

IFT 6113

INTRODUCTION

<http://tiny.cc/ift6113>



Mikhail Bessmeltsev



TEAM

Instructor: Mikhail Bessmeltsev

Email: bmpix@iro.umontreal.ca

Office hour: Fri, 5:00-6:00 pm, Zoom or AA 3357

Web: <http://www-labs.iro.umontreal.ca/~bmpix/>

TA: Ivan Puhachov

Contact: via Piazza

FORMALITIES

Lectures: Tue 9.30-11.30

Thu 16.30-18.30

Web: <http://tiny.cc/ift6113>

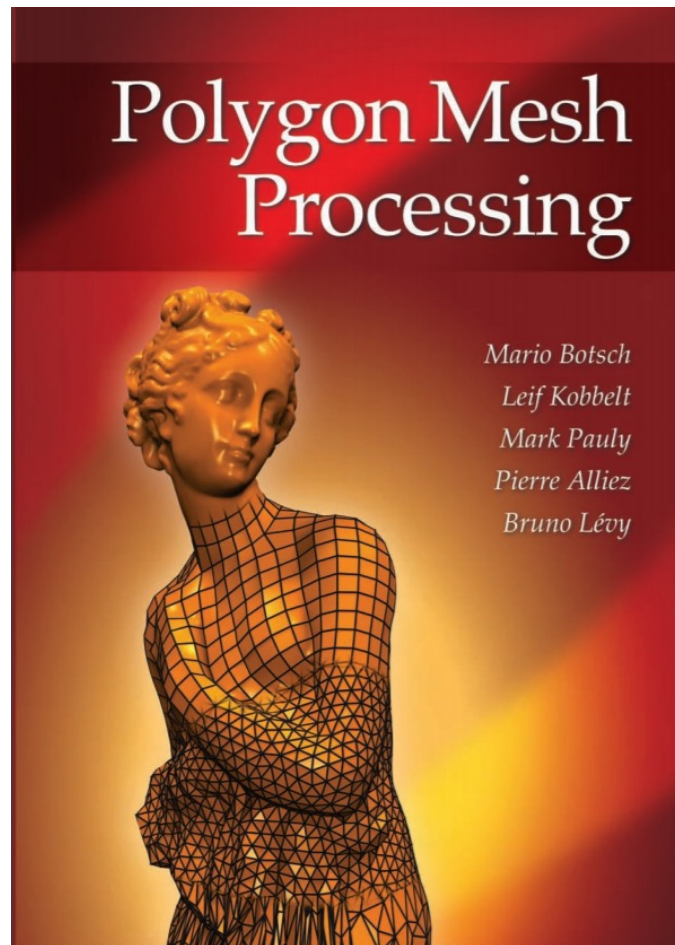
Use Piazza!

<http://piazza.com/umontreal.ca/fall2021/ift6113>

TEXTBOOK (OPTIONAL)

Polygon Mesh Processing by Botsch et al.

Free online at UdeM library



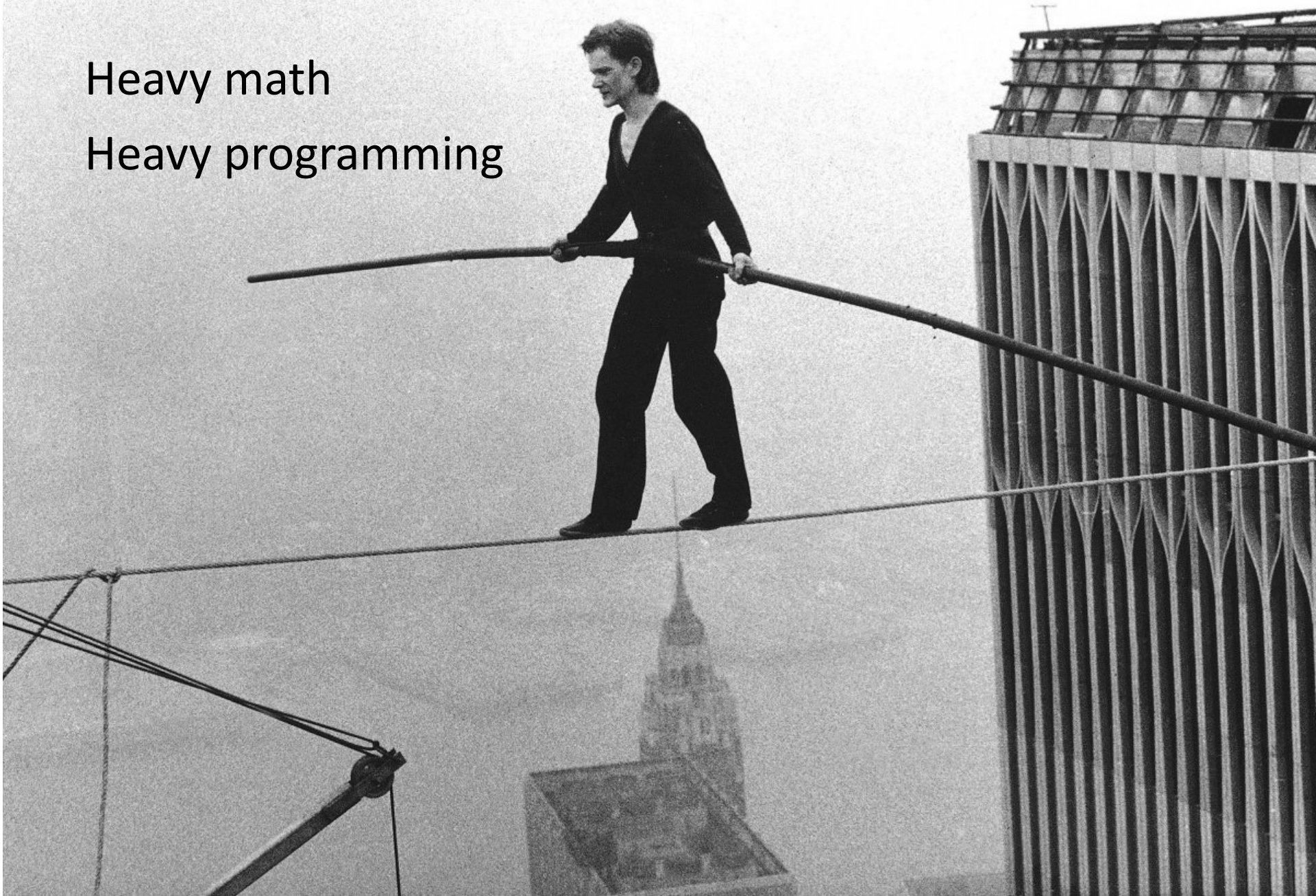
PREREQS

- For sure:
 - Calculus
 - Linear algebra
- Bonus (but not required):
 - Differential Geometry
 - Partial Differential Equations

THIS IS NOT AN EASY COURSE

Heavy math

Heavy programming



BUT INTERESTING

I will help
and be generous with grading

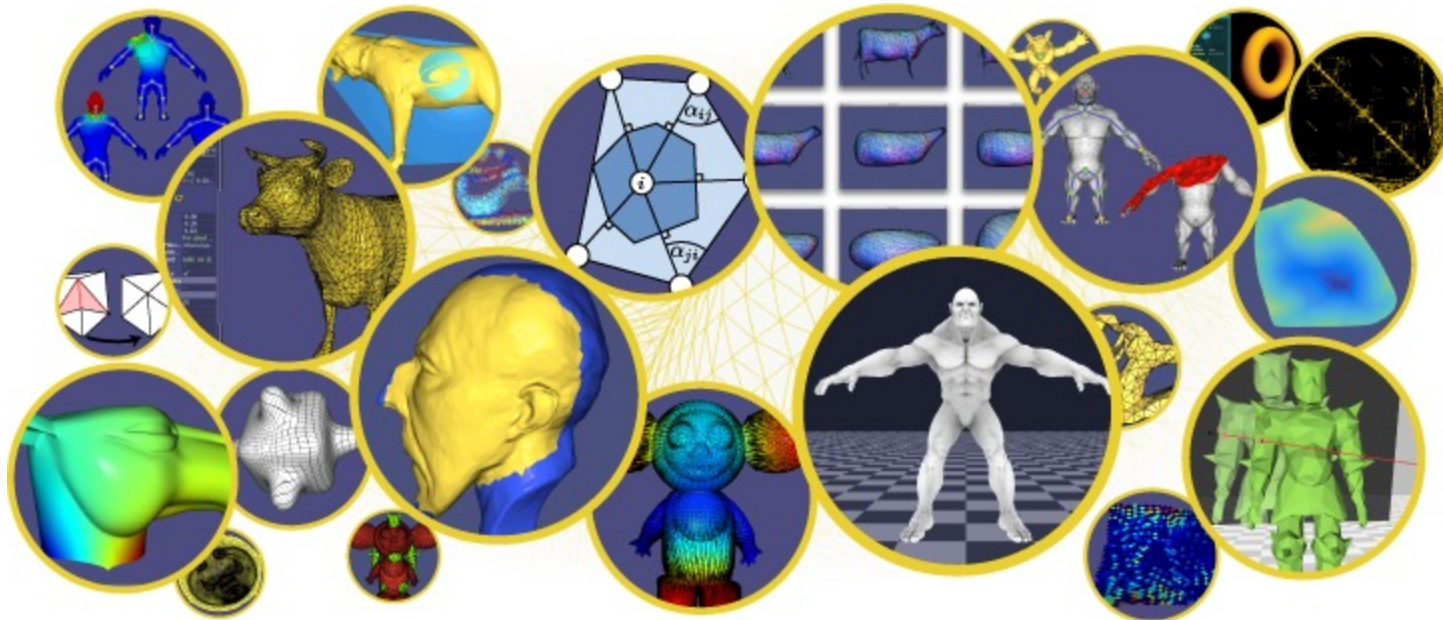
GRADING

Assignments (33%)
Paper Presentation (25%)
Participation (2%)
Final Project (40%)

100%

ASSIGNMENTS (33%)

- Mostly coding, but math also
- ~ 2 weeks long
- Recommended framework: libigl (C++)



<http://libigl.github.io/libigl/>

FACE TO FACE GRADING

- Understand each line of your code
- Plagiarism policy

PAPER PRESENTATION (25%)

- Choose a SIGGRAPH paper on a related topic
 - Give a talk!
 - Others: ask questions
-
- Choose a paper (and post about it in Piazza) by **Oct 29th**

PARTICIPATION (2%)

- Classroom
- Questions/discussion at paper and project presentations

FINAL PROJECT (40%)

1. Choose a recent SIGGRAPH paper
2. Propose an improvement/modification
3. Implement
4. Present!

Important dates:

Project proposal: Oct 16th, by midnight

Final Presentation: Dec 7th, in class

NO EXAMS

WHAT YOU WILL LEARN

Fundamentals:

Mesh data structures
Discrete Differential Geometry
Discrete Laplacians
Vector Fields

Applications:

Shape Deformations
Surface Reconstruction
Mesh Parameterization
Remeshing
Animation/Sketch-based modeling/...
Learning geometry

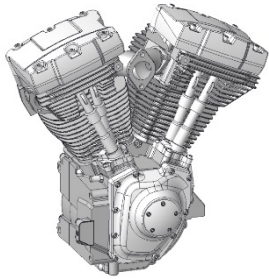
TODAY

1. Geometric modeling?
2. Shape analysis?
3. Studying shapes: what we need
4. Applications

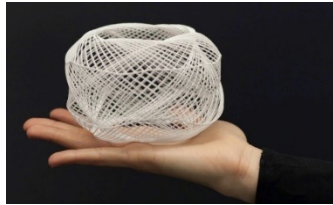
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- 1. Geometric modeling?**
- 2. Shape analysis?**
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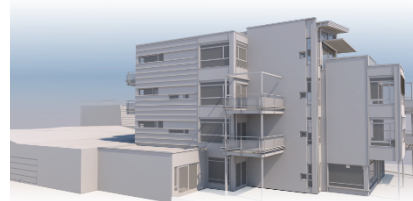
WHAT IS GEOMETRIC MODELING?



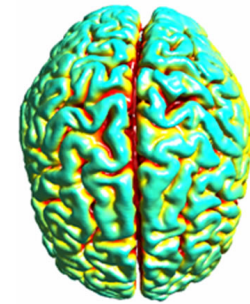
Mechanical Engineering



Fabrication



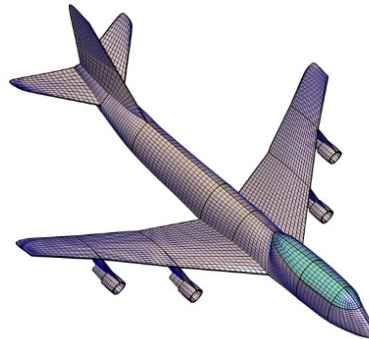
Architecture and civil engineering



Medical



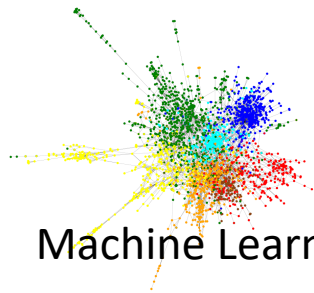
Computer Vision



Geometric Modeling



Computer Graphics



Machine Learning



Augmented/Virtual Reality



Human-Computer Interaction (HCI)

$$\begin{aligned} &\min f(x) \\ \text{s.t. } &g(x) \leq 0 \\ &h(x) = 0 \end{aligned}$$

Numerical Optimization

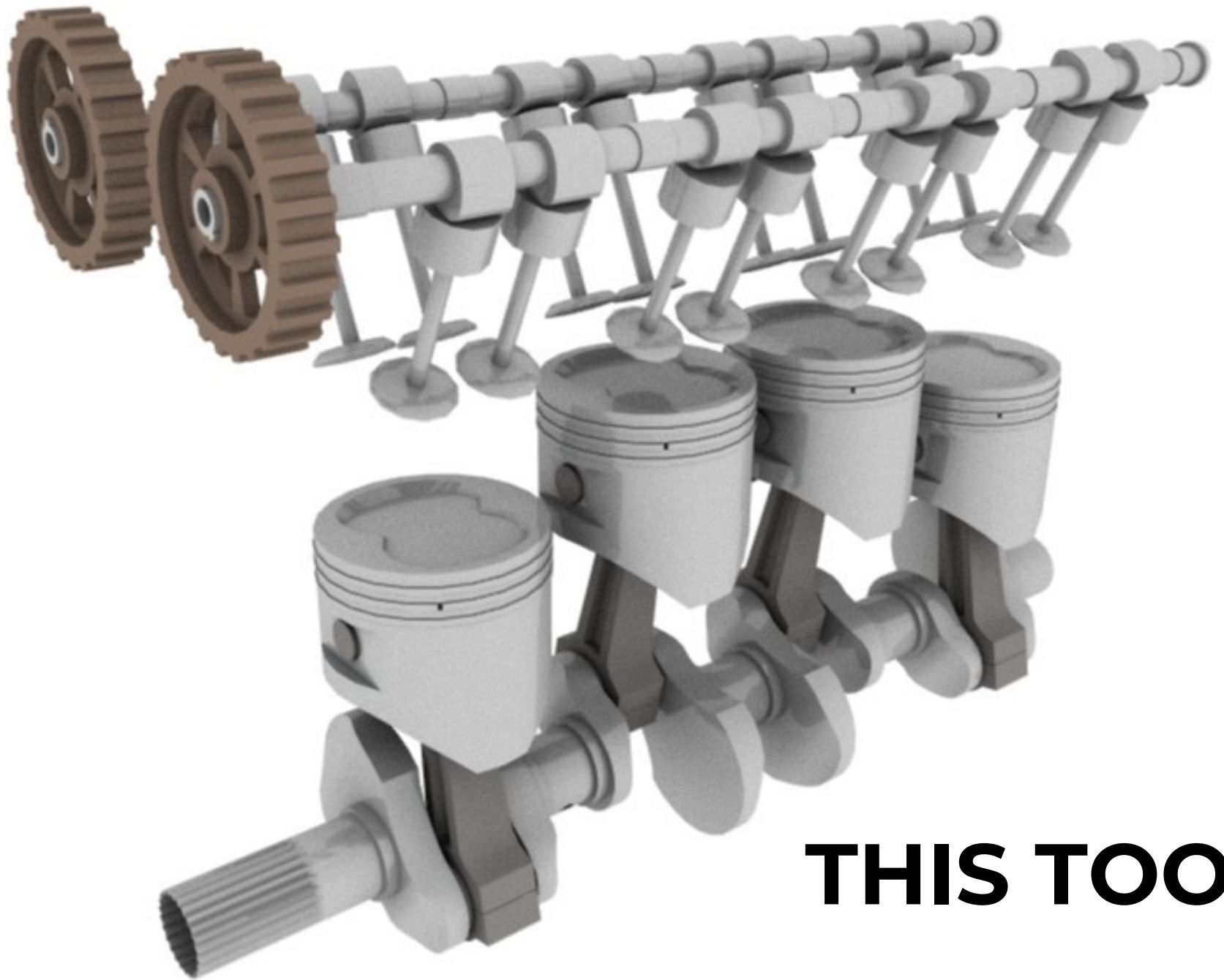
THIS.



