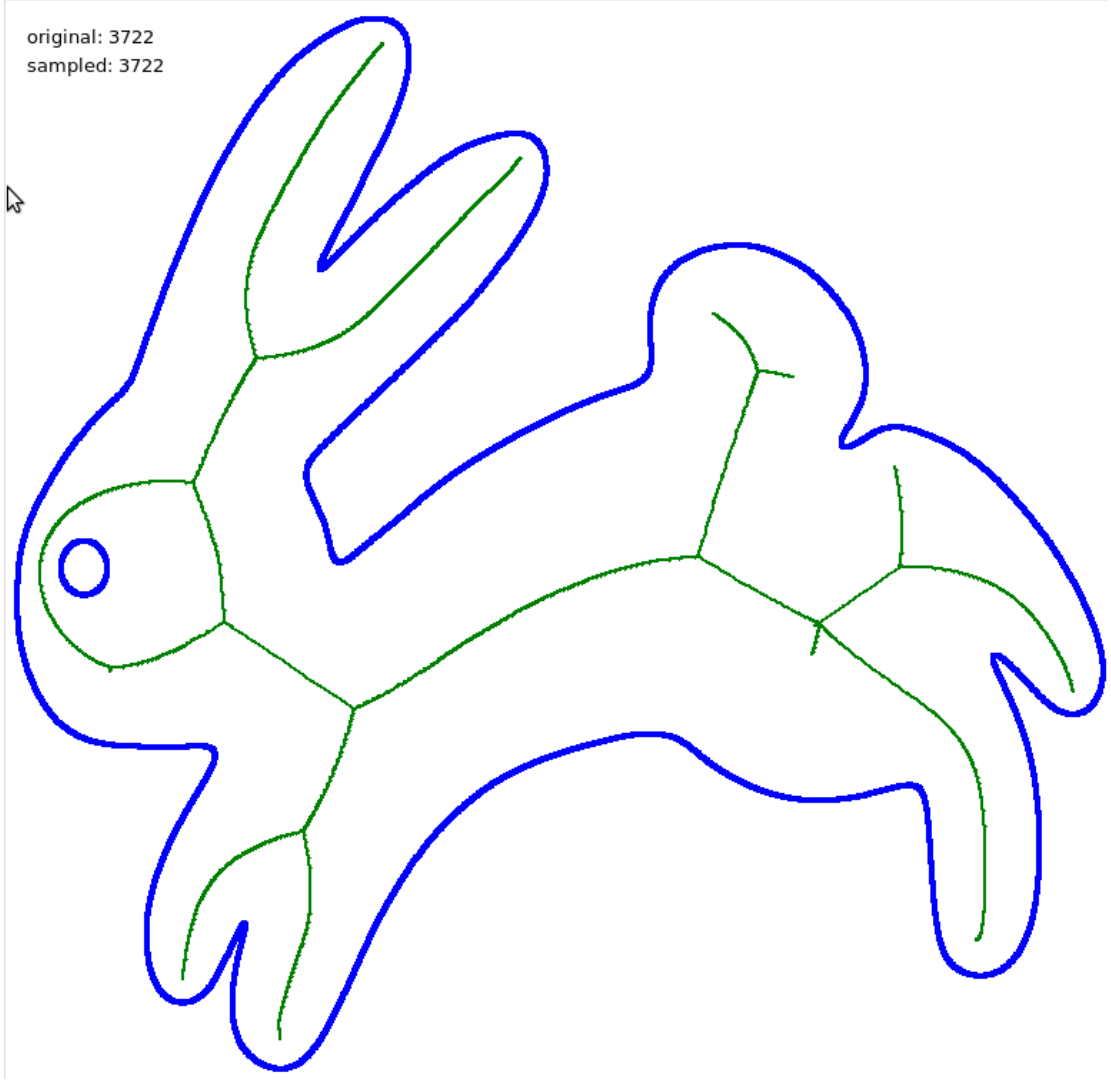
Medial axis vs Voronoi diagram



"On the Evaluation of the Voronoi-Based Medial Axis"
by Adriana Schulz, Francisco Ganacim and Leandro Cruz

Recall:

Medial axis vs Voronoi diagram



"On the Evaluation of the Voronoi-Based Medial Axis"
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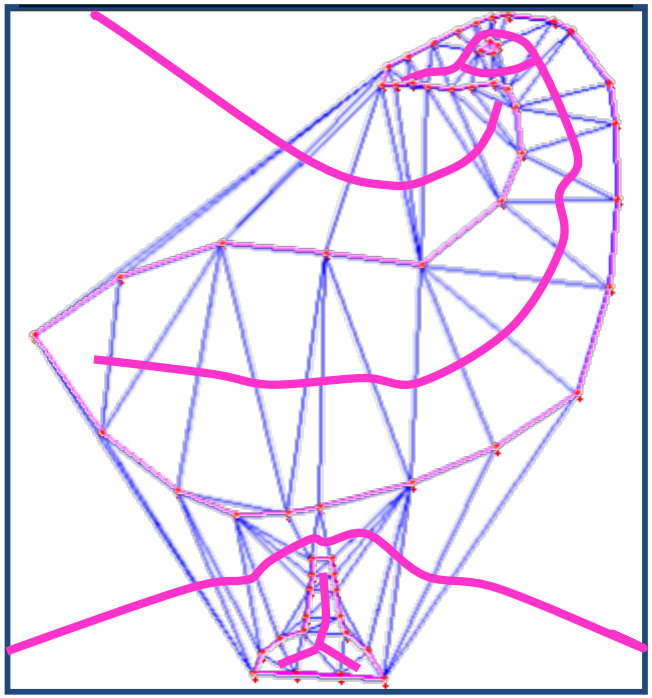
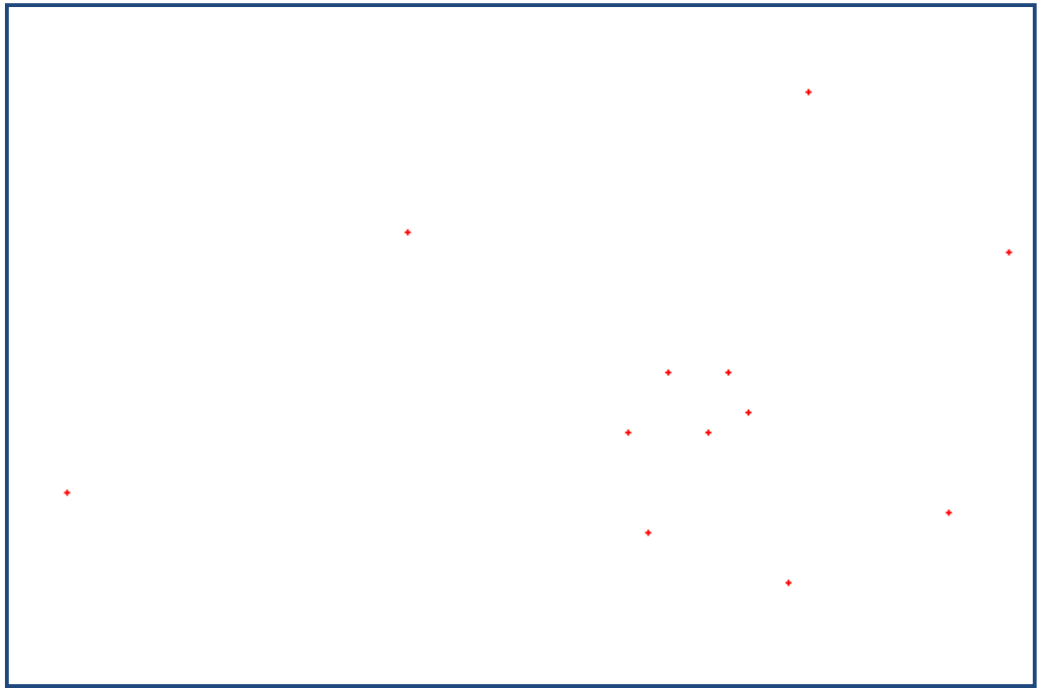
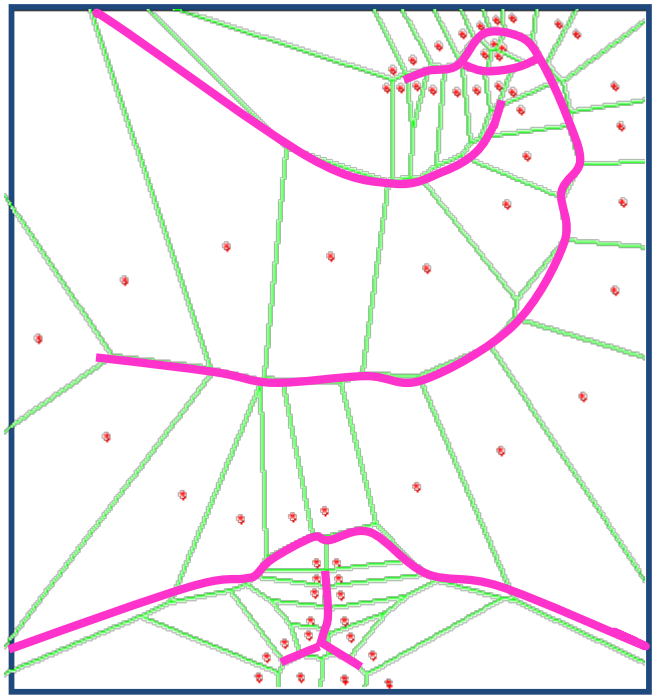
2D: connect the dots