EVENT COVERAGE

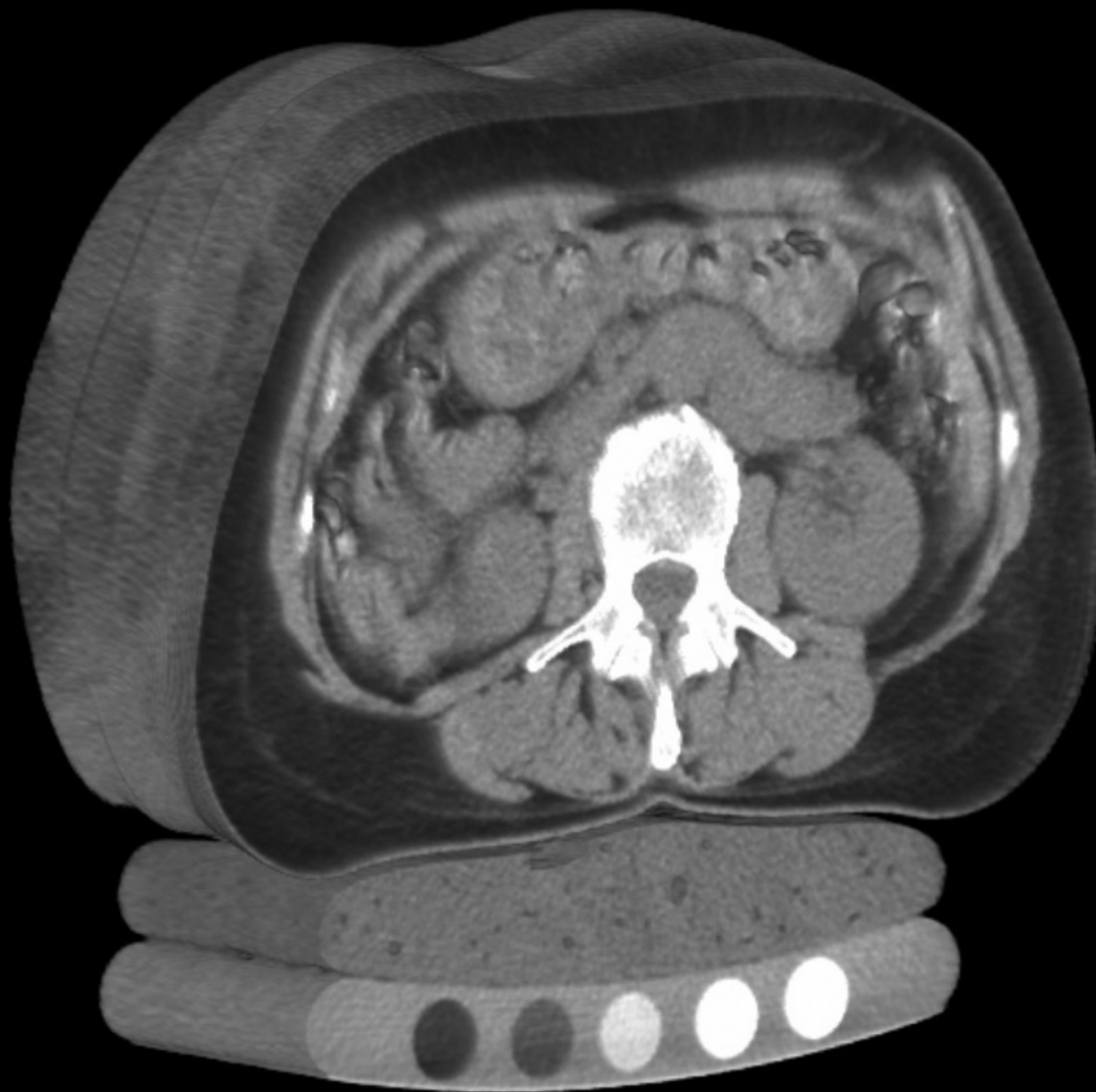
AND THIS.





THIS TOO.

AND EVEN THIS.

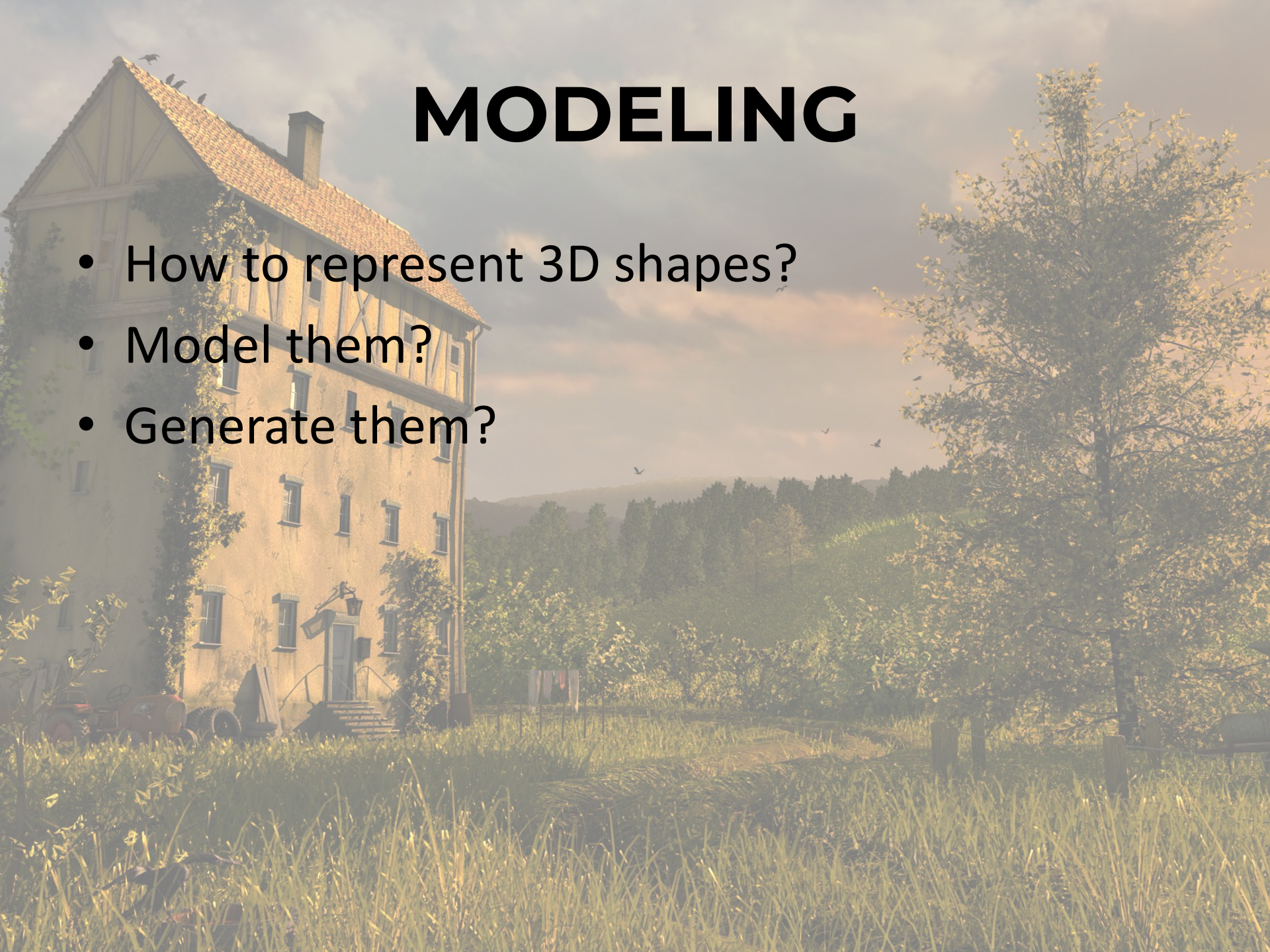


MODELING



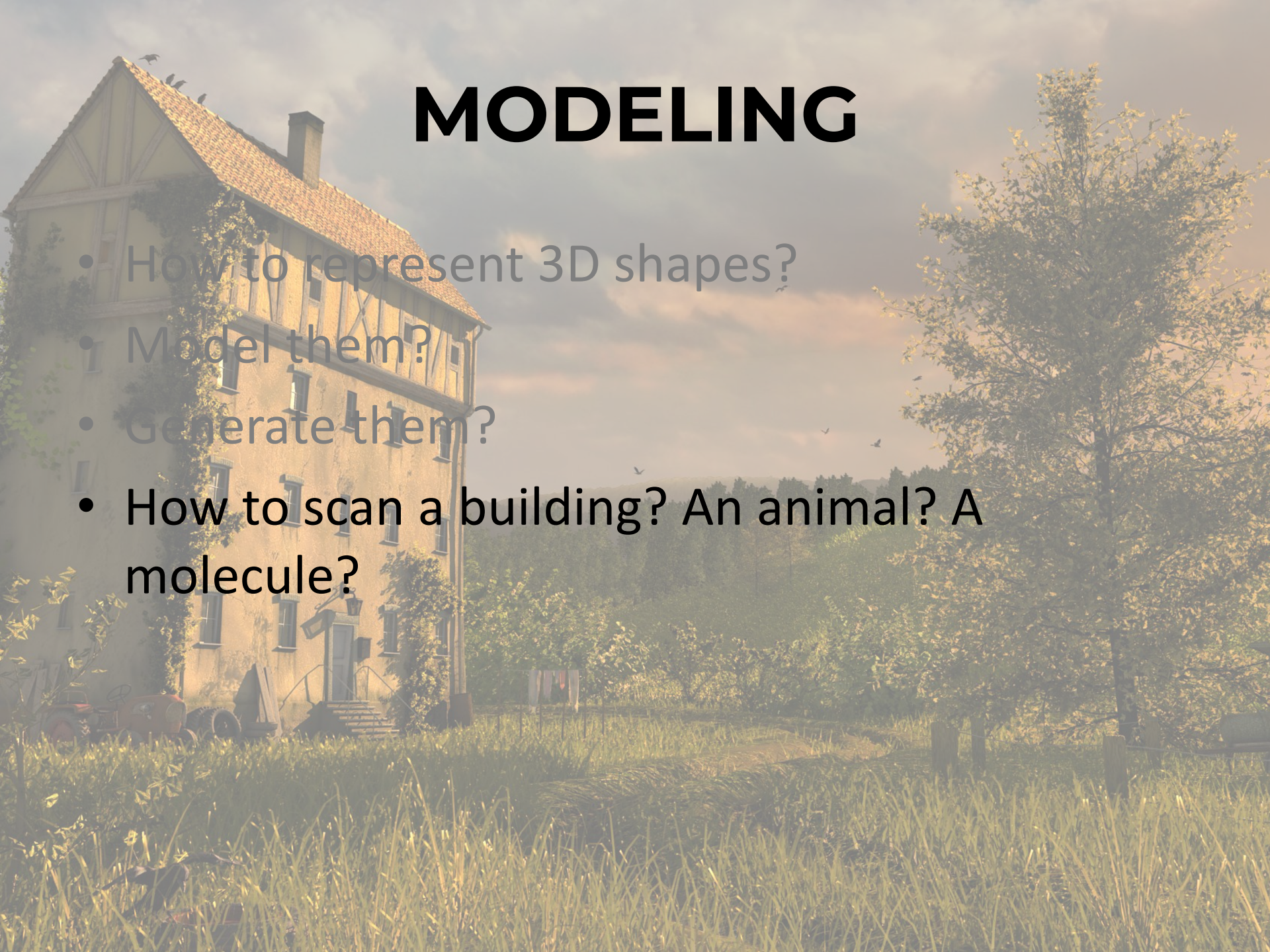
MODELING

- How to represent 3D shapes?
- Model them?
- Generate them?



MODELING

- How to represent 3D shapes?
- Model them?
- Generate them?
- How to scan a building? An animal? A molecule?



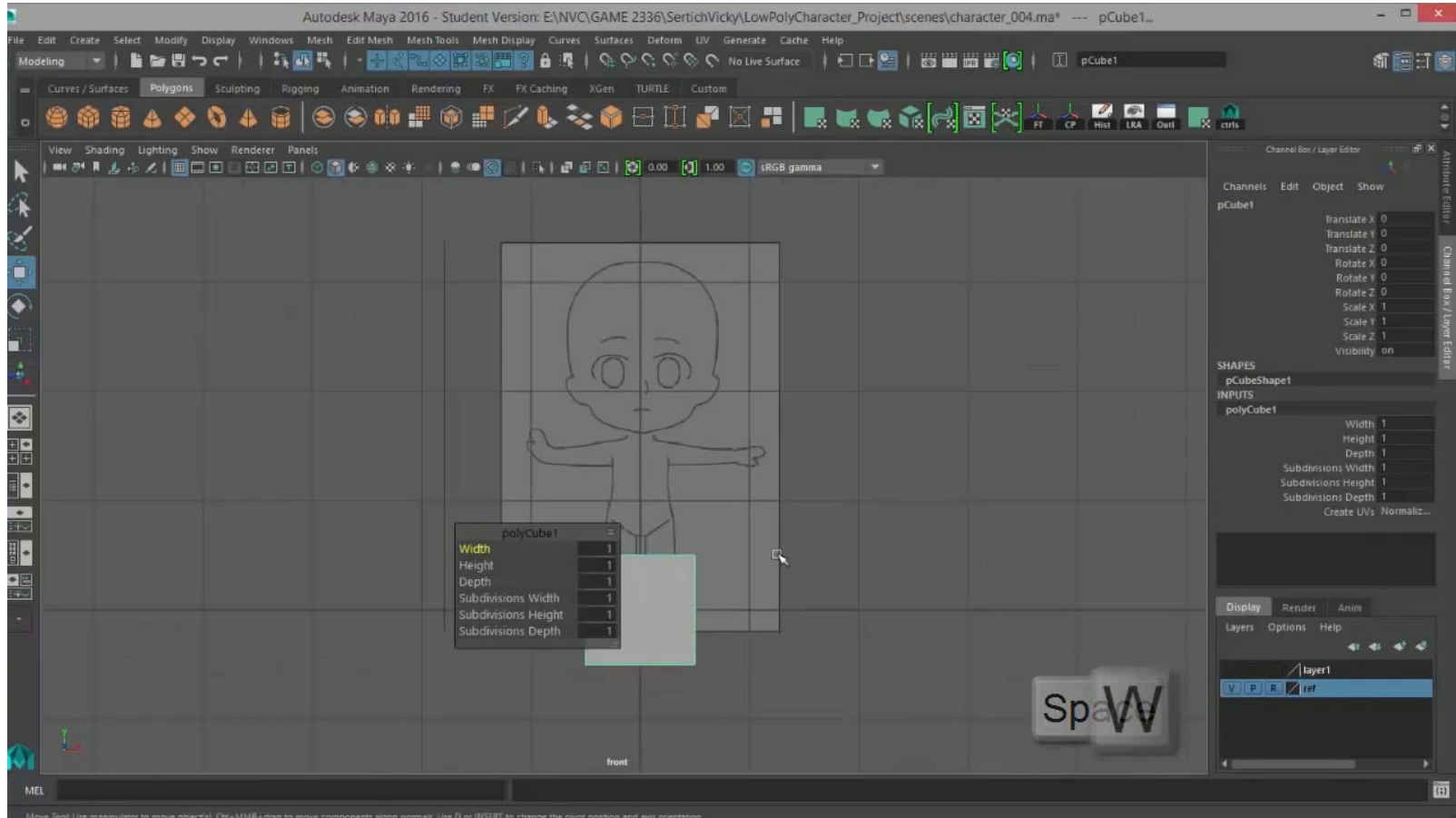
MODELING

- How to represent 3D shapes?
- Model them?
- Generate them?
- How to scan a building? An animal? A molecule?
- How to deform a model?

MODELING

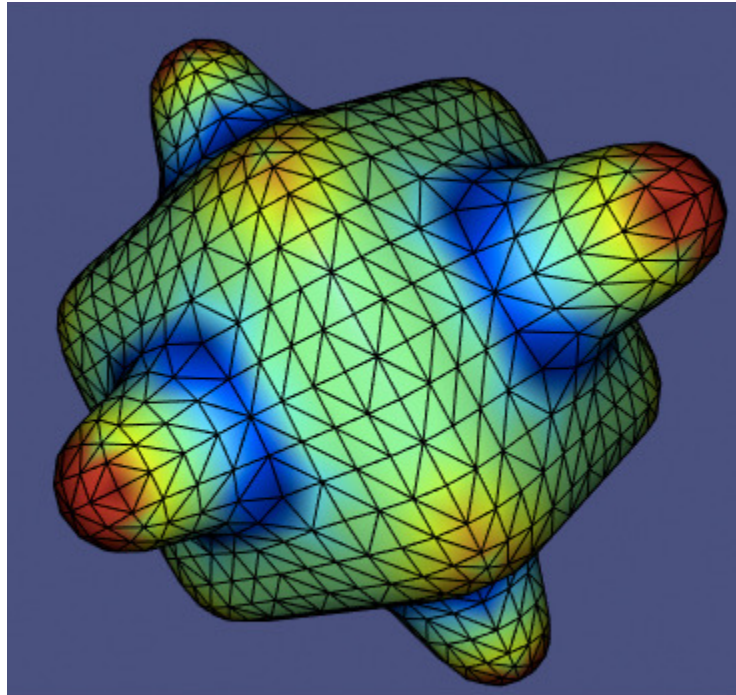
- How to represent 3D shapes?
- Model them?
- Generate them?
- How to scan a building? An animal? A molecule?
- How to deform a model?
-

MODELING SESSION



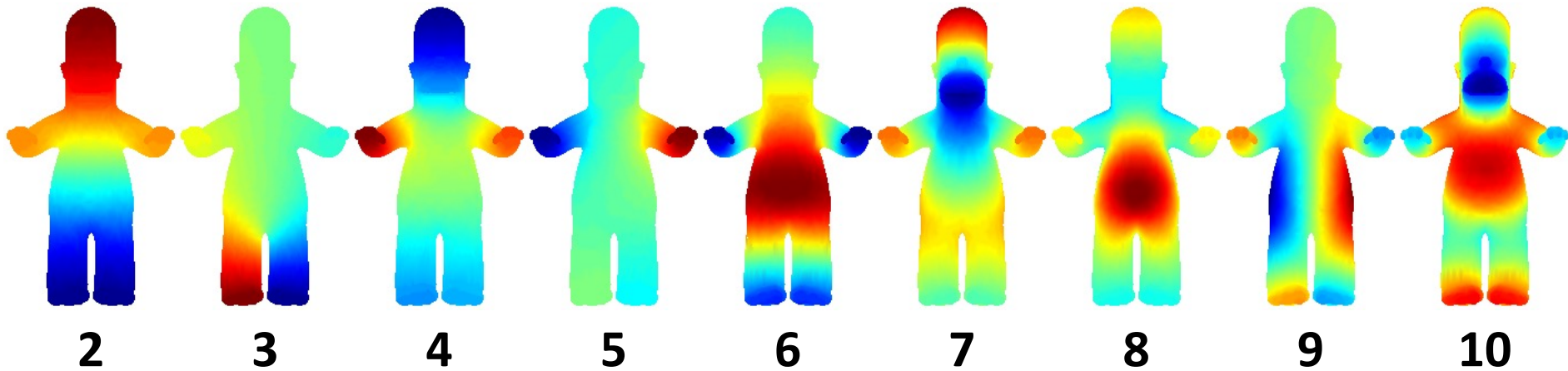
SHAPE ANALYSIS

Which parts of 3D shape are *interesting*?