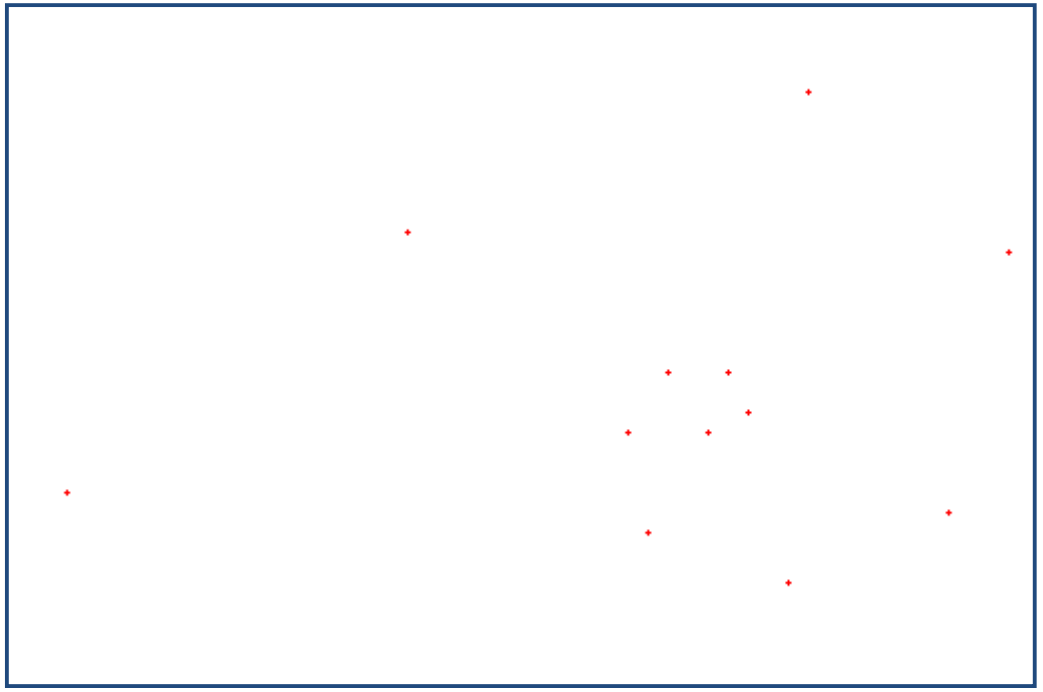




Edges should be “far” from Medial Axis

2D: connect the dots

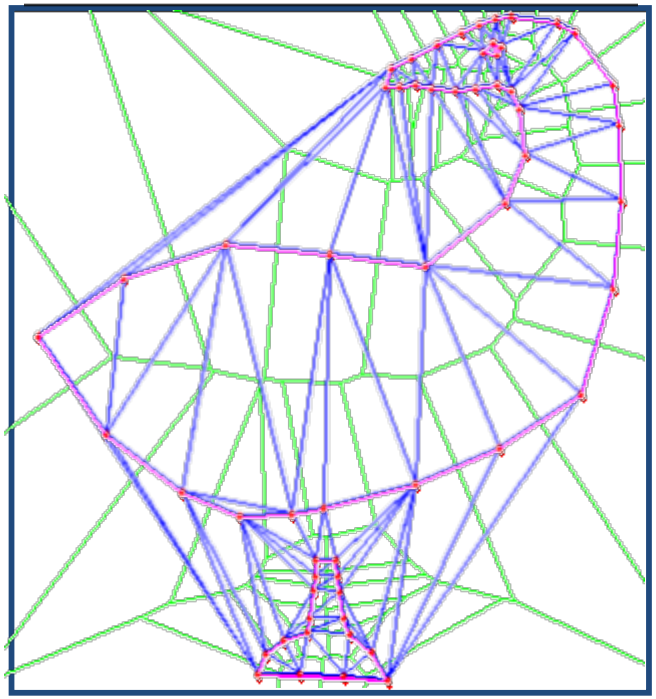


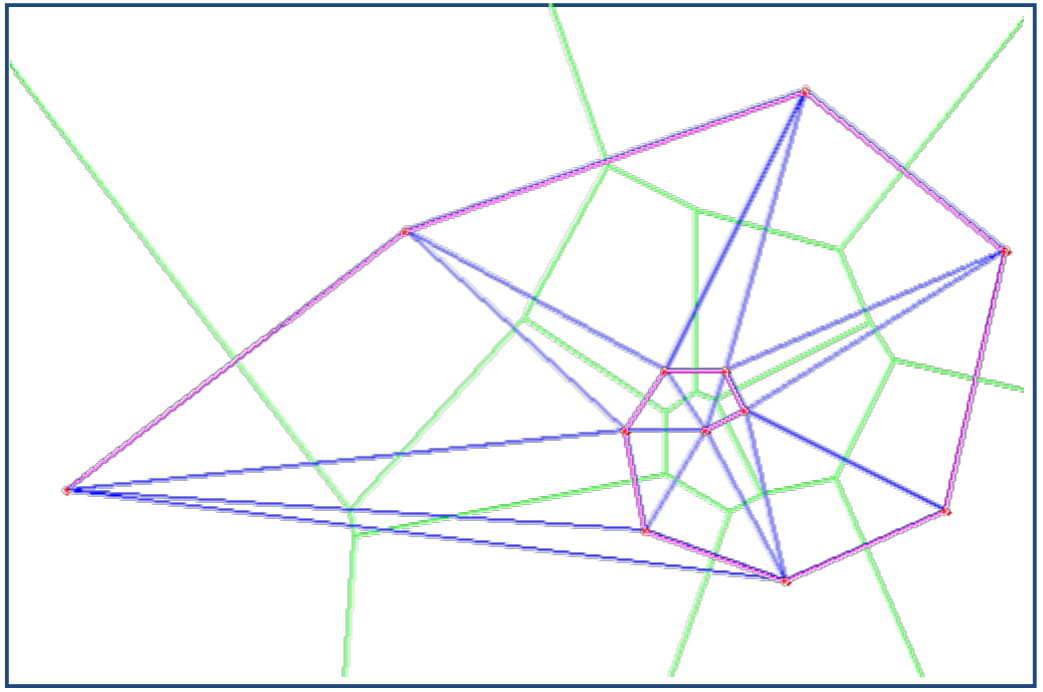


Voronoi diagram approximates Medial Axis

if points are sampled densely enough

2D: connect the dots

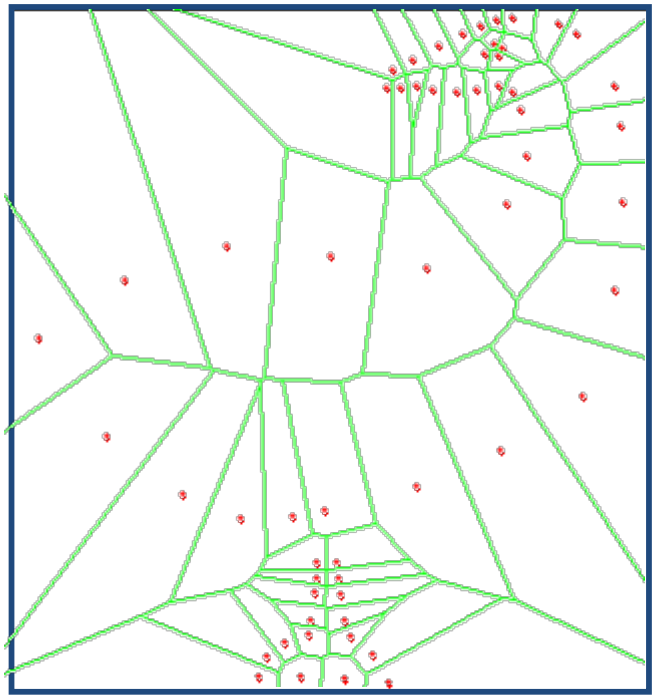


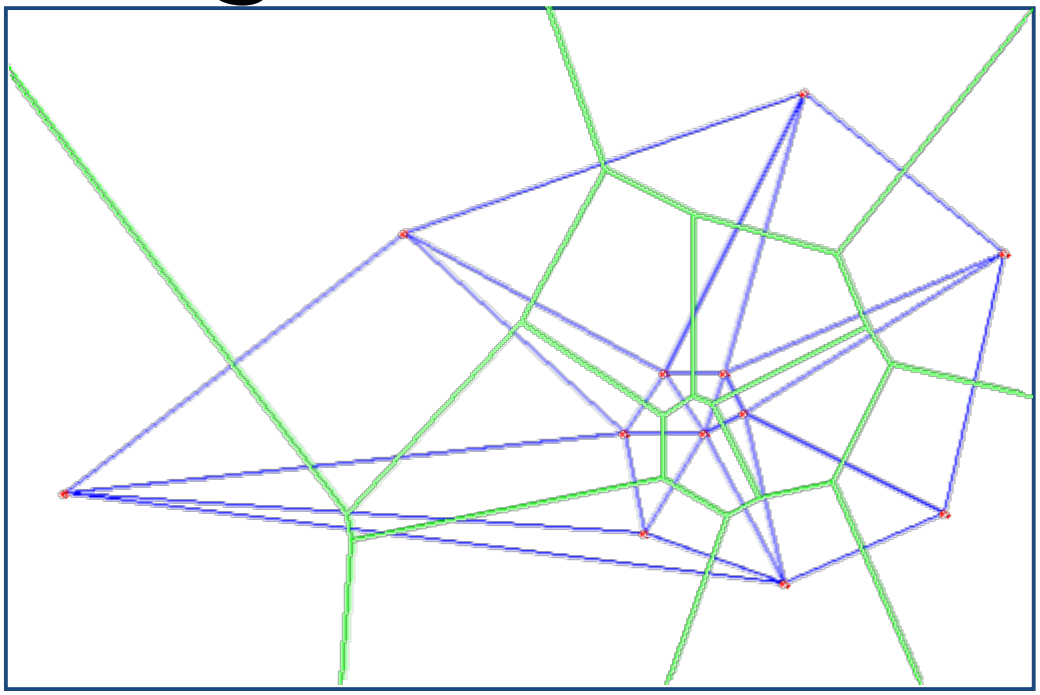


Edge e in **crust** $\Leftrightarrow$

a circumcircle of e contains no other sample points or Voronoi vertices of S

Crust: Algorithm

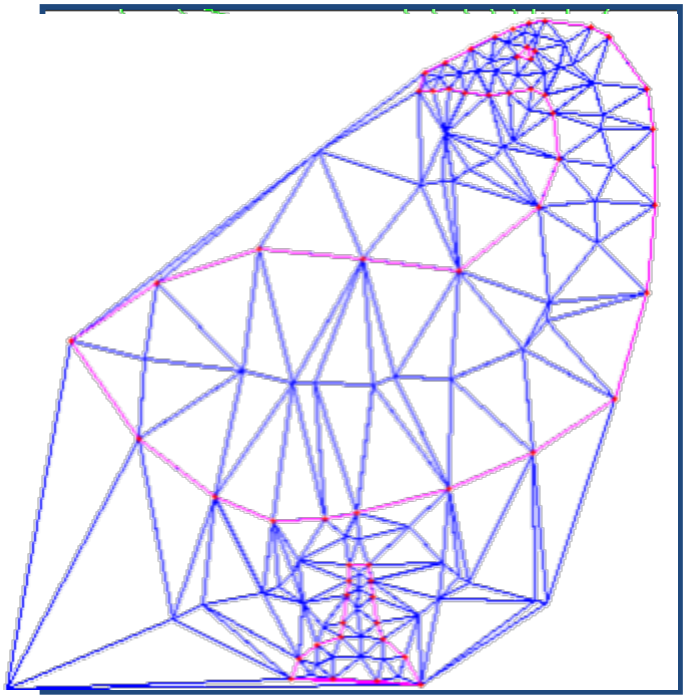