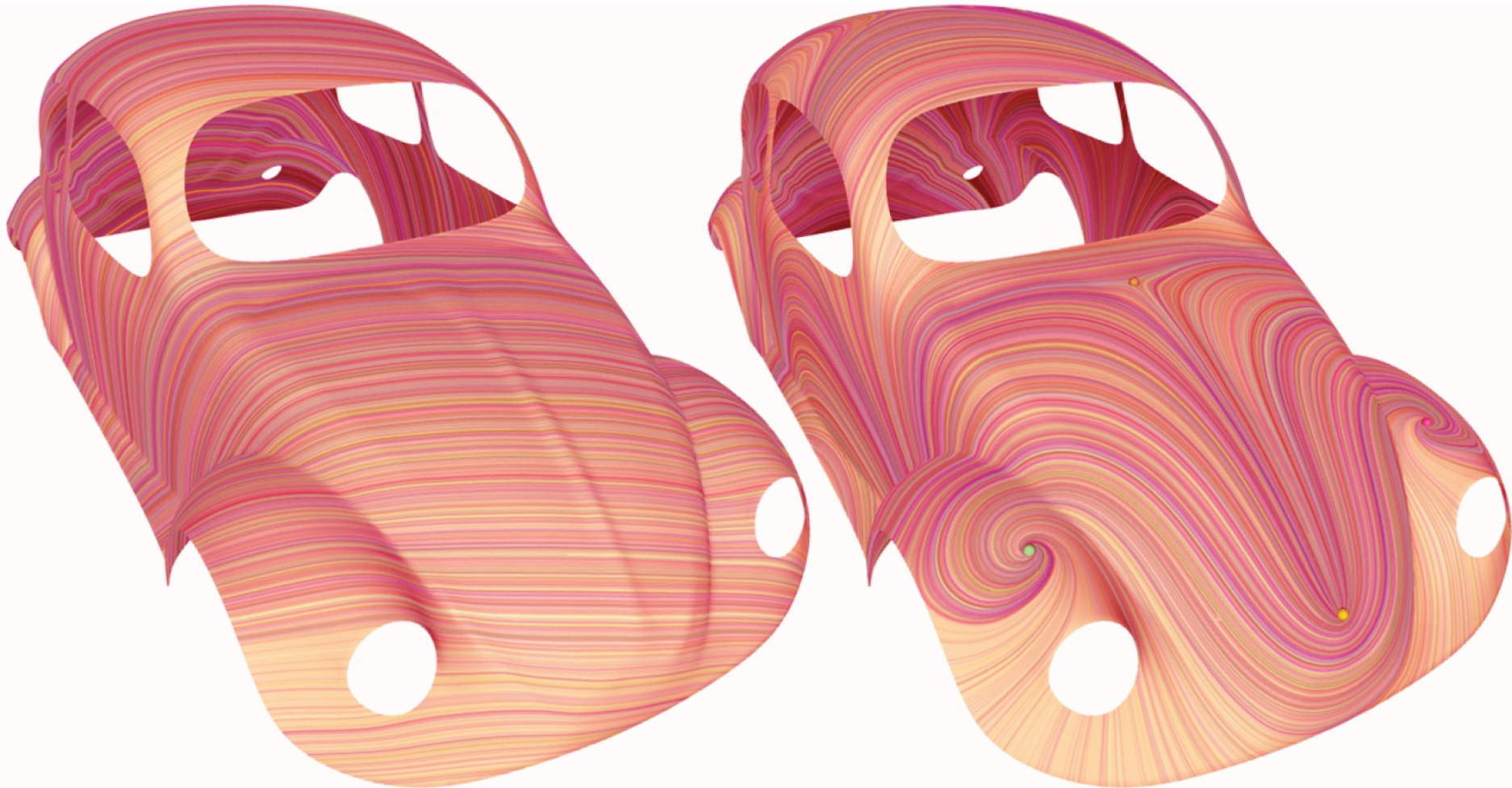
SHAPE ANALYSIS

Can you **hear** the **shape** of a drum?



SHAPE ANALYSIS

How to create vector fields and how to use them?



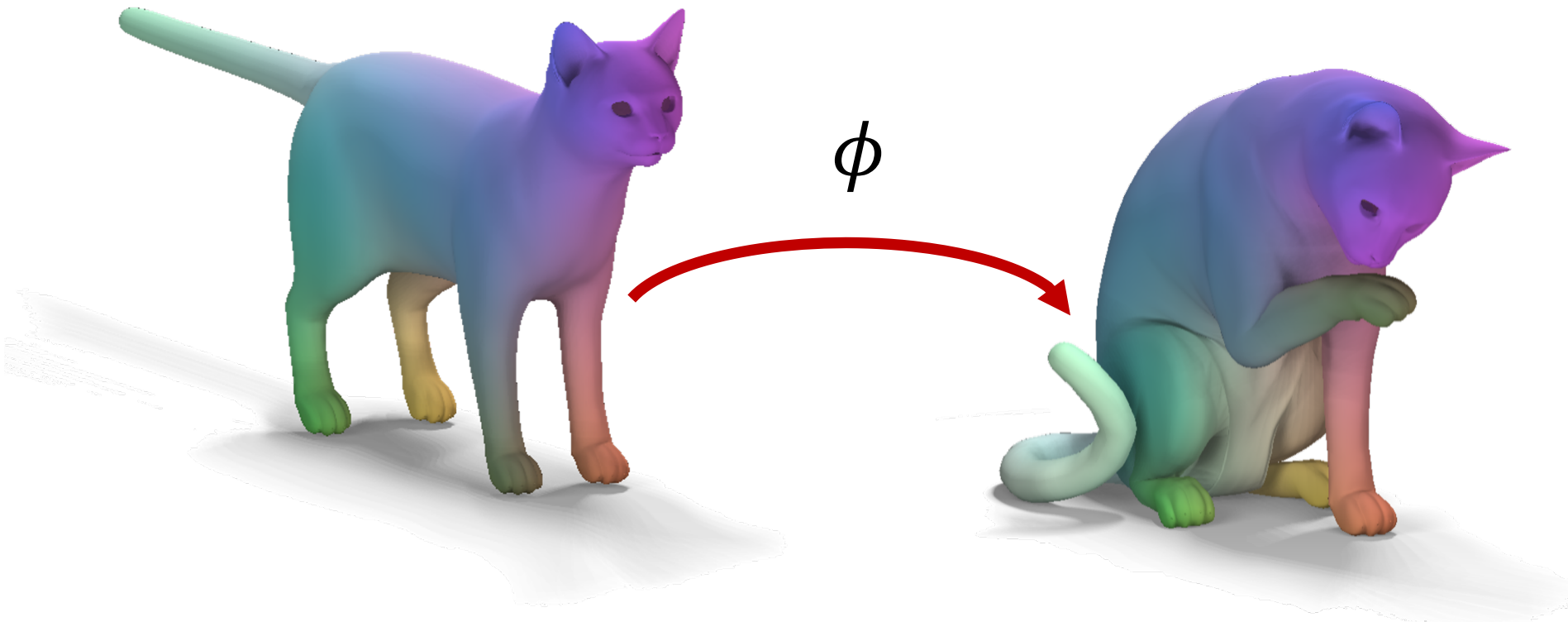
SHAPE ANALYSIS

How can we *compare* shapes?



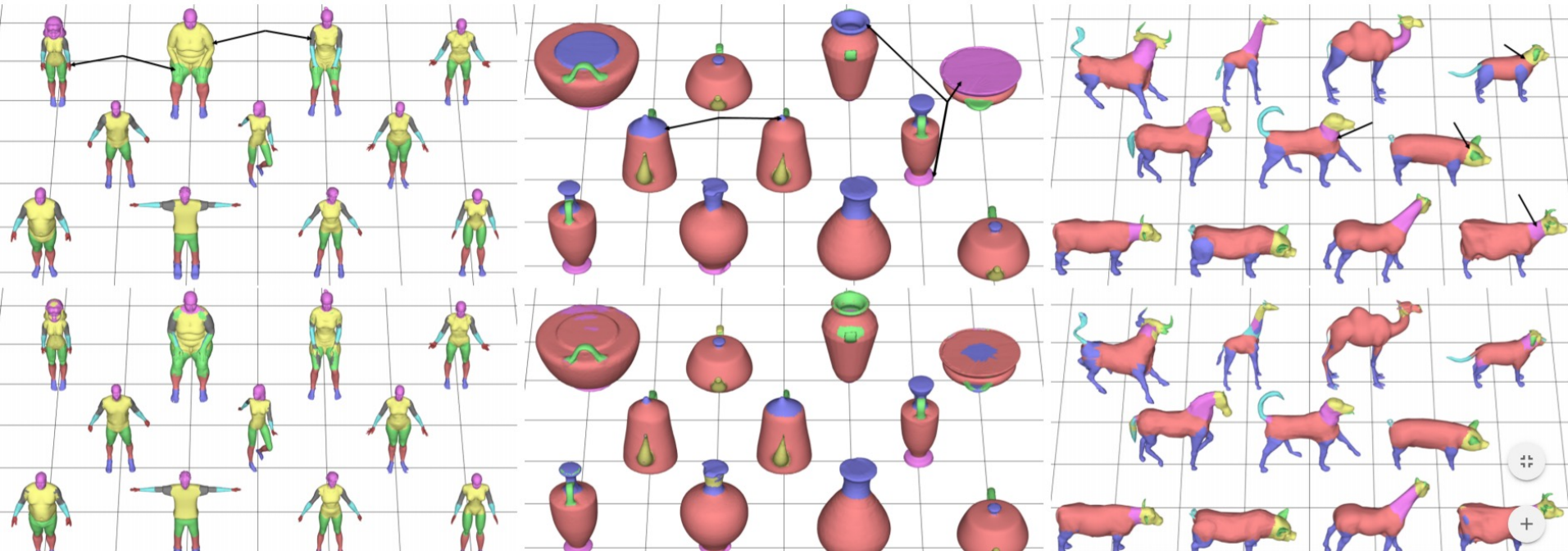
SHAPE ANALYSIS

- How can we *compare* shapes?
- How to map them onto each other?



SHAPE ANALYSIS

Segment shapes?



<https://arxiv.org/pdf/1705.11050.pdf>

ROADMAP

- Data Structures $\int dx$
- Recap of differential geometry $\iiint dx dy dz$
- Mesh Simplification & Saliency $\iint dx dy$
- Vector Fields $\iiint dx dy dz$
- Mesh Parameterization $\iint dx dy$
- Shape Deformation $\iint dx dy$
- 2D modeling $\int dx$
- Remeshing $\iint dx dy$
- Learning $\int dx$



SOURCES

Based on two courses:

Many slides are derived from theirs



6.838 (Justin Solomon)

MIT CSAIL

http://groups.csail.mit.edu/gdpgroup/6838_spring_2021.html



524 (Alla Sheffer)

UBC CS

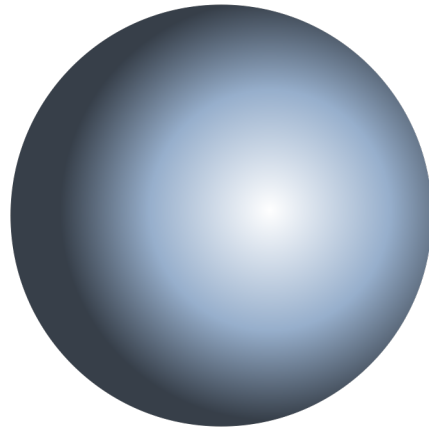
<https://www.cs.ubc.ca/~sheffa/dgp/>

TODAY

1. Geometric modeling?
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4. Applications

WE WANT TO STUDY SHAPES

What (pure) mathematicians think:



EUCLIDEAN GEOMETRY

1. duplicare est que diametro recte autem bisectae sunt minima. et quae ipsiusque recte in omnibus lineis est. dicitur quod si in lineis bisectis. quae ipsiusque sunt aequales.

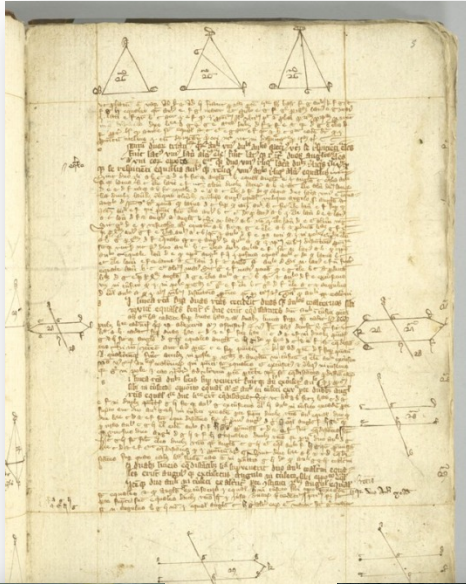
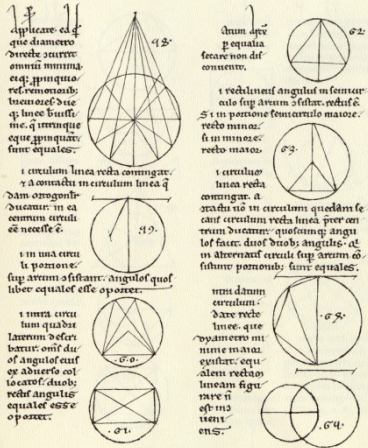
2. si circuli linea recta commingatur. et a centro in circulum linea. quae diametrum dividit. ducatur. in eadem circuli est necessitas.

3. in una circuli positio. super arcum astantur. anguli quoslibet esse oppositos.

4. si una circuli quadrata laterum adest. batur. omni dicitur. anguli autem aduersi collocatione. duobus rectis angulis equaliter esse oppositos.

5. si unum arcum astantur. quae a centro in circulum linea. quae diametrum dividit. ducatur. in eadem circuli est necessitas.

6. si unum arcum astantur. quae a centro in circulum linea. quae diametrum dividit. ducatur. in eadem circuli est necessitas.



This page shows a manuscript with handwritten text in Arabic script and several geometric diagrams. The diagrams include triangles, circles, and lines, with various points and lines labeled. The text appears to be a commentary or translation of Euclidean geometry.




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16 EVCLID. ELEM. GEOM.
verò tangantur quod sub tota fecante, & exteriori inter punctum & contactum peripheriam alio puncto comprehensit, recta tangens quælibet, quod à tangente distat, batur, quadrato.

Theorem. 31. Propositio 37.

Si extra circulum sumatur punctum ali- quod, ab eoque puncto in circulum circuli duo recte lineæ, quarum altera circulum fecerit, altera in eum incidat, sit autem, quod sub tota fecante, & exteriori inter punctum, & contactum peripheriam assumpta, comprehendit, aequali, quod ab incidente describitur quadrato, in circuli circulo quadrato.

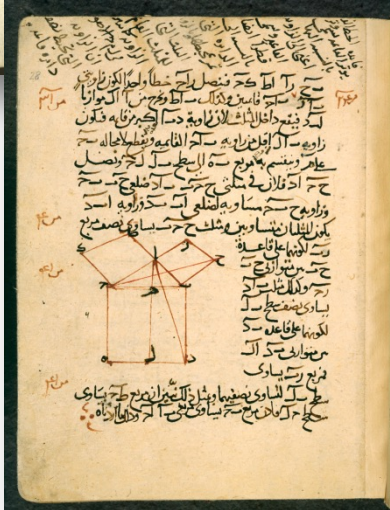
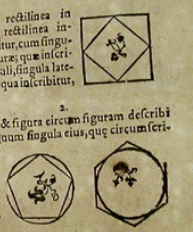


LIBER IV.
EVCLIDIS
ELEMENTVM
QVARTVM.
DEFINITIONES.

1. Figura rectilinea in circulo inferri dicitur, cum singuli eius figure, que inferri buntur, anguli, in quibus latera eius in qua inferri buntur, tangunt.

2. Similiter & figura circuli figuram describi dicitur, quum singula eius, que circumferri buntur, latera singulorum eius figure, quoslibet angulos rectos, circum quam illa describitur.

3. Figura rectilinea in circulo inferri dicitur, quum singuli eius figure, que inferri buntur, anguli





This page shows a manuscript with handwritten text in Arabic script and several geometric diagrams. The diagrams include a square and a hexagon inscribed in a circle, with various points and lines labeled. The text appears to be a commentary or translation of Euclidean geometry.

17 EVCLID. ELEM. GEOM.
verò tangantur quod sub tota fecante, & exteriori inter punctum & contactum peripheriam alio puncto comprehensit, recta tangens quælibet, quod à tangente distat, batur, quadrato.

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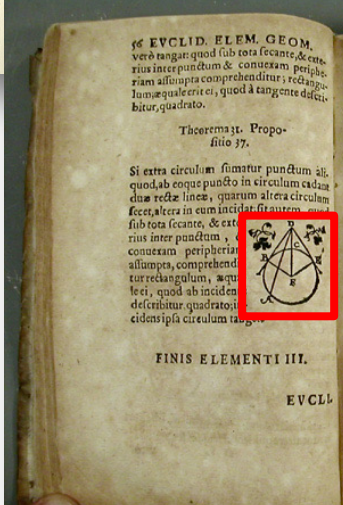
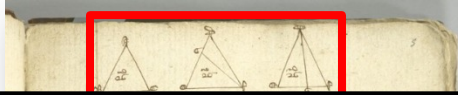
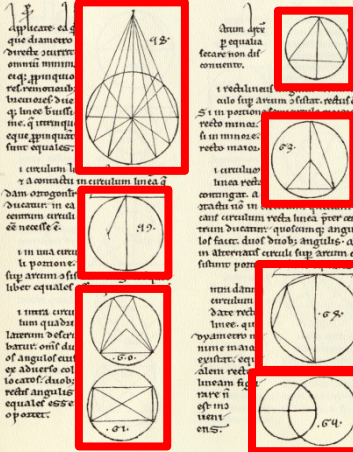
Si extra circulum sumatur punctum ali- quod, ab eoque puncto in circulum circuli duo recte lineæ, quarum altera circulum fecerit, altera in eum incidat, sit autem, quod sub tota fecante, & exteriori inter punctum, & contactum peripheriam assumpta, comprehendit, aequali, quod ab incidente describitur quadrato, in circuli circulo quadrato.



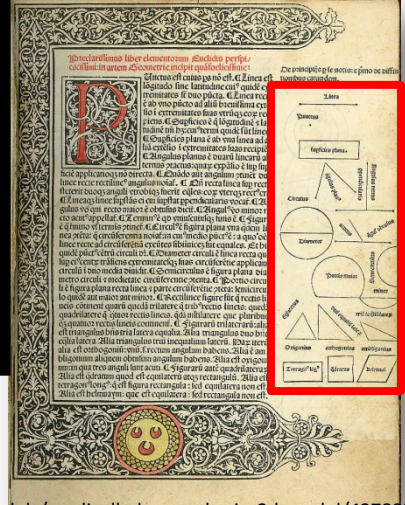
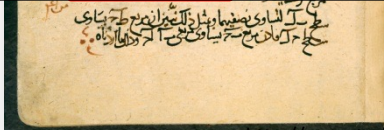
EUCLIDEAN GEOMETRY



EUCLIDEAN GEOMETRY

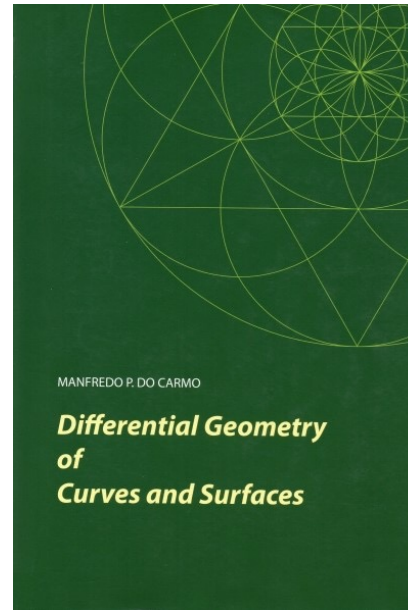


gerini, circun quam illa detrahatur.
EVCLD
Figura redilinea in circulo inferribi dicitur, quoniam singuli eius figure, quae inferribitur, anguli

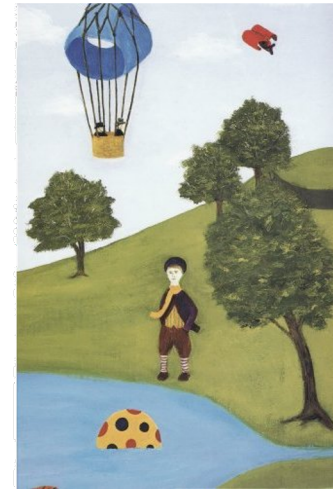
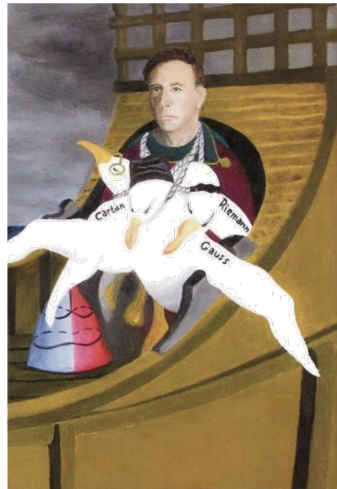
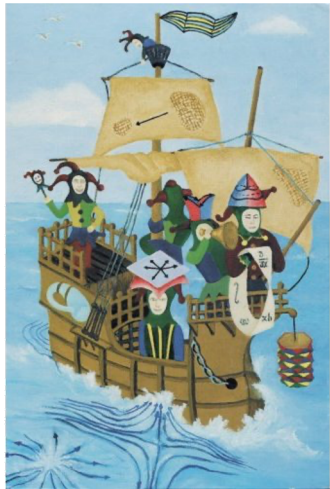


DIFFERENTIAL GEOMETRY

Easy intro:

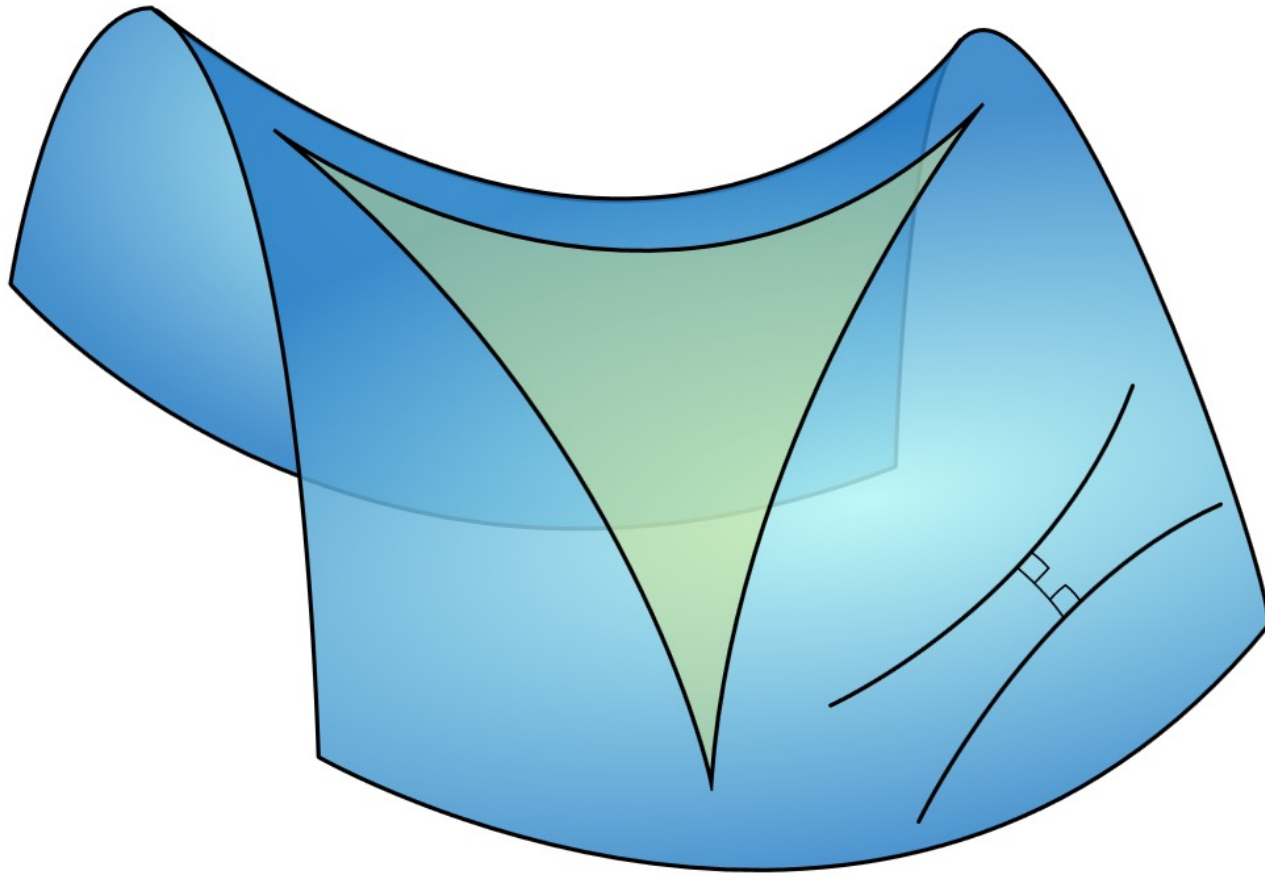


Harder intro (5 volumes):



Spivak: *A Comprehensive Introduction to Differential Geometry*

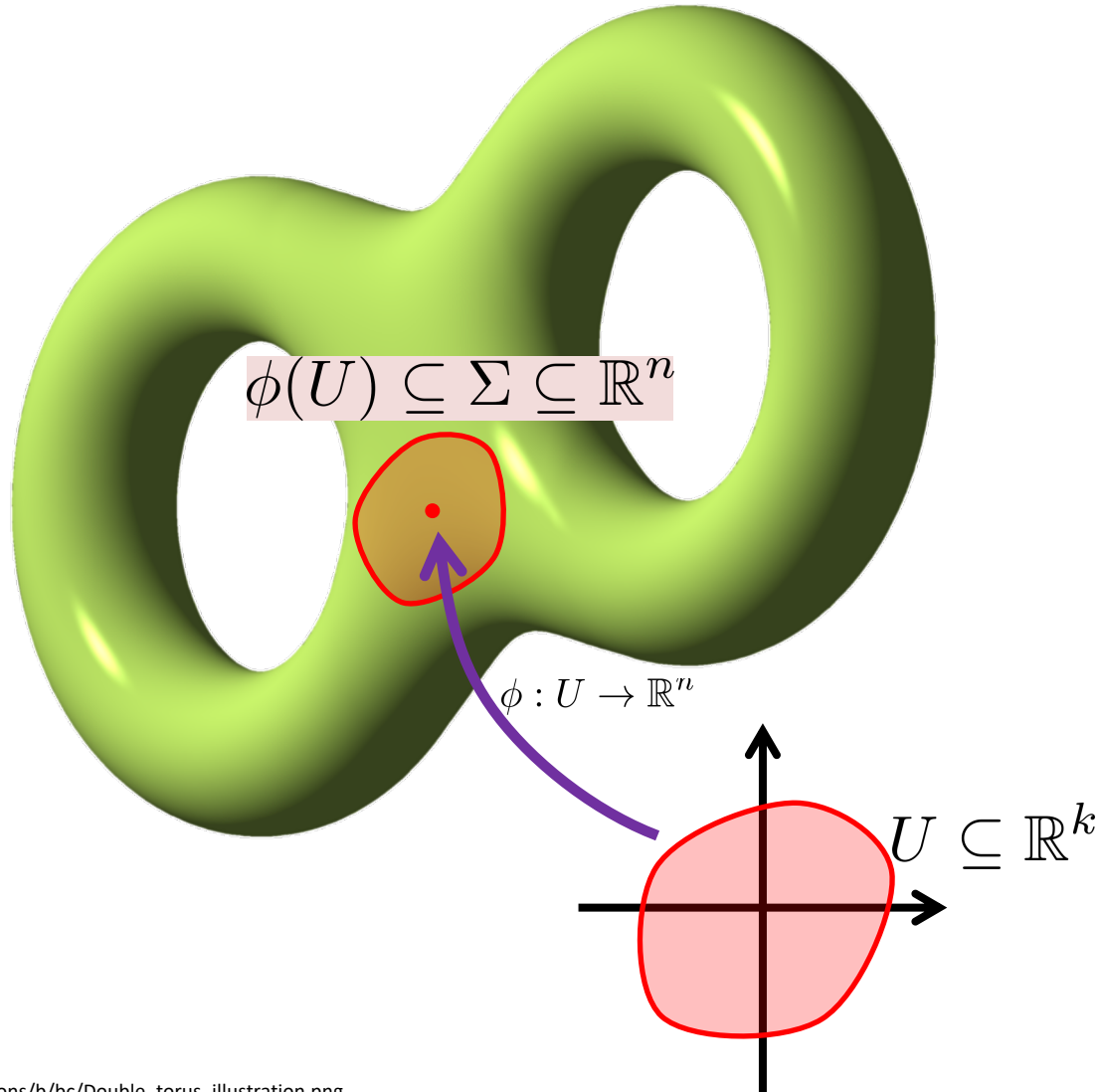
DIFFERENTIAL GEOMETRY



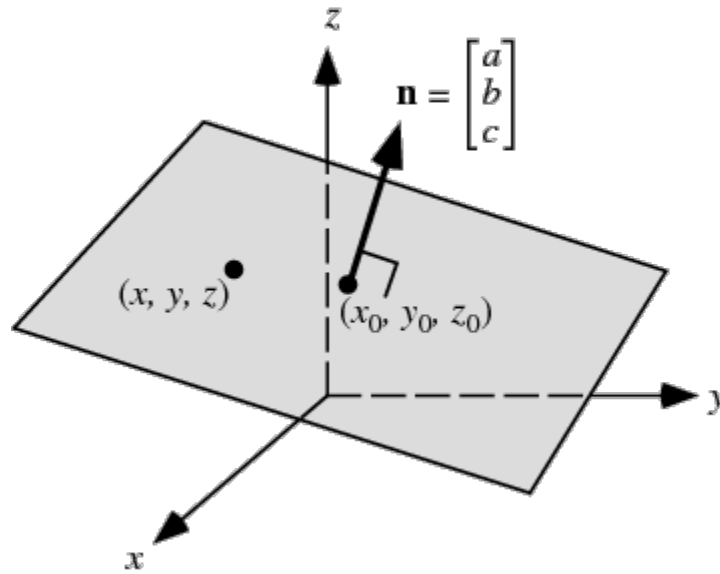
http://en.wikipedia.org/wiki/Differential_geometry