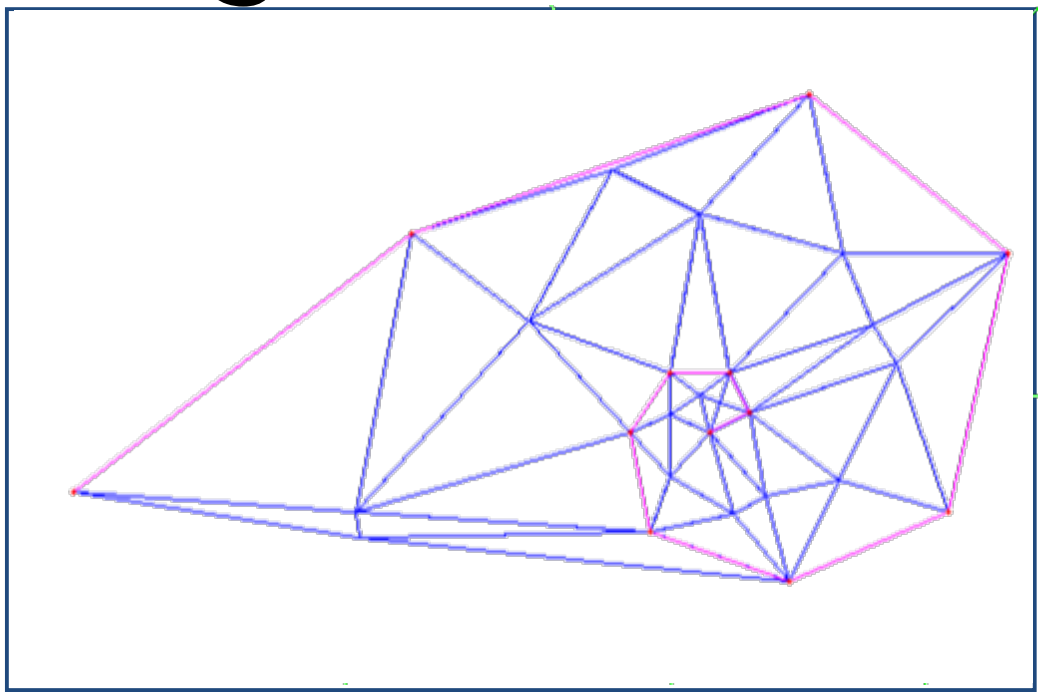


Compute Voronoi diagram of S

$V = \{\text{Voronoi vertices}\}$

Crust: Algorithm



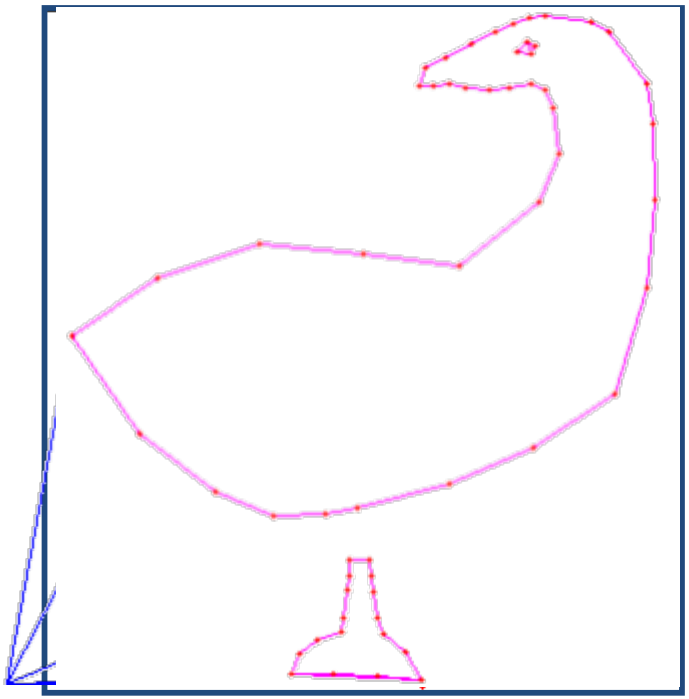


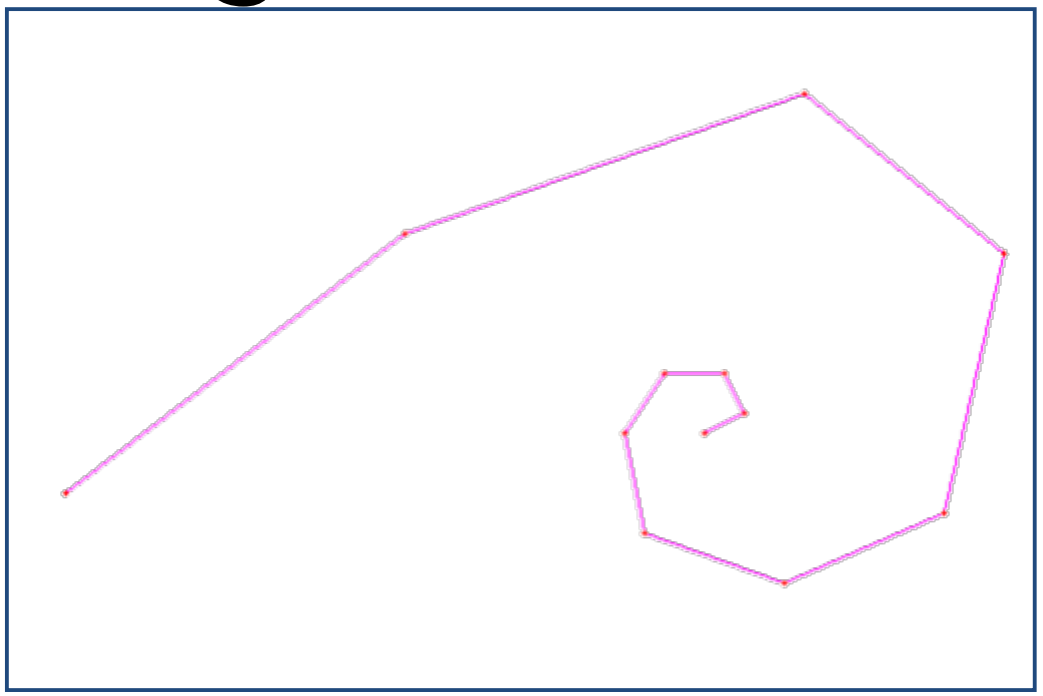
Compute Voronoi diagram of S

$$V = \{\text{Voronoi vertices}\}$$

Compute Delaunay Triangulation of $S \cup V$

Crust: Algorithm





Compute Voronoi diagram of S

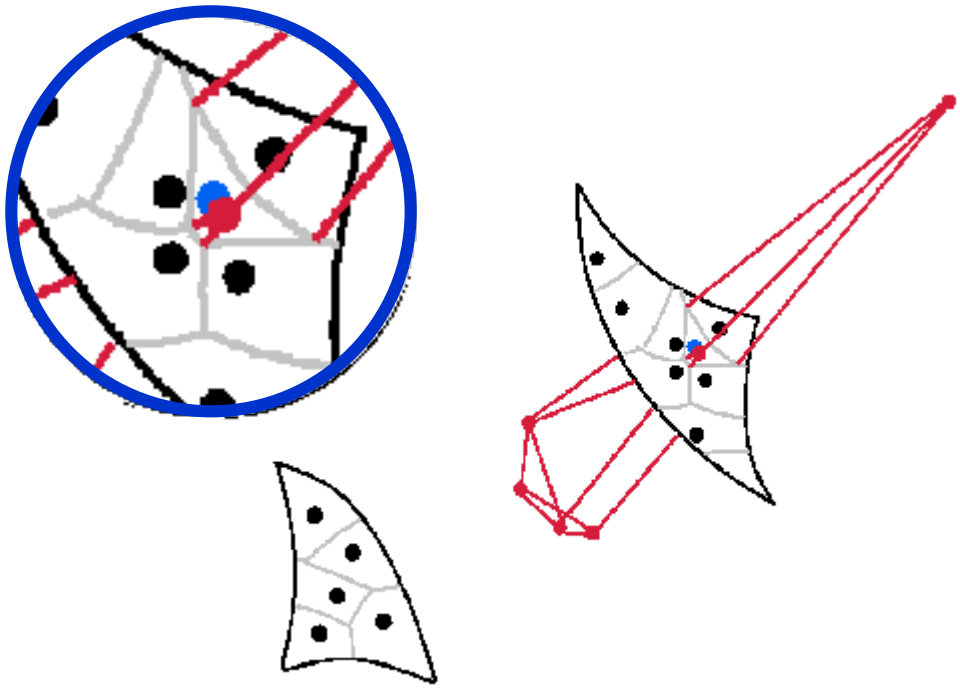
$$V = \{\text{Voronoi vertices}\}$$

Compute Delaunay Triangulation of $S \cup V$

Crust = all edges between points of S

3D Crust Algorithm