Study of smooth manifolds

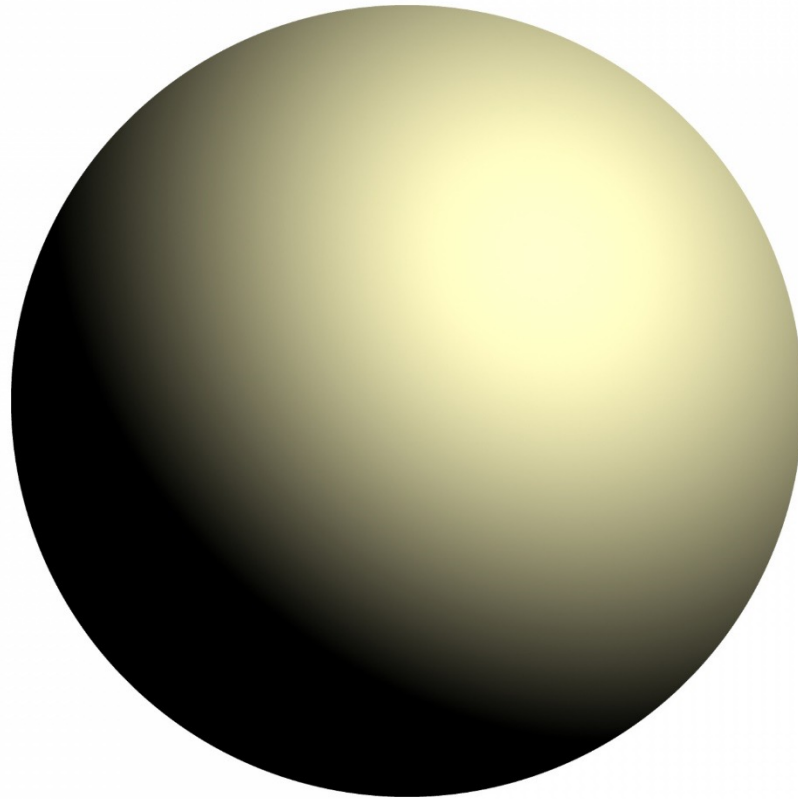
MANIFOLD



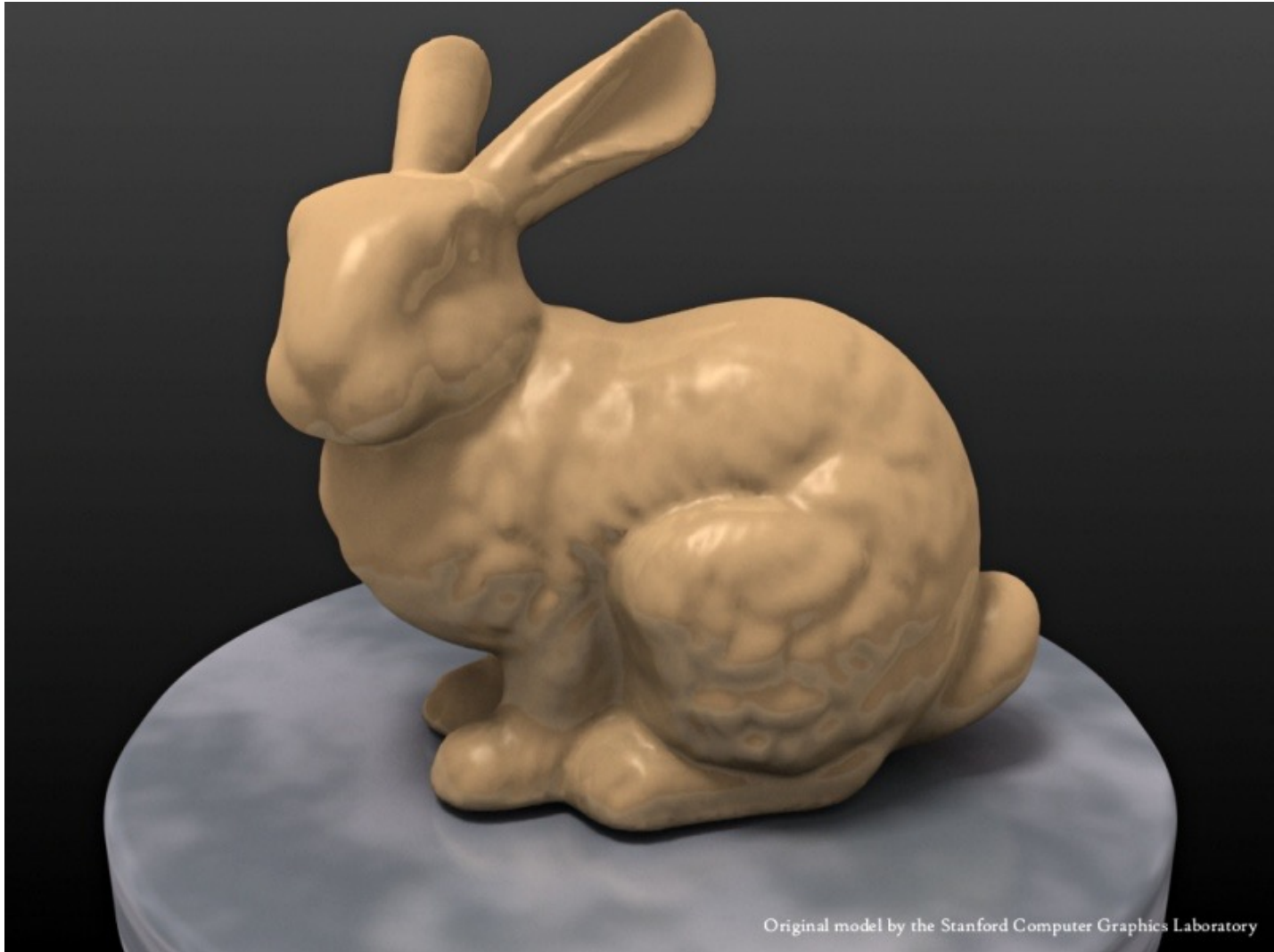
MANIFOLD?



MANIFOLD?



MANIFOLD?

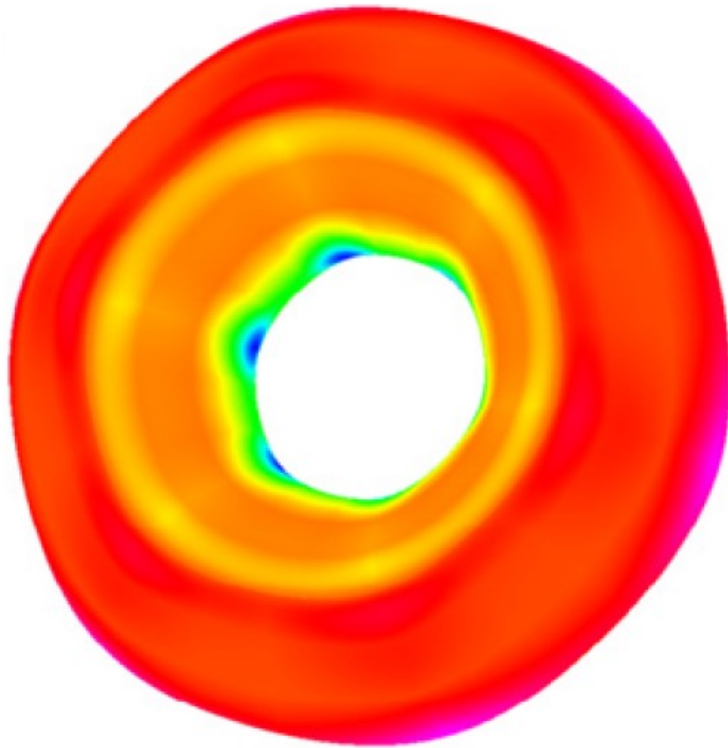


Original model by the Stanford Computer Graphics Laboratory

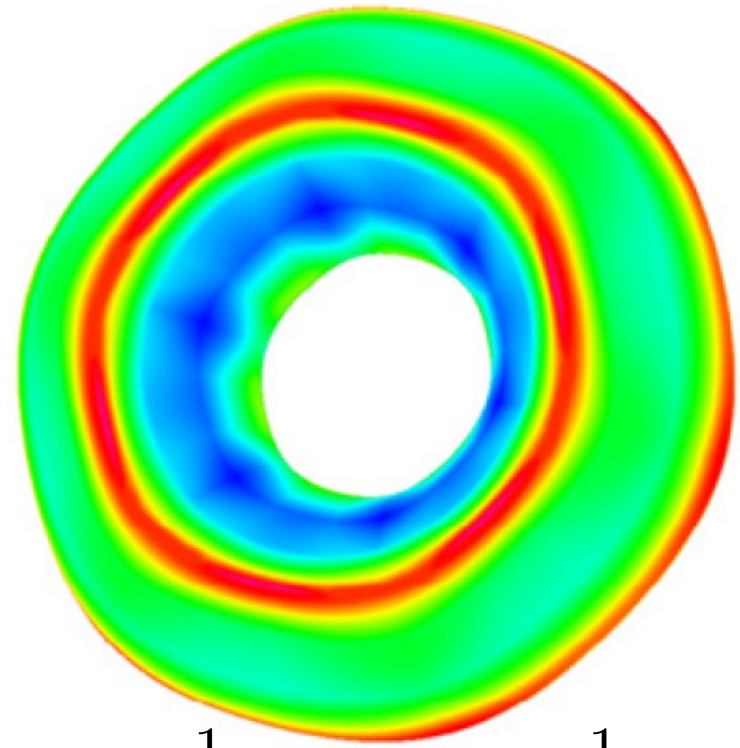
MANIFOLD?



DIFFERENTIAL GEOMETRY TOOLBOX



$$K := \kappa_1 \kappa_2 = \det \mathbb{II}$$



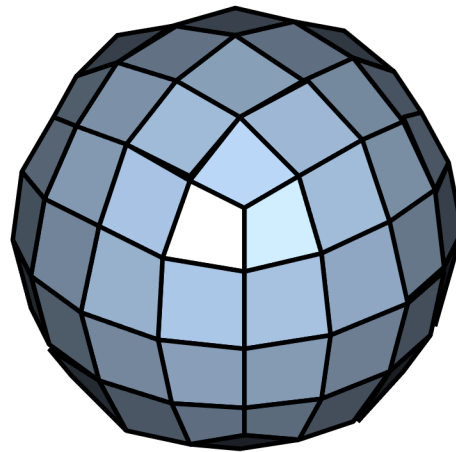
$$H := \frac{1}{2}(\kappa_1 + \kappa_2) = \frac{1}{2} \operatorname{tr} \mathbb{II}$$

<http://www.sciencedirect.com/science/article/pii/S0010448510001983>

Curvature and shape properties

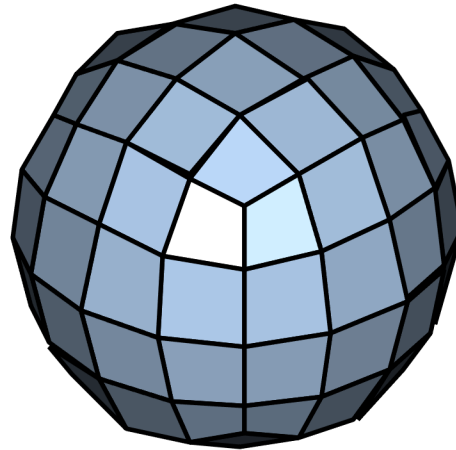
WE WANT TO STUDY SHAPES

What graphics people think:



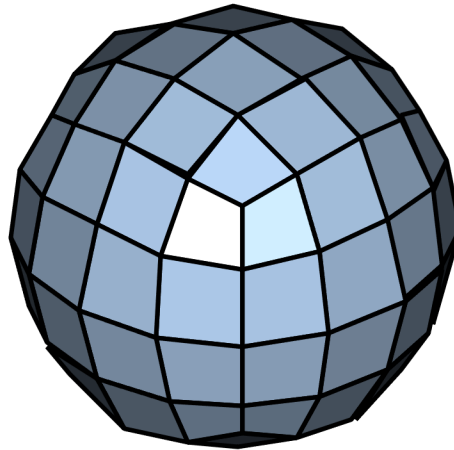
WE WANT TO STUDY SHAPES

- Collection of **flat triangles/quads**
- Approximates a **smooth surface**

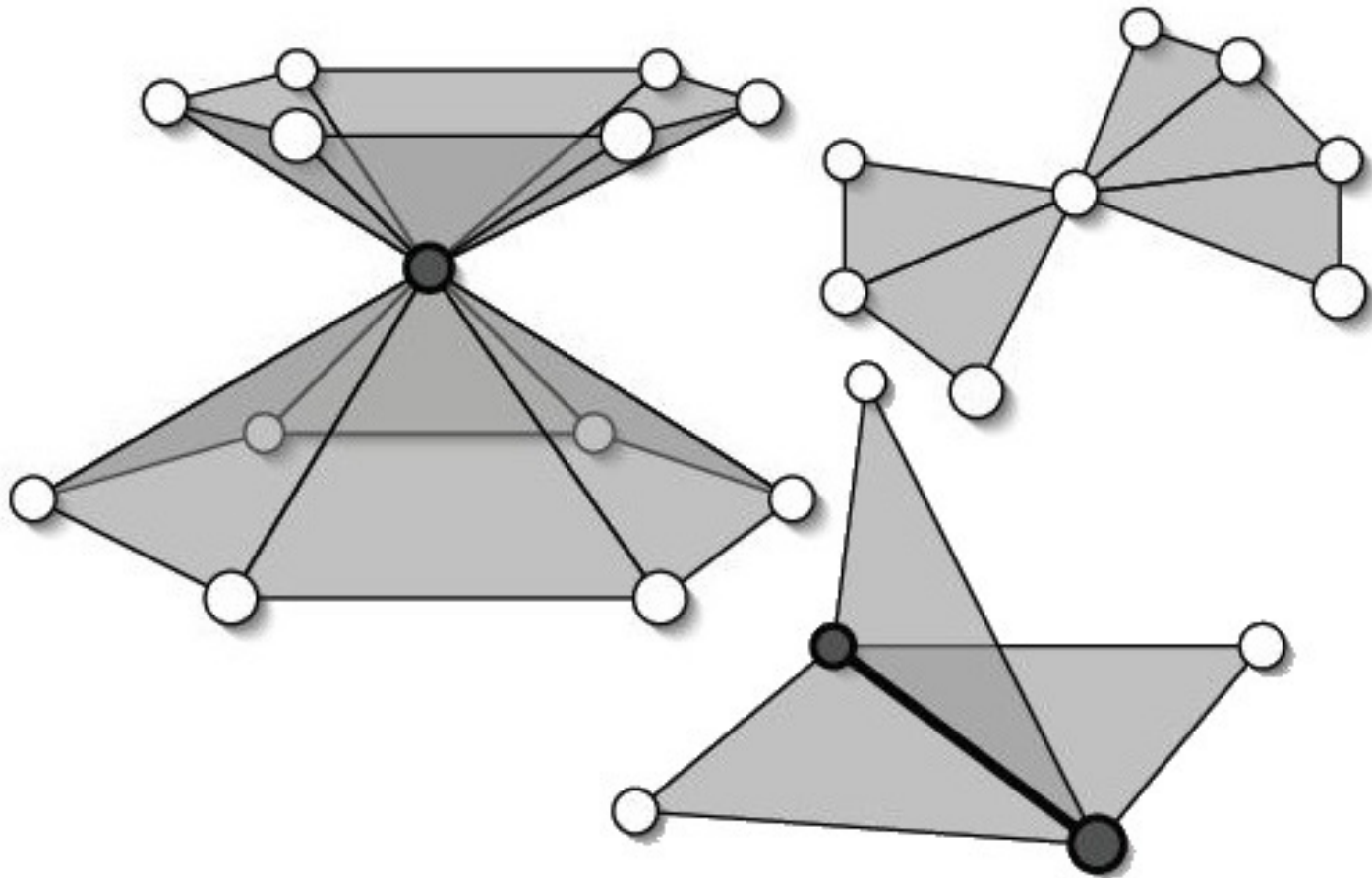


WE WANT TO STUDY SHAPES

- Can such a mesh have curvature?



MANIFOLD?

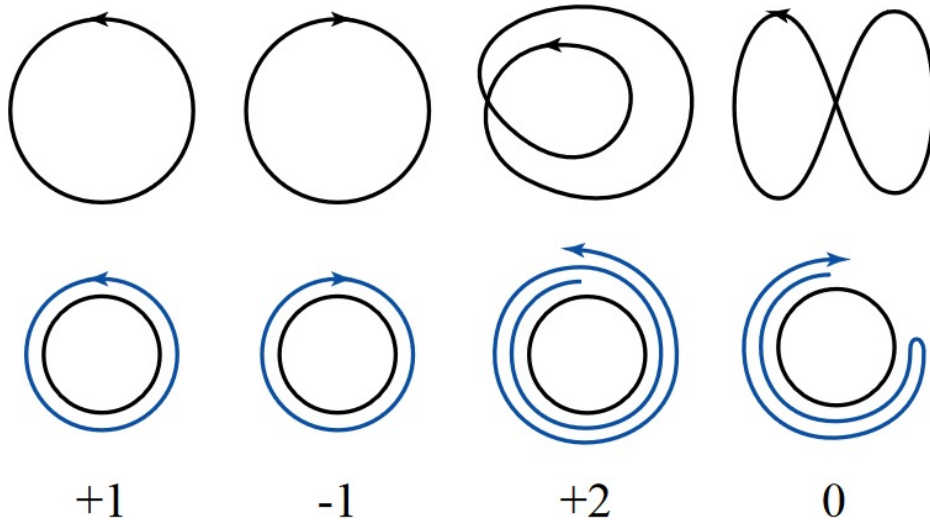


DISCRETE DIFFERENTIAL GEOMETRY

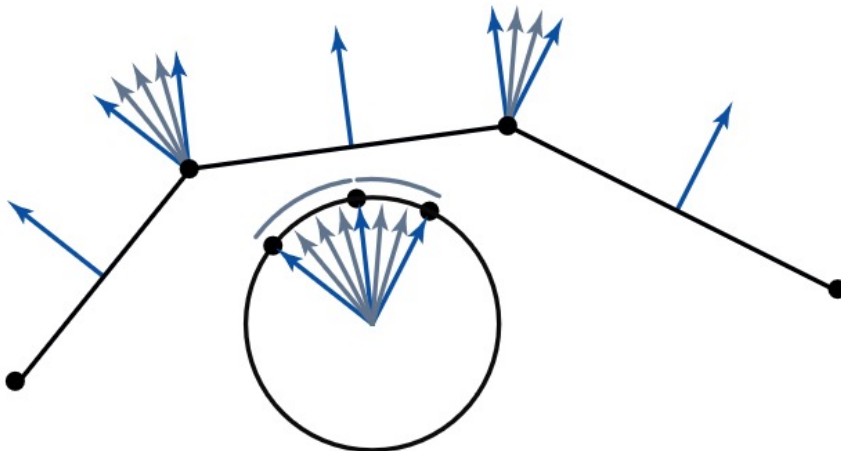
Structure preservation:

- Keeping properties from the continuous abstraction exactly true in a discretization.

EXAMPLE: TURNING NUMBERS



$$\int_{\Omega} \kappa ds = 2\pi k$$



$$\sum_i \alpha_i = 2\pi k$$

DISCRETE DIFFERENTIAL GEOMETRY

Eurographics Symposium on Geometry Processing (2007)
Alexander Belyaev, Michael Garland (Editors)

Discrete Laplace operators: No free lunch

Max Wardetzky¹

Saurabh Mathur²

Felix Kälberer¹

Eitan Grinspun² †

¹Freie Universität Berlin, Germany

²Columbia University, USA

the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators.

operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators. Finally, we present a family of operators that includes and extends well-known and widely-used operators.

1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tau00, Zha04, FH05, Sor05].