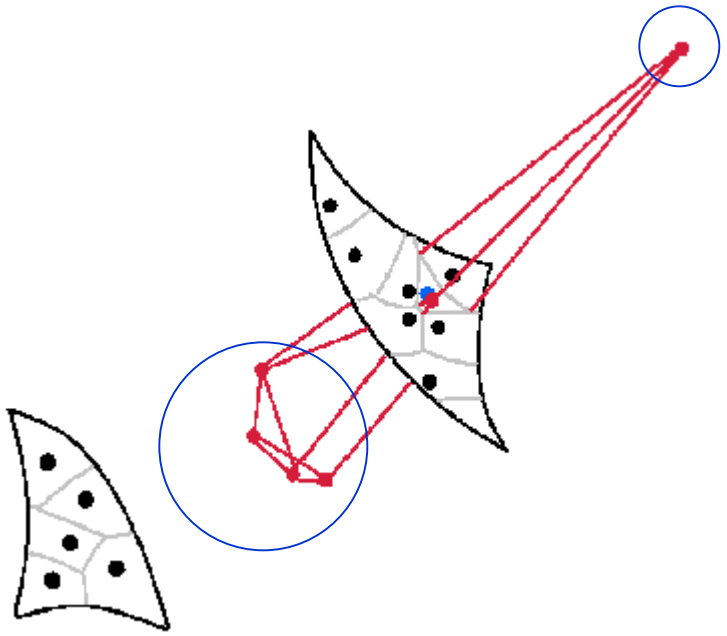
- Extend 2D approach
- Voronoi vertex is equidistant from 4 sample points
- BUT in 3D not all Voronoi vertices are near medial axis (regardless of sampling density)



3D Crust Algorithm

Some vertices of the Voronoi cell are near medial axis

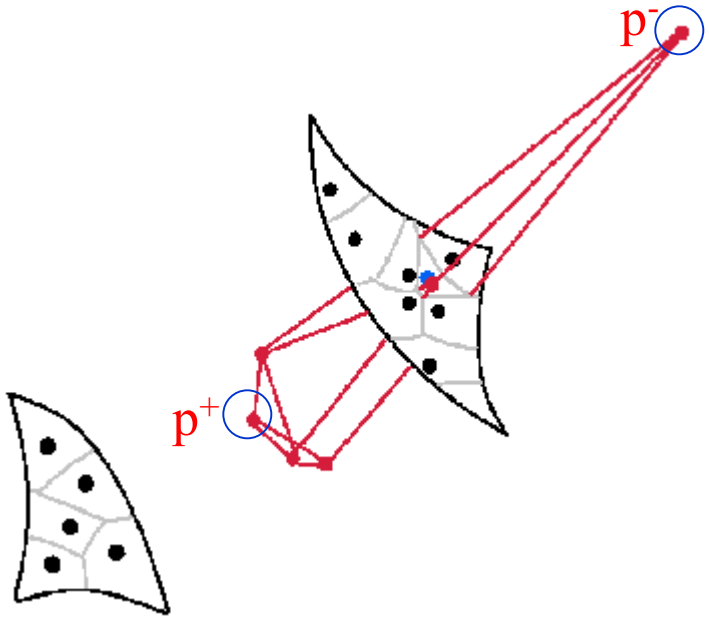
Intuitively – cell is closed not just from the sides but also from “top” & “bottom”



3D Crust Algorithm

Solution: use only two farthest vertices of V_s- one on each side of the surface

- Call vertices *poles* of s (p^+, p^-)



3D Crust Algorithm

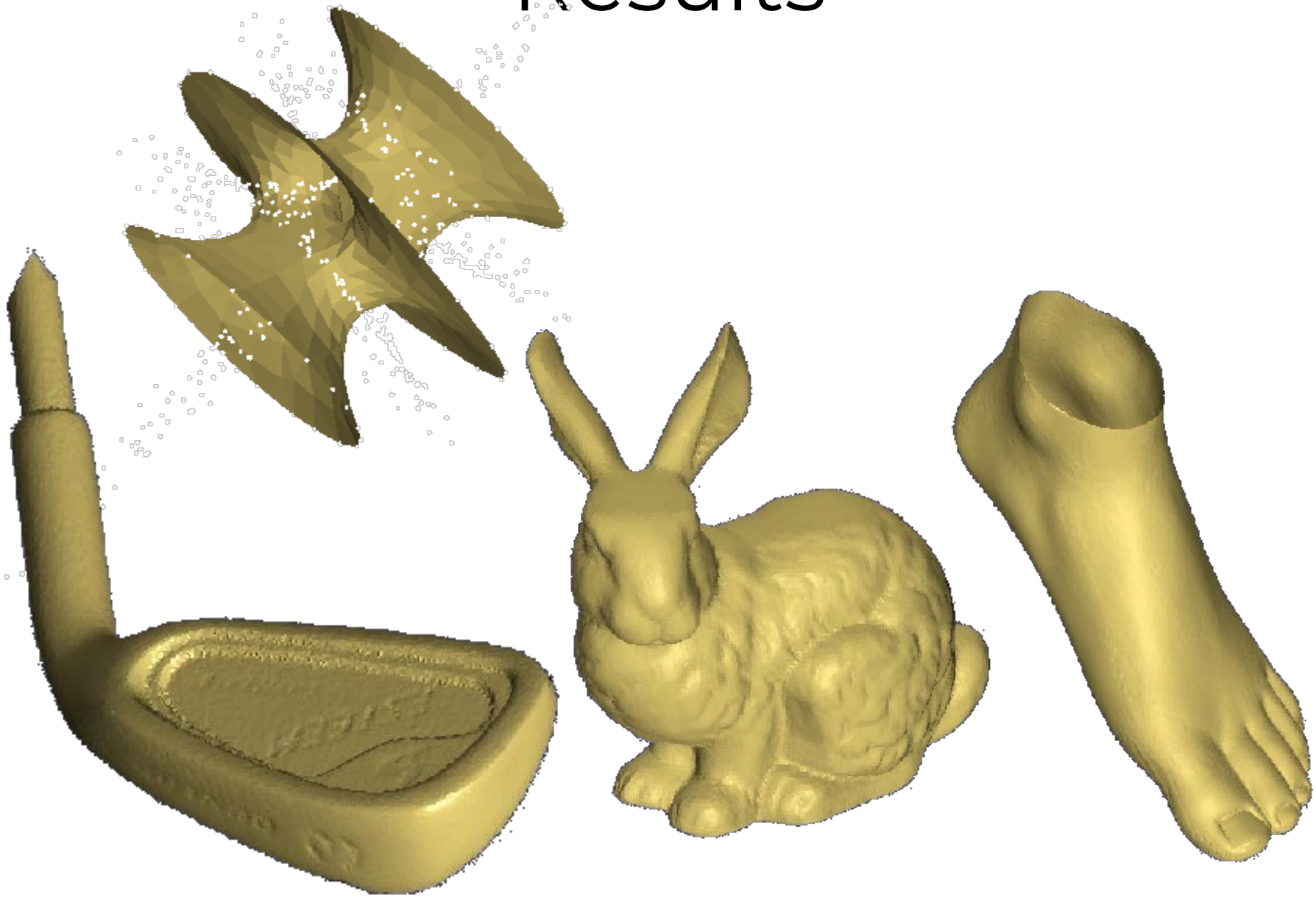
- Compute Voronoi diagram of S
- For each $s_i \in S$, compute

$$P = \{p_i^+, p_i^-\}$$

- Compute Delaunay triangulation T of $S \cup P$

Crust = all triangles in T with vertices in S

Results



Problems & Modifications

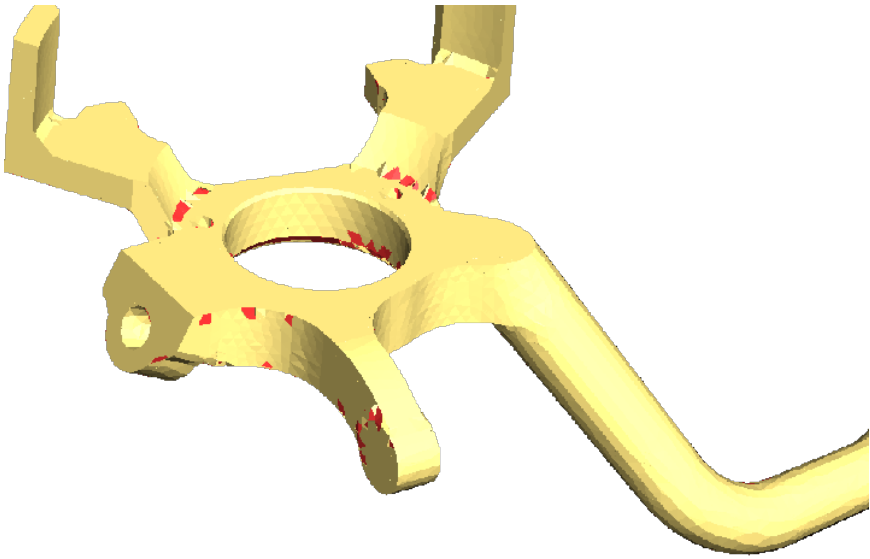
Correct in the absence of noise

Slow-*ish*

Need dense samples

Problems at sharp corners

Noise

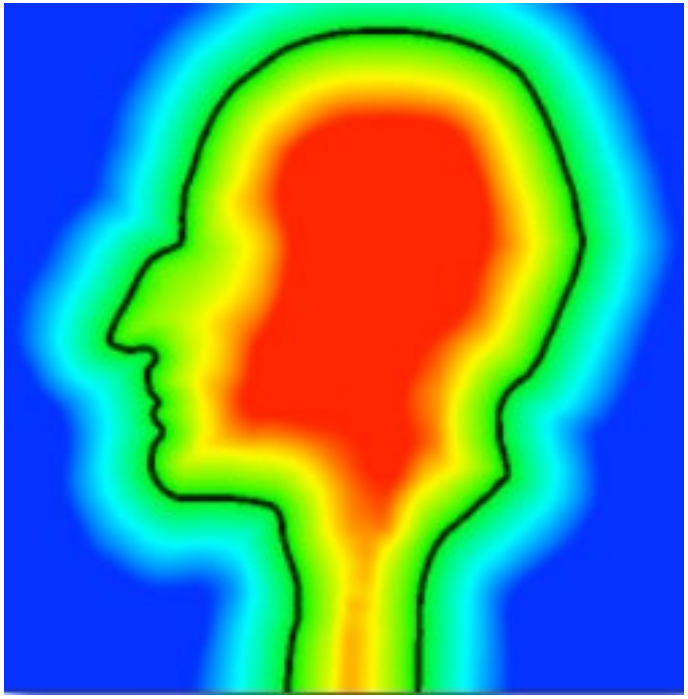


Methods

- Explicit, *or reconstruction circa 1998*
  - Zippering
  - Delaunay/Voronoi-based
- **Implicit**
  - Signed distance function
  - Poisson
- Data-driven

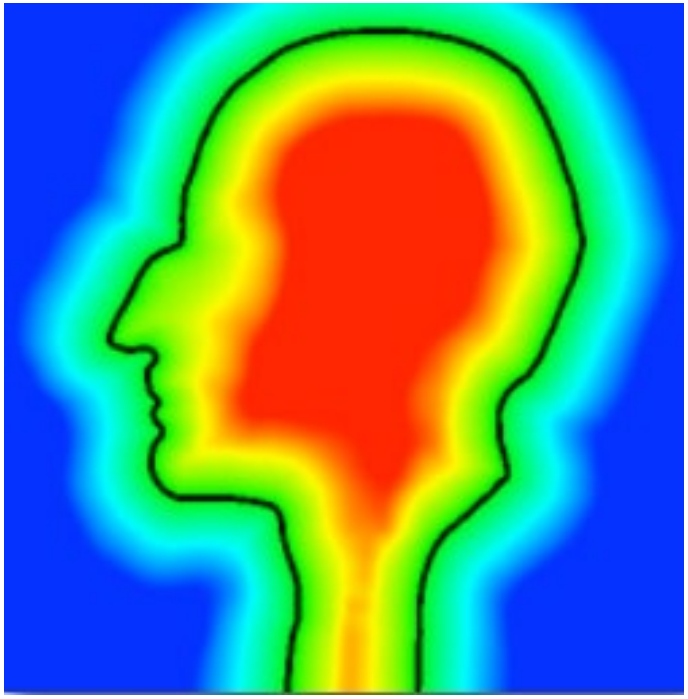
Implicit Reconstruction

1. Estimate signed distance function $d: \mathbb{R}^3 \rightarrow \mathbb{R}$
2. Extract an isosurface $d = 0$



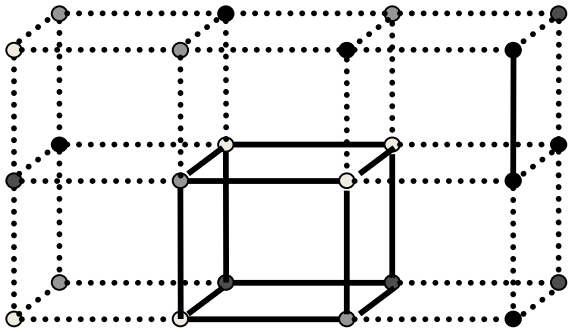
Implicit Reconstruction

1. Estimate signed distance function $d: \mathbb{R}^3 \rightarrow \mathbb{R}$
2. **Extract an isosurface $d = 0$**



Marching Cubes

- Each voxel:
  - Has values at 8 corners
  - Has 256 possible configurations
    - 15 after counting symmetries and rotations
  - Either
    - Inside isosurface
    - Outside isosurface
    - Intersects isosurface