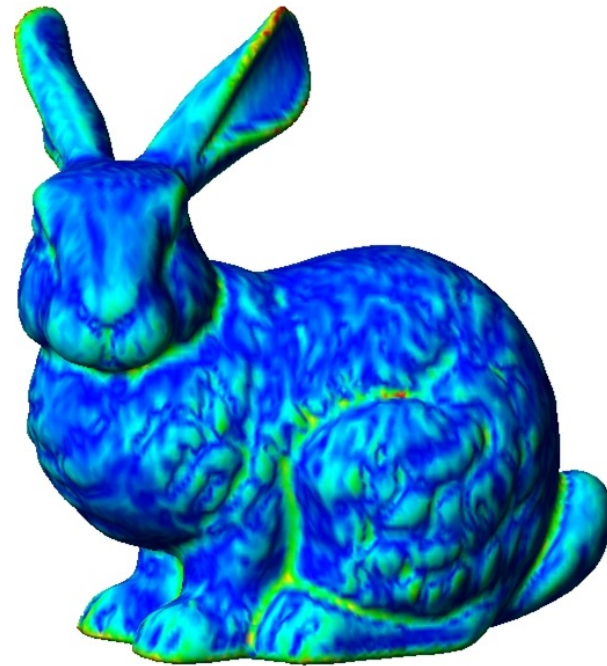
In applications one often requires certain structural prop-

1.1. Properties of smooth Laplacians

Consider a smooth surface S , possibly with boundary, equipped with a Riemannian metric, *i.e.*, an intrinsic notion of distance. Let the intrinsic L^2 inner product of functions u and v on S be denoted by $(u, v)_{L^2} = \int_S uv \, dA$, and let $\Delta = -\operatorname{div} \operatorname{grad}$ denote the intrinsic smooth Laplace-Beltrami operator [Ros97]. We list salient properties of this operator:

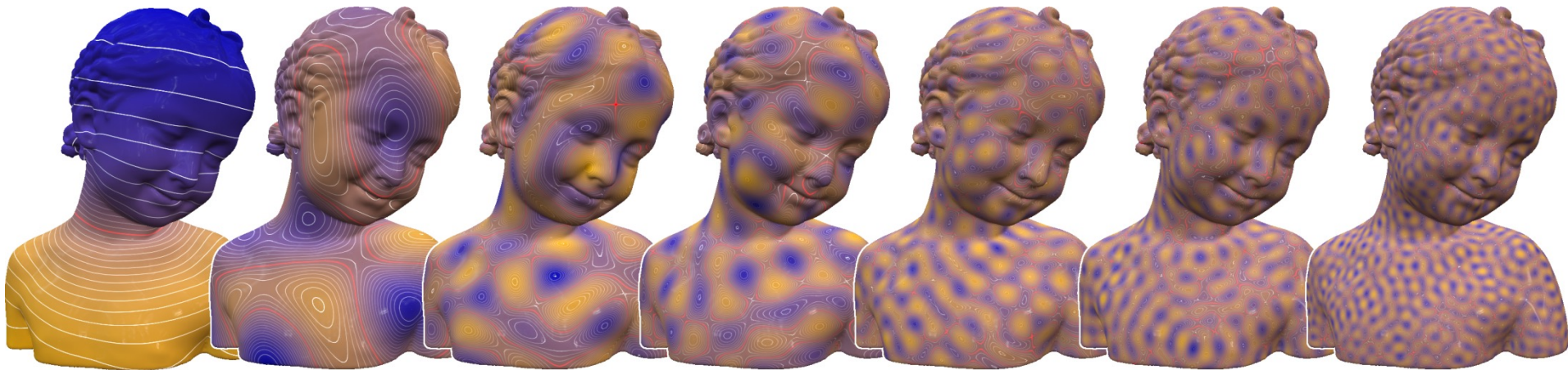
(NULL) $\Delta u = 0$ whenever u is constant.

THEORY: GRAD



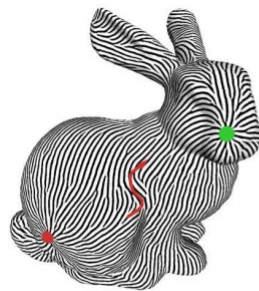
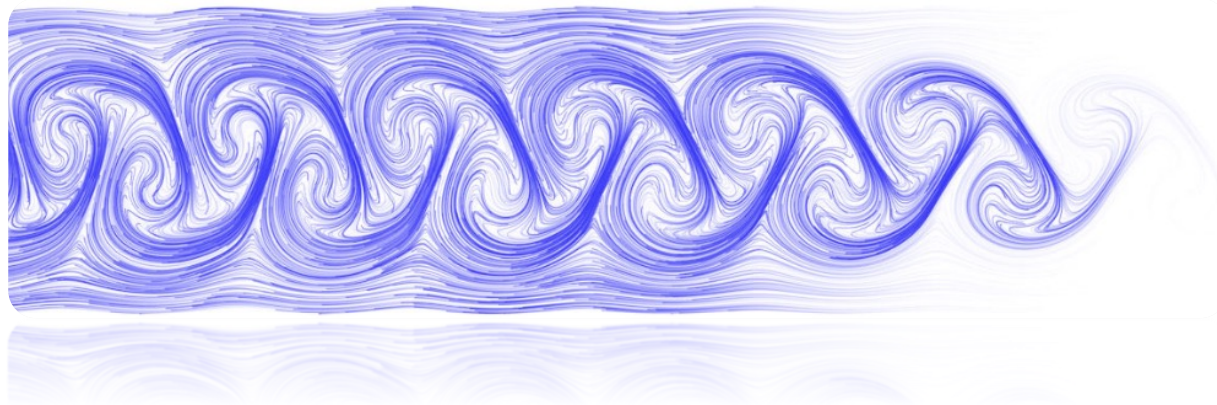
THEORY: LAPLACIAN

$$\Delta$$



Vallet and Lévy.
Spectral Geometry Processing with Manifold Harmonics. EG 2008

THEORY: VECTOR FIELDS AND FLOWS



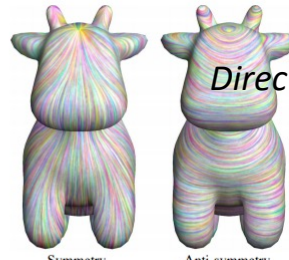
(a) [FSDH07]



(b) [BCBSG10]



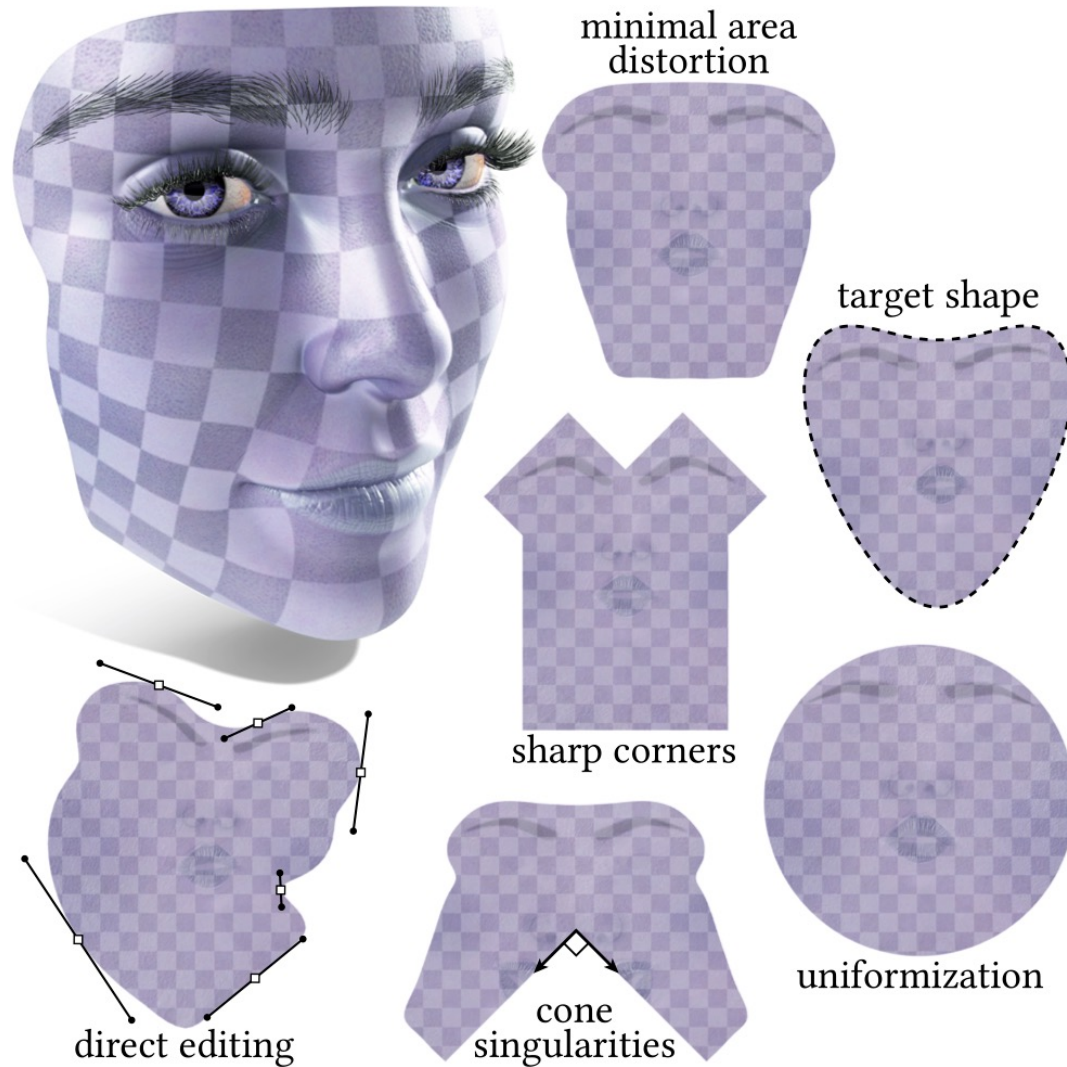
(c) [KCPS13]



(d) [ABCCO13]

Vaxman et al.
Directional field synthesis, design, and processing.
EG STAR 2016.

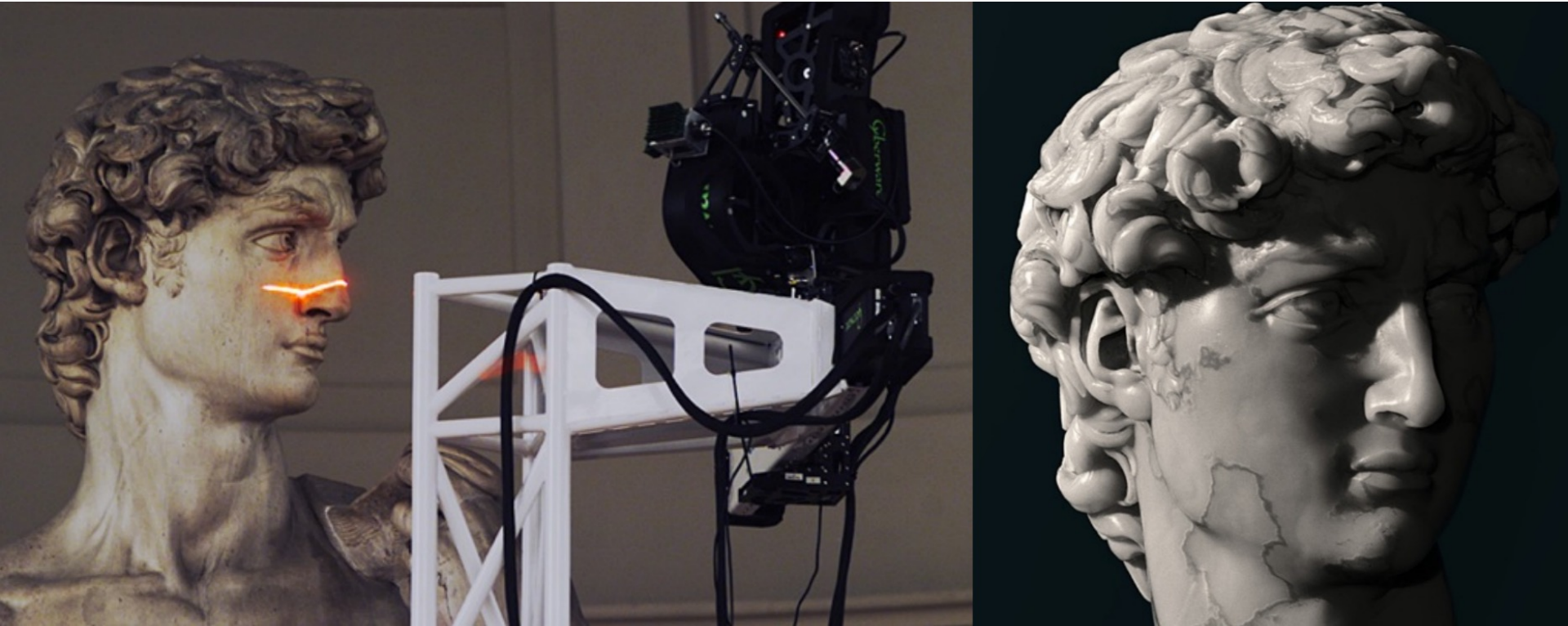
NUMERICS



TODAY

1. Geometric modeling?
2. Shape analysis?
3. Studying shapes: what we need
- 4. Applications**

APPLICATION: 3D RECONSTRUCTION



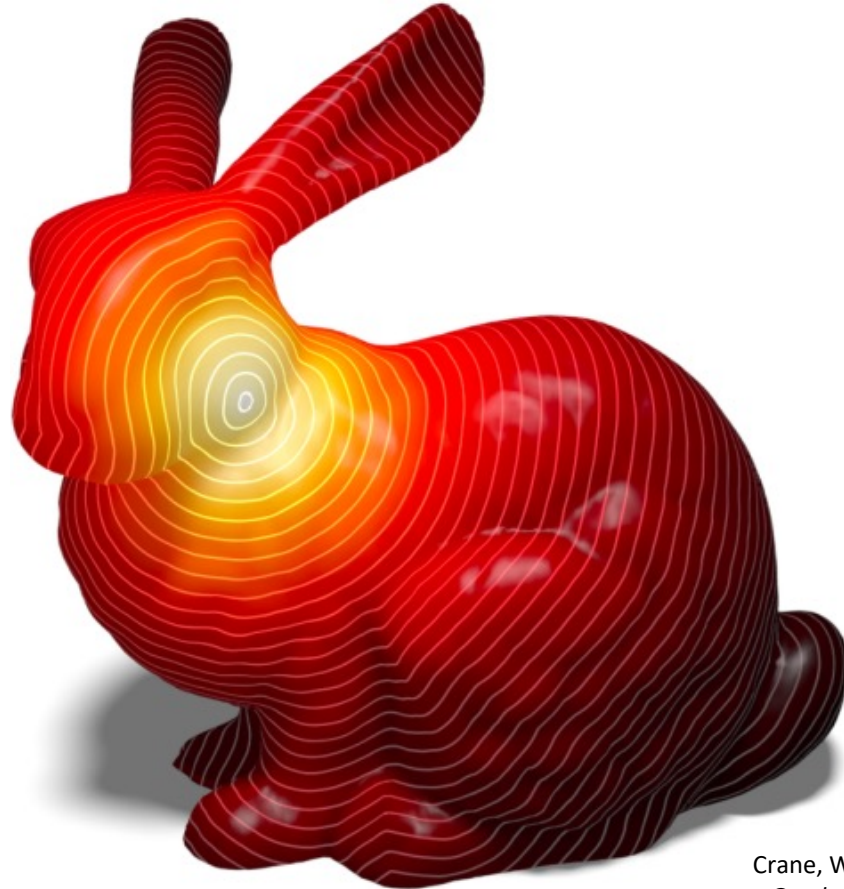
Depth maps [?] → points [?] → triangle mesh

APPLICATIONS: MESH DEFORMATION



How to propagate deformations to **unconstrained parts of shape?**

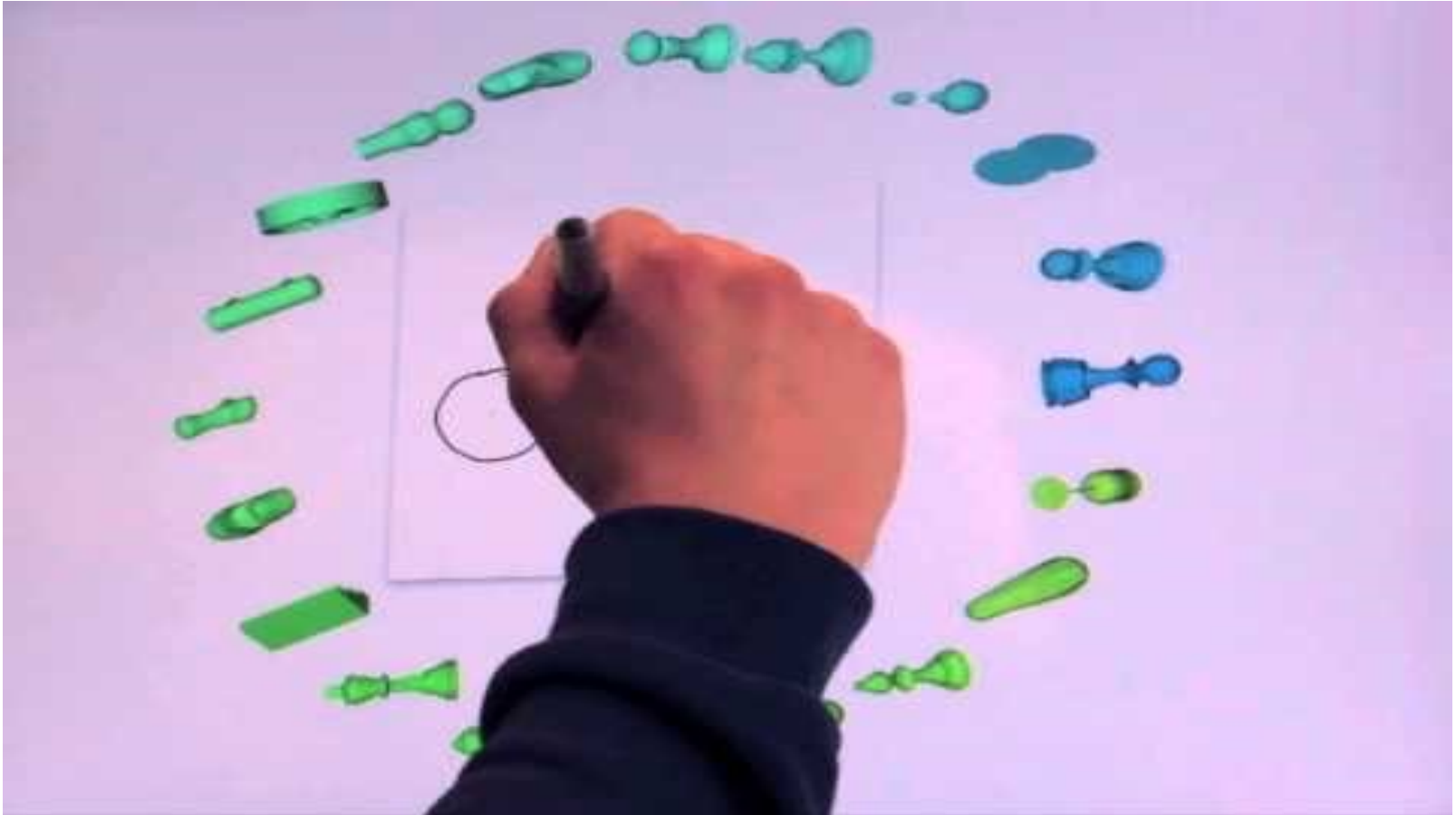
APPLICATIONS: HEAT AND GEODESICS



Crane, Weischedel, Wardetzky.
Geodesics in heat. TOG 2013.