- Can extract triangulation independently per voxel

Marching Cubes

For each *intersecting* voxel contains triangles of the isosurface

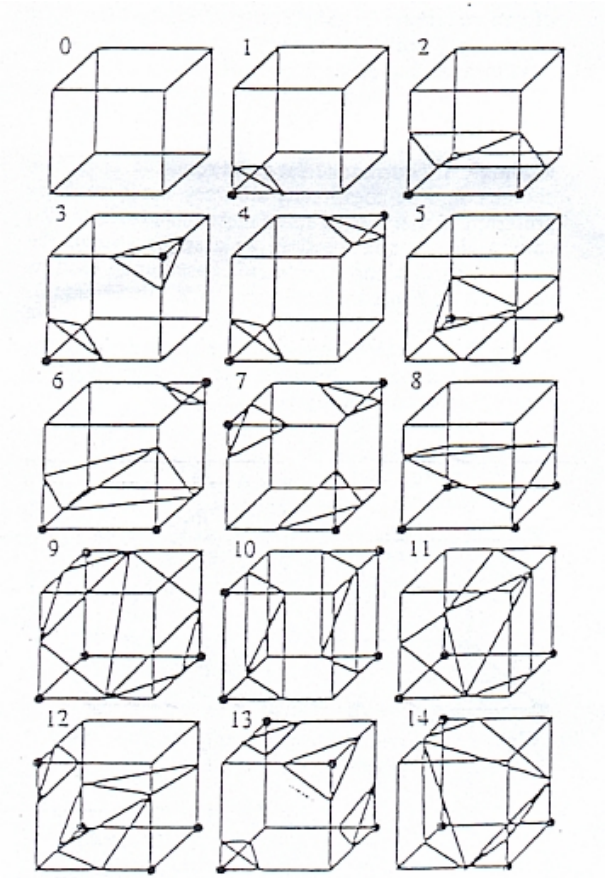
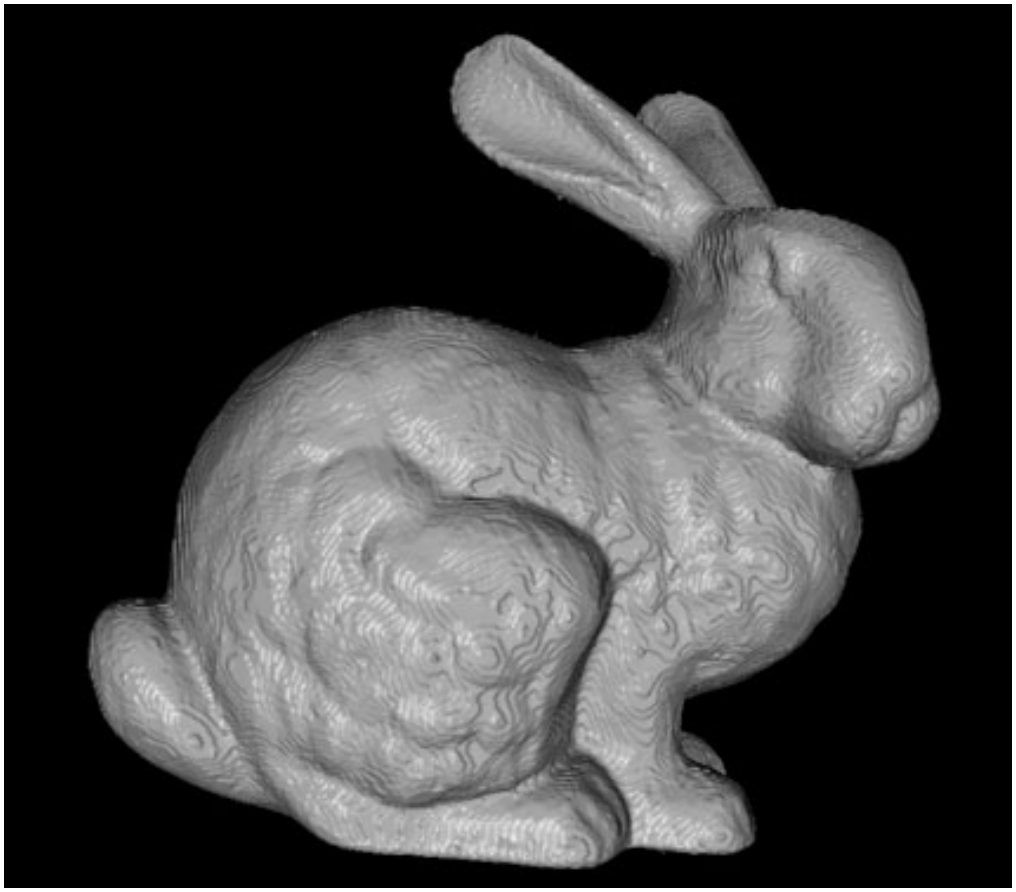


Figure 2. Configurations.

Configurations

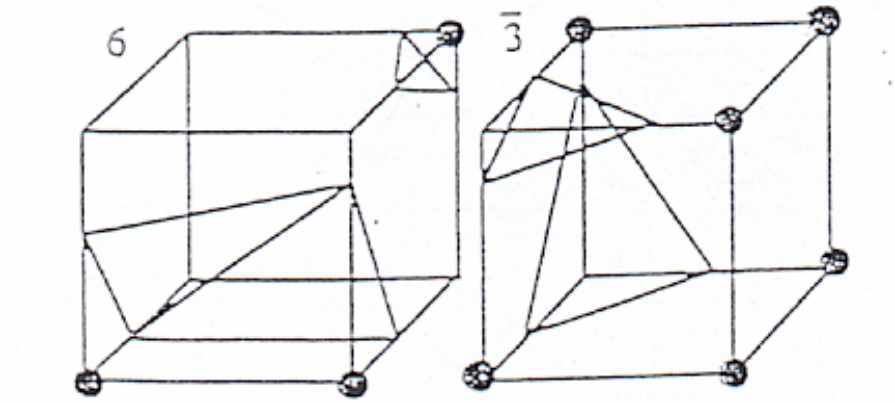
- For each configuration add 1-4 triangles to isosurface
- Isosurface vertices computed by:
  - Interpolation along edges (according to grid values)

Example



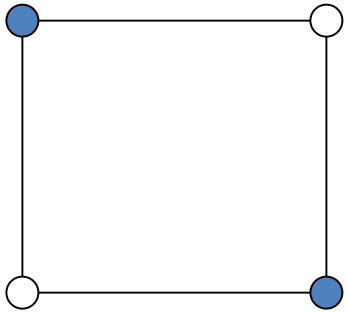
Problem

Can produce non-manifold results and wrong genus

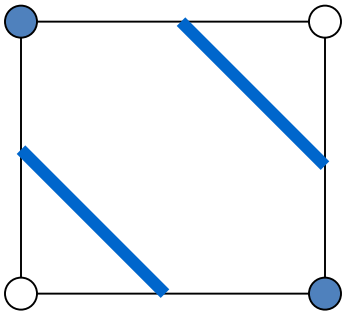


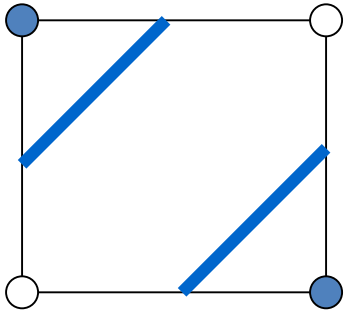
– What if those two are adjacent?
  - Each is ambiguous
– Consistency?

Ambiguous Faces



- Two locally valid interpretations





- Source of MC consistency problem

Solution

For those cases, store multiple triangulations

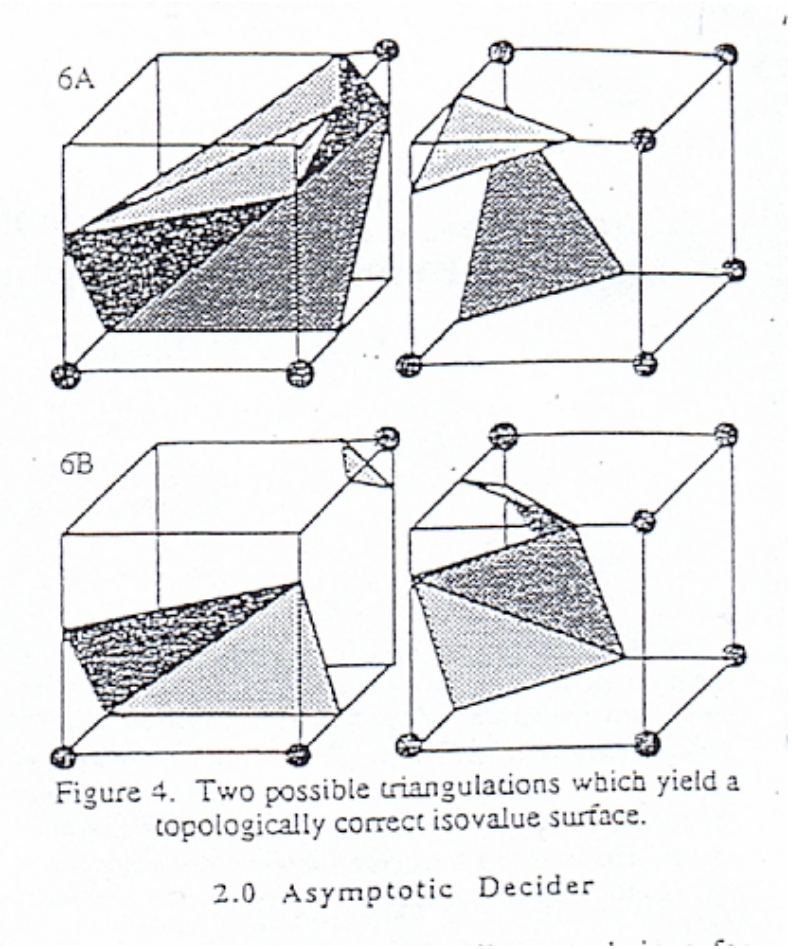


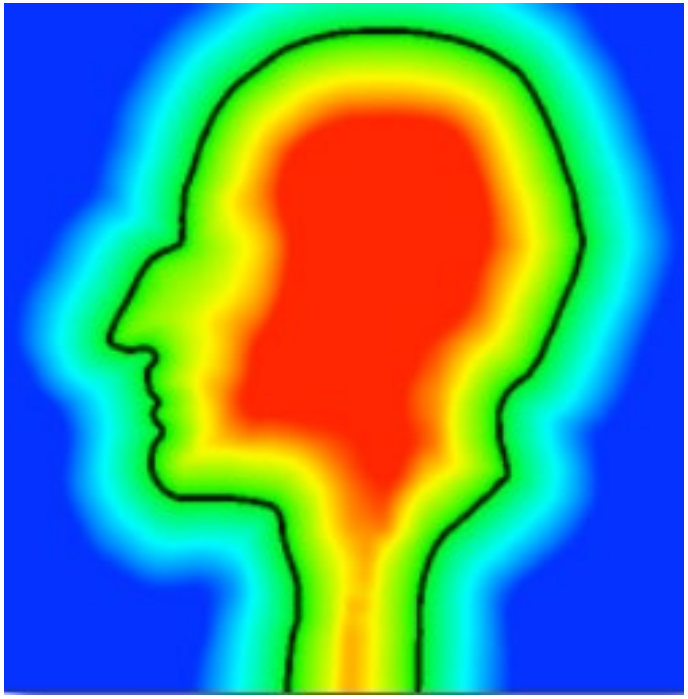
Figure 4. Two possible triangulations which yield a topologically correct isovalue surface.

2.0 Asymptotic Decider

Choose one that agrees with neighbor voxels

Implicit Reconstruction

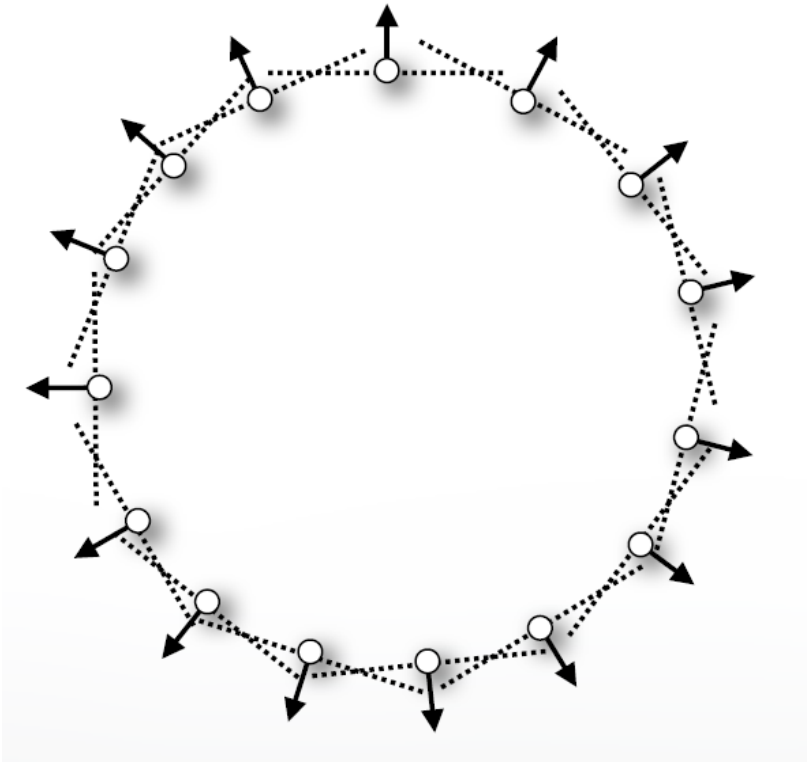
1. **Estimate signed distance function $\boldsymbol{d}$: $\mathbb{R}^3 \rightarrow \mathbb{R}$**
2. Extract an isosurface $d = 0$



Signed distance function

Distance to points is not enough

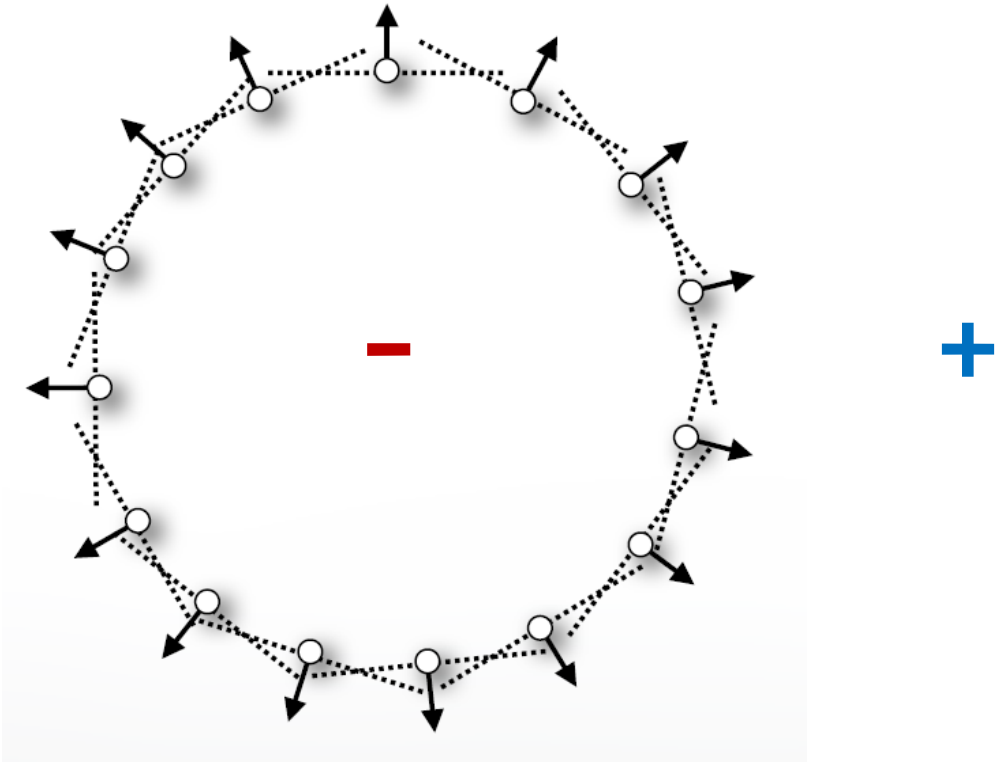
Need more structure



Signed distance function

How can we tell inside from outside?

Estimate normals.



Estimating normals

- Fit a plane into neighborhood of each point
  - Neighborhood = k nearest neighbors
- Determine *consistent* normal orientation

Estimating normals

- Fit a plane into neighborhood of each point
  - Neighborhood = k nearest neighbors
    - Use spatial decompositions (BSP-trees)
- Determine *consistent* normal orientation

Fitting plane

$$\min_{c \in \mathbb{R}^3, \|n\|=1} \sum_i (n^T (p_i - c))^2$$

On the board, time permitting

Estimating normals

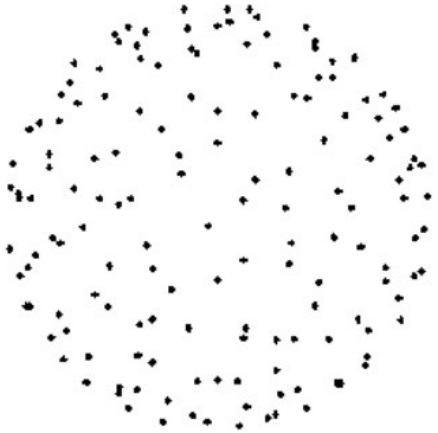
- Fit a plane into neighborhood of each point
  - Neighborhood = k nearest neighbors
- Determine *consistent* normal orientation

Estimating normals

- Fit a plane into neighborhood of each point
  - Neighborhood = k nearest neighbors
- Determine *consistent* normal orientation
  - Make sure $n_i \cdot n_j > 0$ for neighbors

Signed Distance Function

- Distance to tangent planes
  - [Hoppe et al. '92]