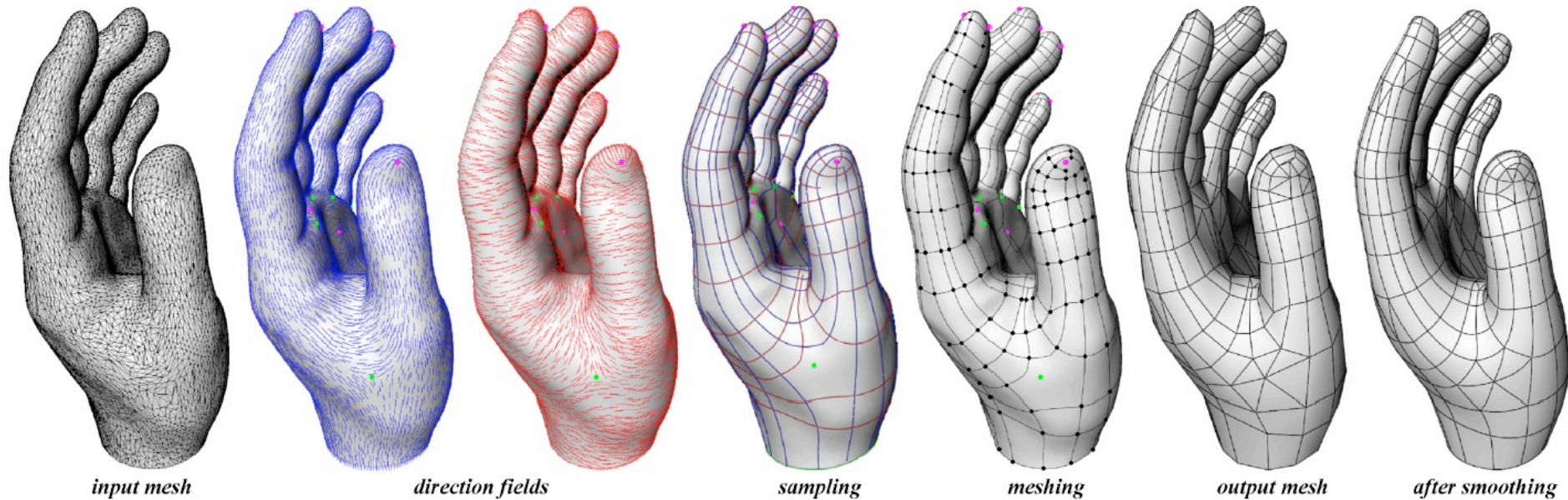
How to compute distances
between points on a surface?

APPLICATIONS: RETRIEVAL



How to **compare** shapes of
different modalities?

APPLICATIONS: REMESHING



Pierre Alliez, David Cohen-Steiner, Olivier Devillers, Bruno Lévy, and Mathieu Desbrun. 2003. Anisotropic polygonal remeshing. ACM Trans. Graph. 22, 3 (July 2003)

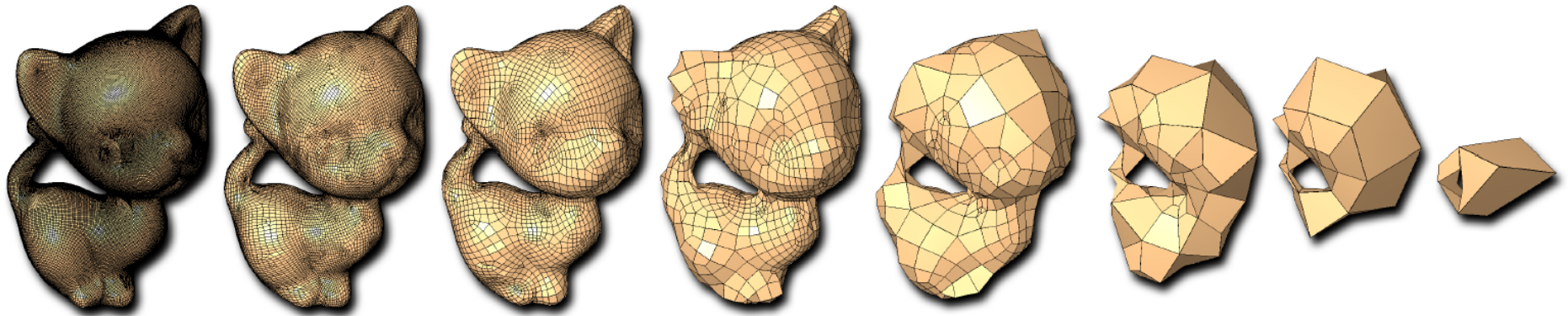
How to **convert** any mesh
into a **nice** one?

APPLICATIONS: QUADMESHING



How to make a **mesh** out of **quads**?

APPLICATIONS: MESH SIMPLIFICATION

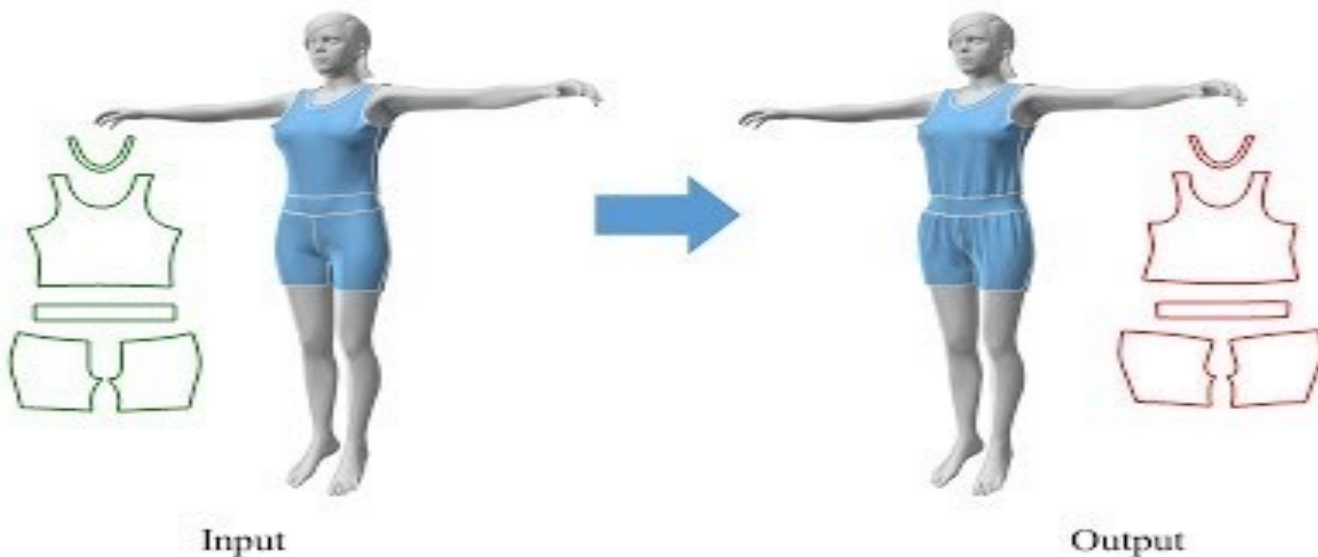


Joel Daniels, Cláudio T. Silva, Jason Shepherd, and Elaine Cohen. 2008. Quadrilateral mesh simplification. ACM Trans. Graph. 27, 5, Article 148 (December 2008)

How to **compress** geometry?

APPLICATIONS: CLOTH MODELING

FoldSketch



APPLICATIONS: SKETCH-BASED MODELING

...perceptual validation

#1 Humans are consistent in perceiving regularity cues in a design sketch.

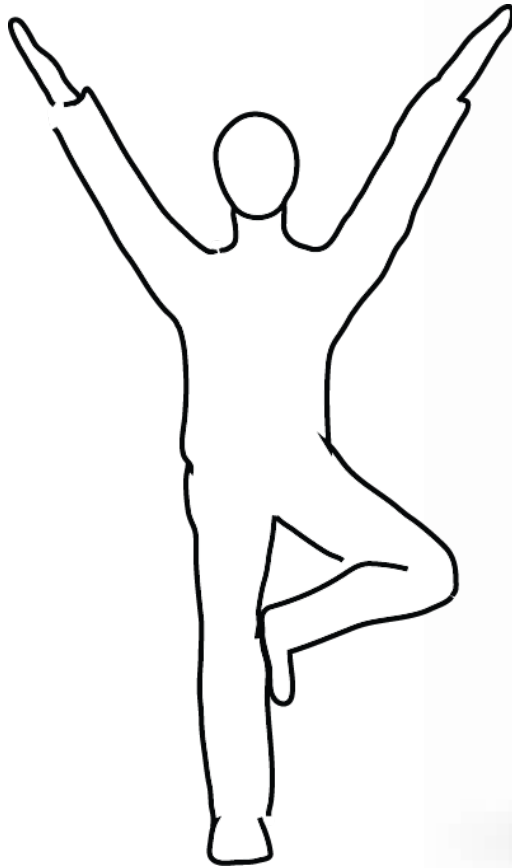
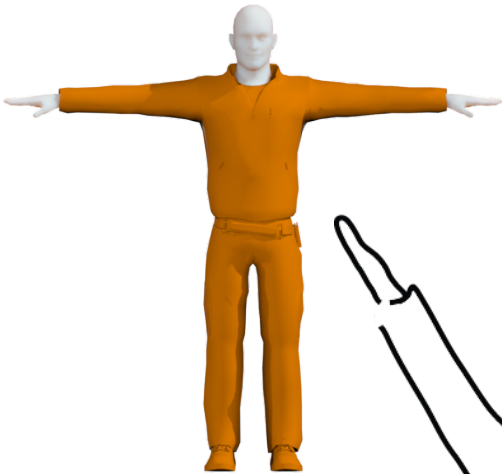


APPLICATIONS: SKETCH-BASED MODELING

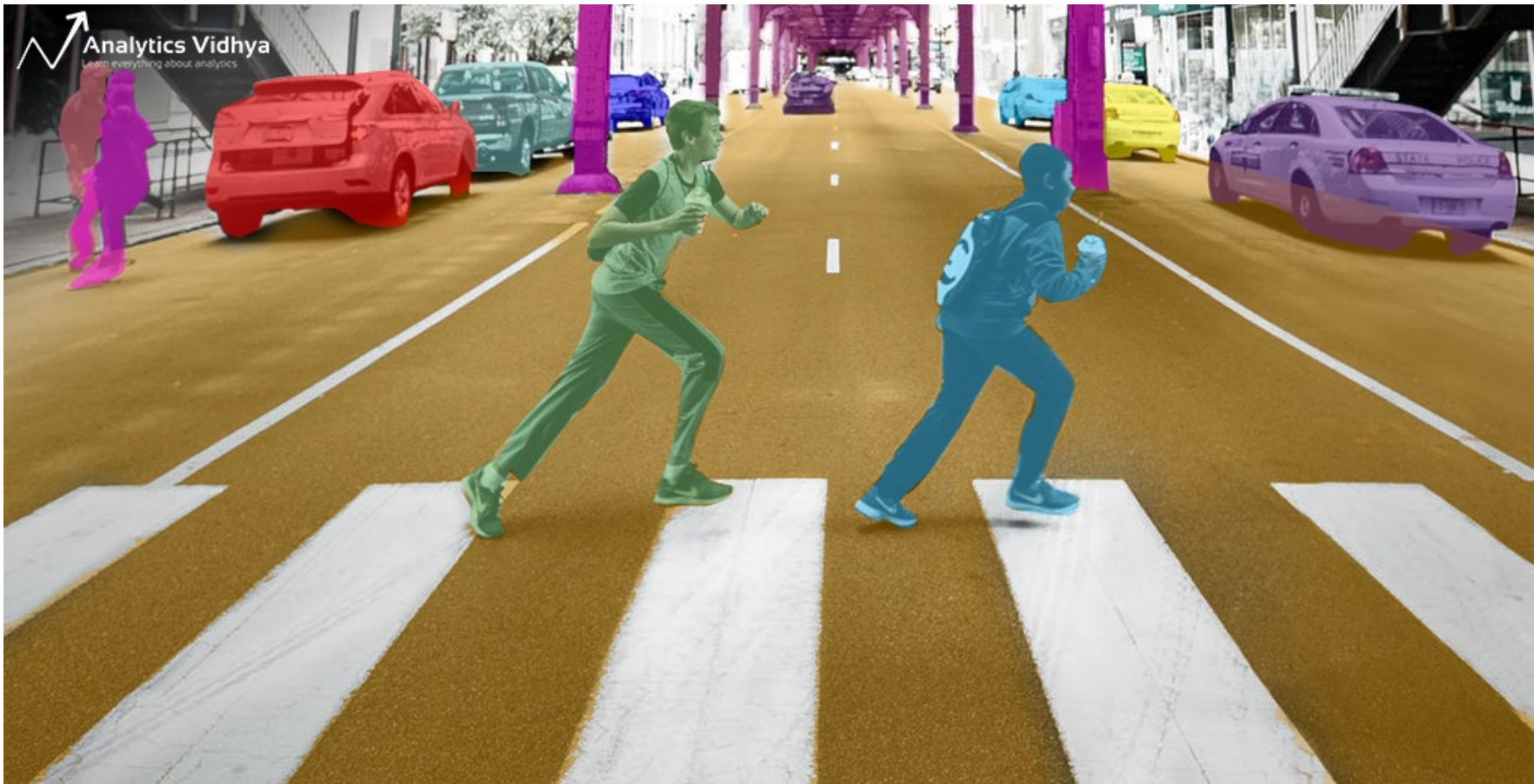
Input:

rigged character model
+ gesture Drawing

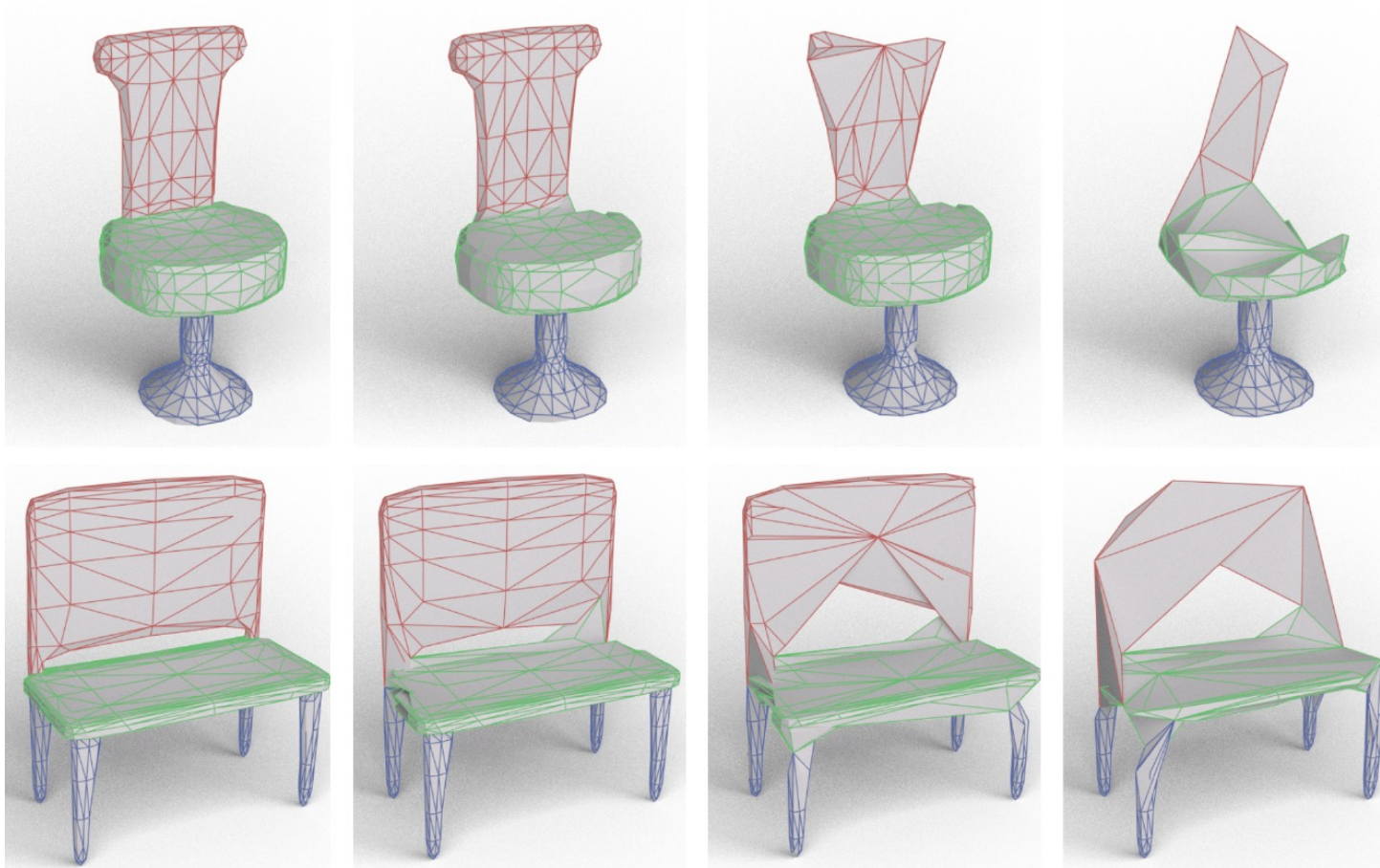
Result



APPLICATIONS: LEARNING



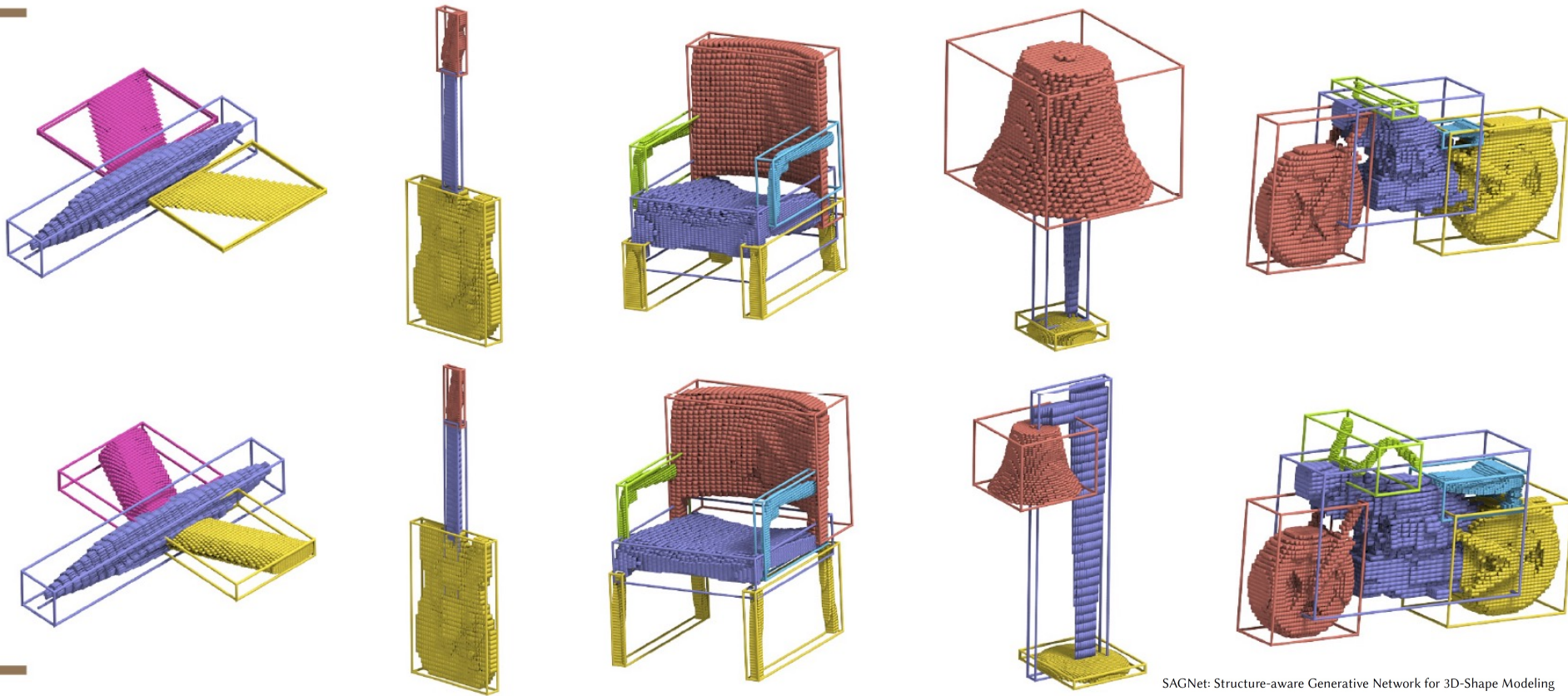
APPLICATIONS: LEARNING



"MeshCNN: A Network with an Edge" by Rana Hanocka, Amir Hertz, Noa Fish, Raja Giryes, Shachar Fleishman, Daniel Cohen-Or, SIGGRAPH 2019

How to bring **learning** approaches
from images to **3D geometry**?

APPLICATIONS: LEARNING



SAGNet: Structure-aware Generative Network for 3D-Shape Modeling

ZHIJIE WU, Shenzhen University
XIANG WANG, Shenzhen University
DI LIN, Shenzhen University
DANI LISCHINSKI, The Hebrew University of Jerusalem
DANIEL COHEN-OR, Shenzhen University
HUI HUANG*, Shenzhen University

How to bring **learning** approaches
from images to **3D geometry**?

</INTRO>

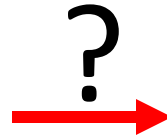
<http://tiny.cc/ift6113>

Piazza!

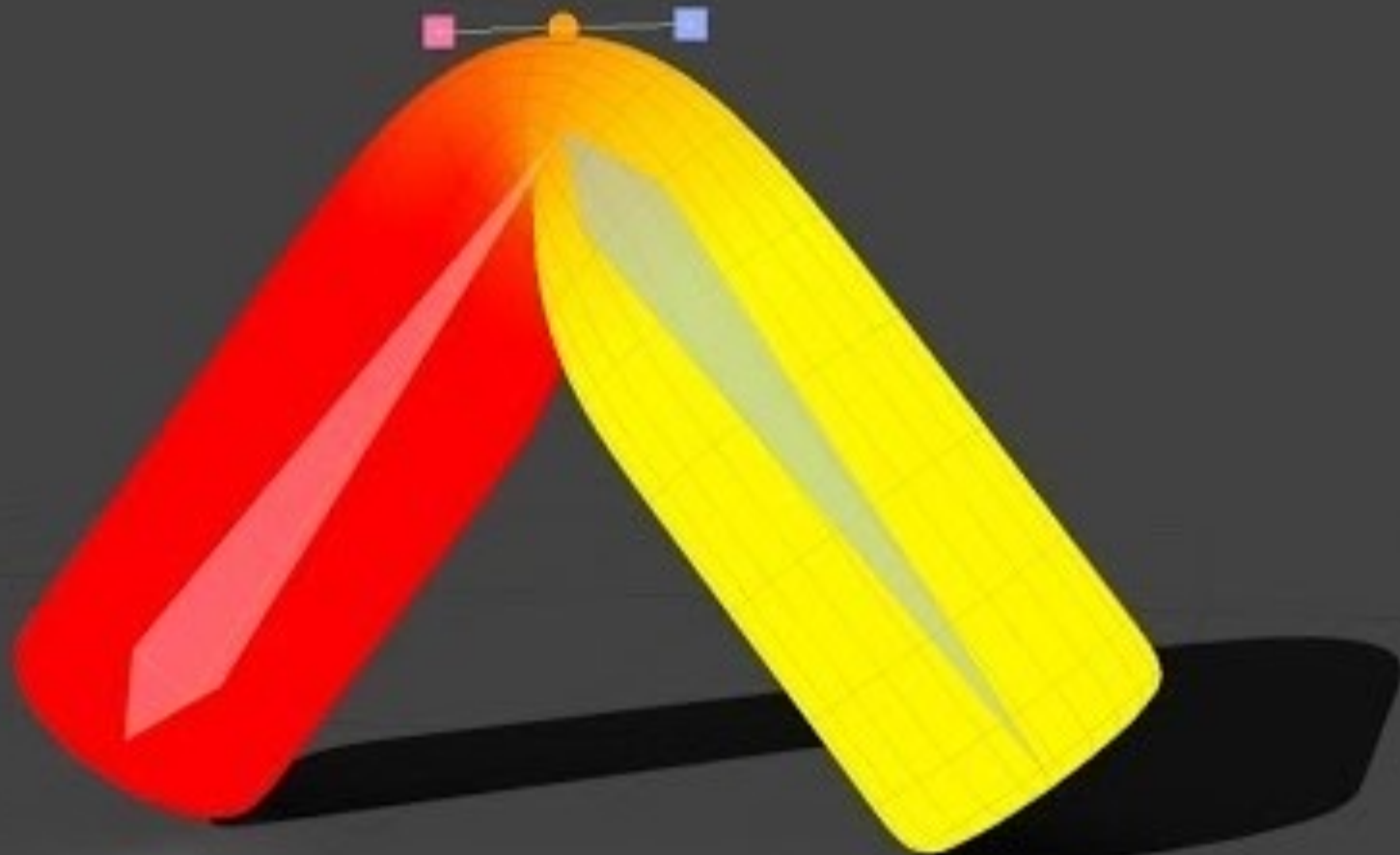
Office hour: Fri, 5-6 pm, Zoom



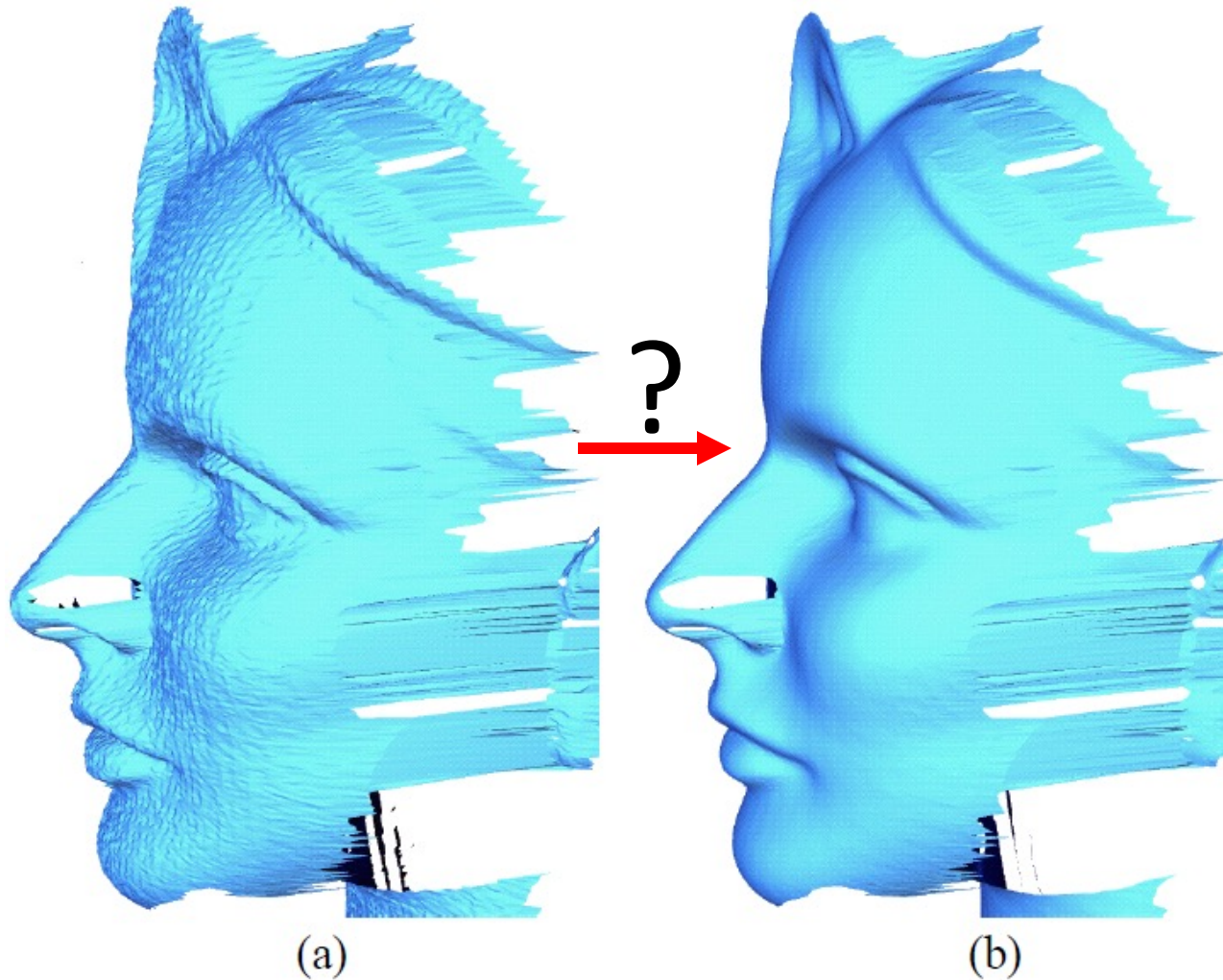
WARM-UP PROBLEMS



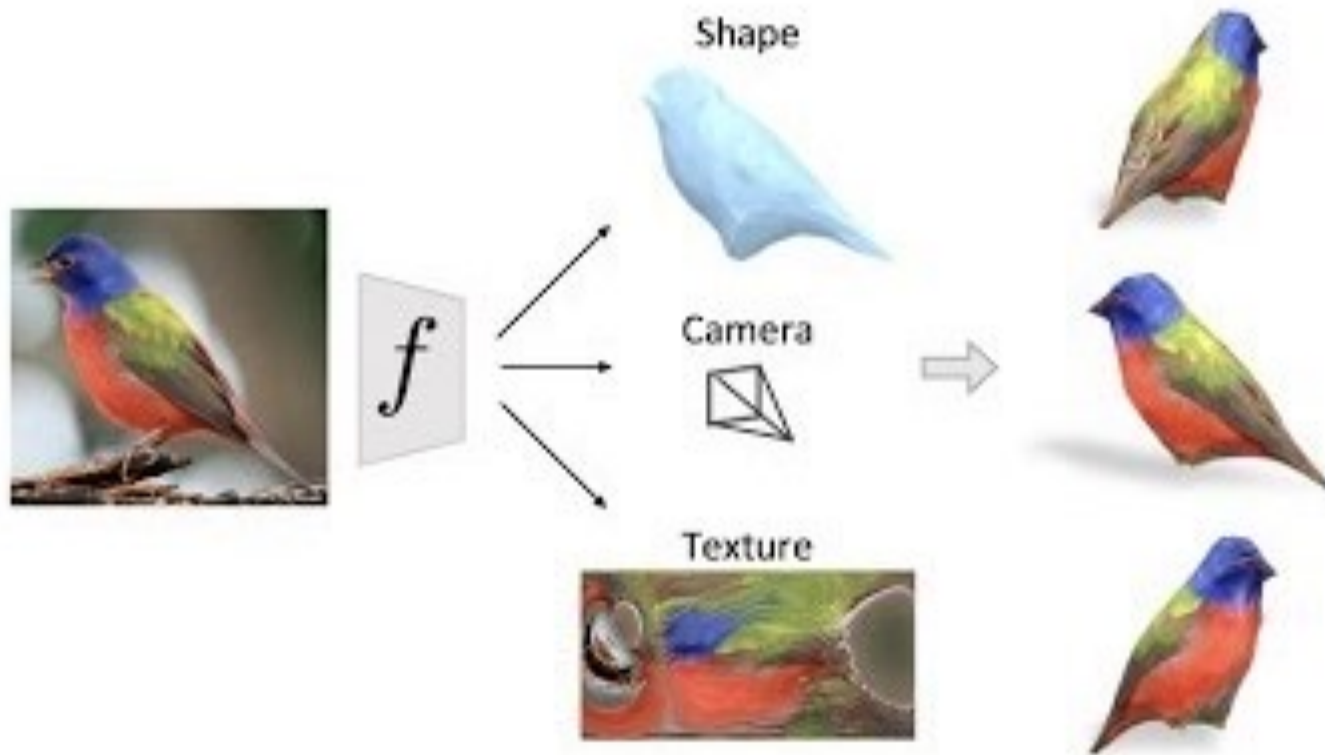
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WARM-UP PROBLEMS



WARM-UP PROBLEMS



LIBIGL & EIGEN

- Eigen: a linear algebra library
- libigl: geometry processing library