150 samples

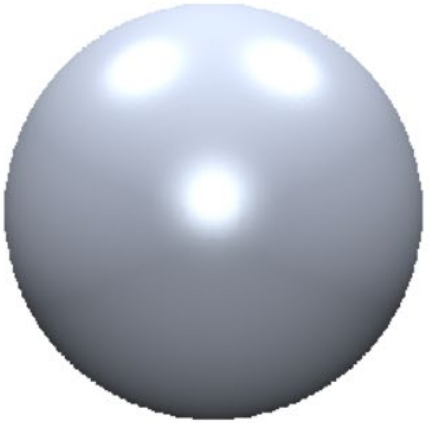
reconstruction
on 50^3 grid

Signed Distance Function

- Smoother: RBF basis

Hoppe '92

Compact RBF
Wendland C^2



Global RBF
Triharmonic

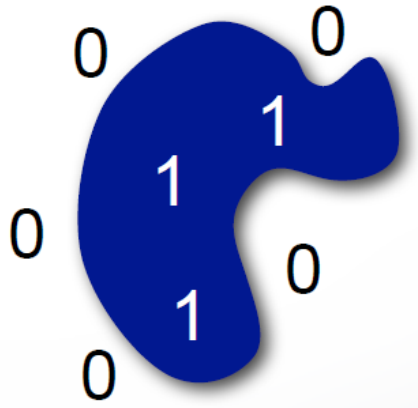
Signed Distance Function

- Poisson surface reconstruction
  - [Kazhdan et al. ‘06]

Signed Distance Function

- Poisson surface reconstruction
  - Solve for indicator function

$$\chi_M(p) = \begin{cases} 1 & \text{if } p \in M \\ 0 & \text{if } p \notin M \end{cases}$$



Indicator function

χ_M

Oriented points

?

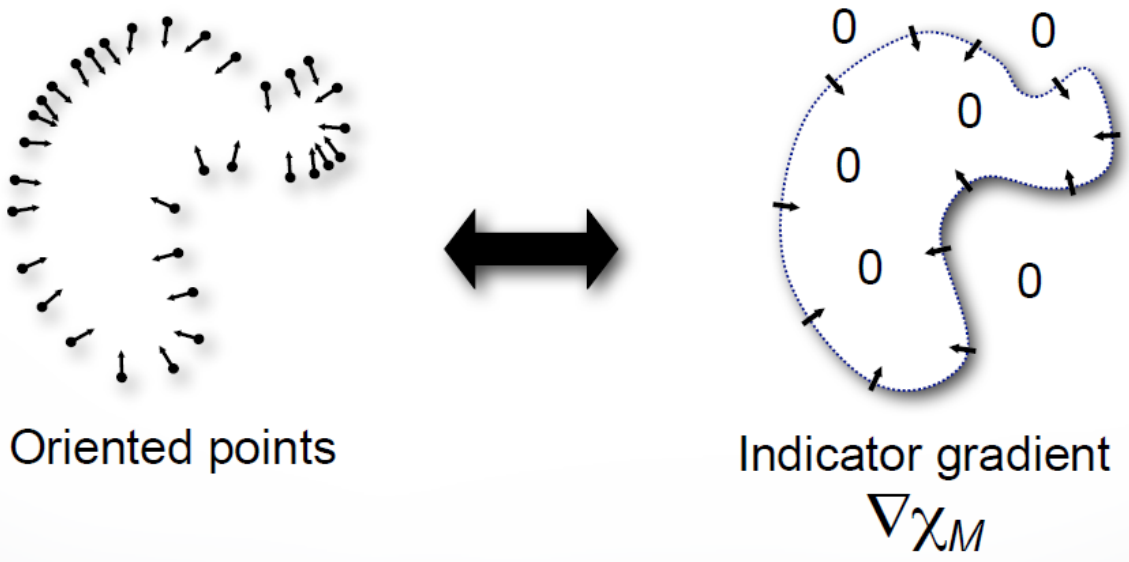
Indicator function

χ_M

Idea

Oriented normals =

gradient of an indicator function?



Idea

Oriented normals ⇒ vector field $\vec{V}$

Find indicator function:

$$\min_{\chi} \left\| \vec{V} - \nabla \chi \right\|^2$$

Idea

Oriented normals ⇒ vector field $\vec{V}$

Find indicator function:

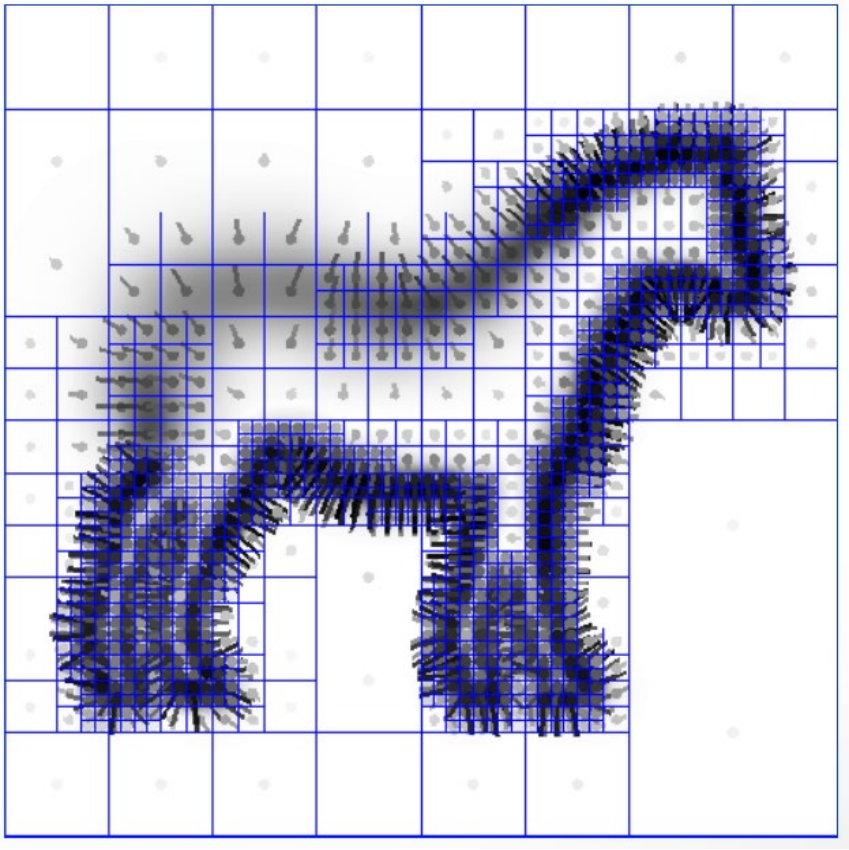
$$\min_{\chi} \left\| \vec{V} - \nabla\chi \right\|^2$$

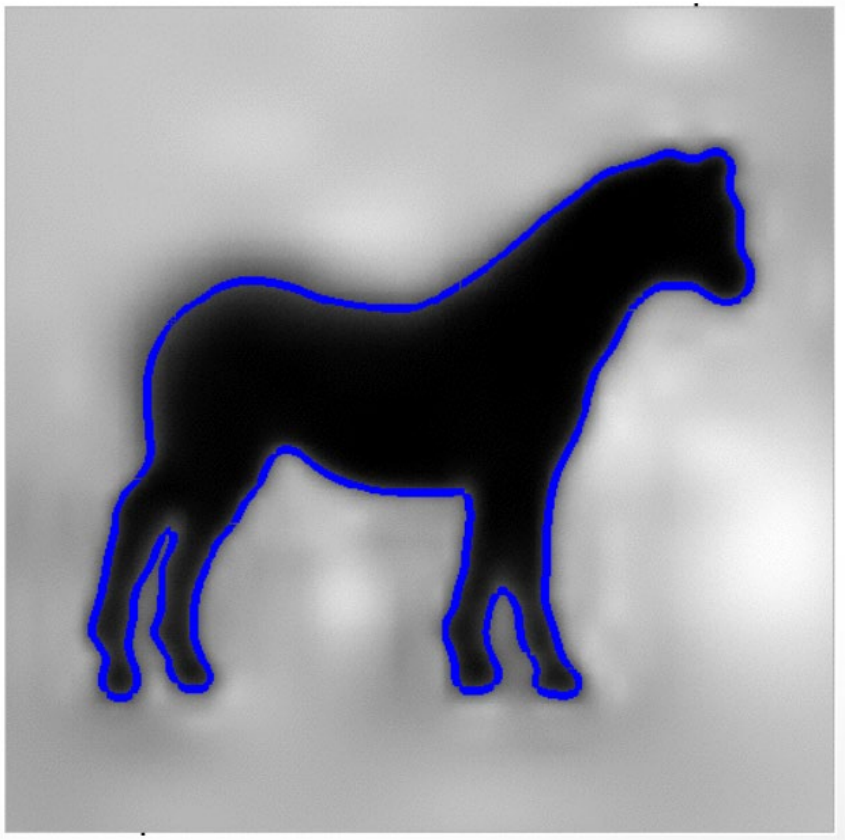
Differentiate,

Poisson equation

$$\Delta\chi = \nabla \cdot \vec{V}$$

Process





Results