

# IFT 6113

# INTRODUCTION

<http://tiny.cc/ift6113>



## Mikhail Bessmeltsev

Some slides from Justin Solomon and Alla Sheffer

**Instructor:** Mikhail Bessmeltsev

**Email:** bmpix@iro.umontreal.ca

**Office hour:** Fri, 4:30-5:30 pm, Zoom or  
AA 3357

**Web:** <http://www-labs.iro.umontreal.ca/~bmpix/>

**Contact:** Piazza (preferred)

# FORMALITIES

**Lectures:** Tue, Thu 8.30-10.30

**Web:** <http://tiny.cc/ift6113>

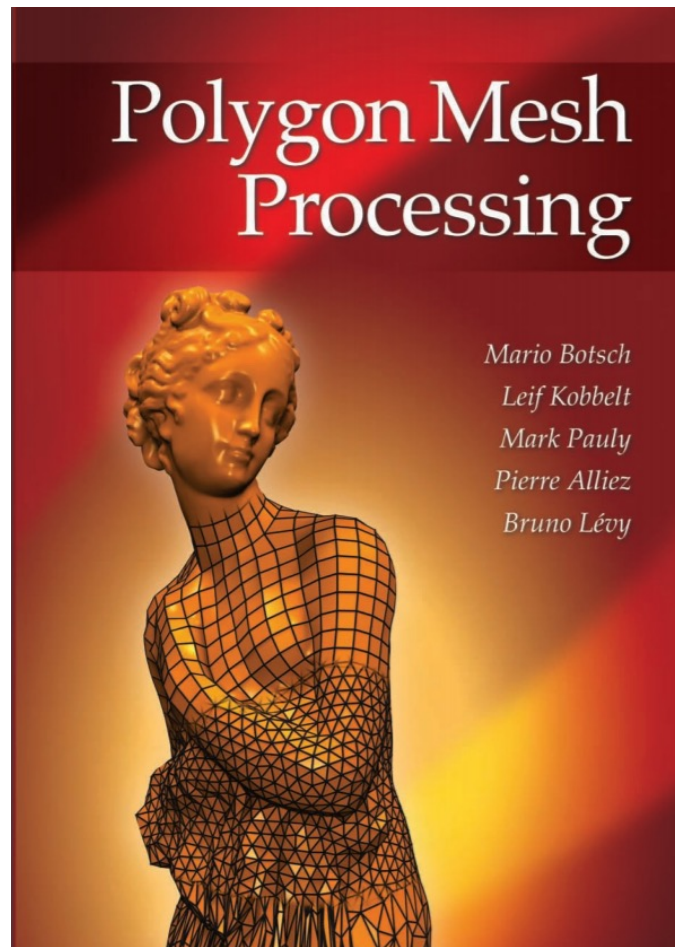
Use Piazza!

<http://piazza.com/umontreal.ca/fall2022/ift6113>

# TEXTBOOK (OPTIONAL)

Polygon Mesh Processing by Botsch et al.

Free online at UdeM library





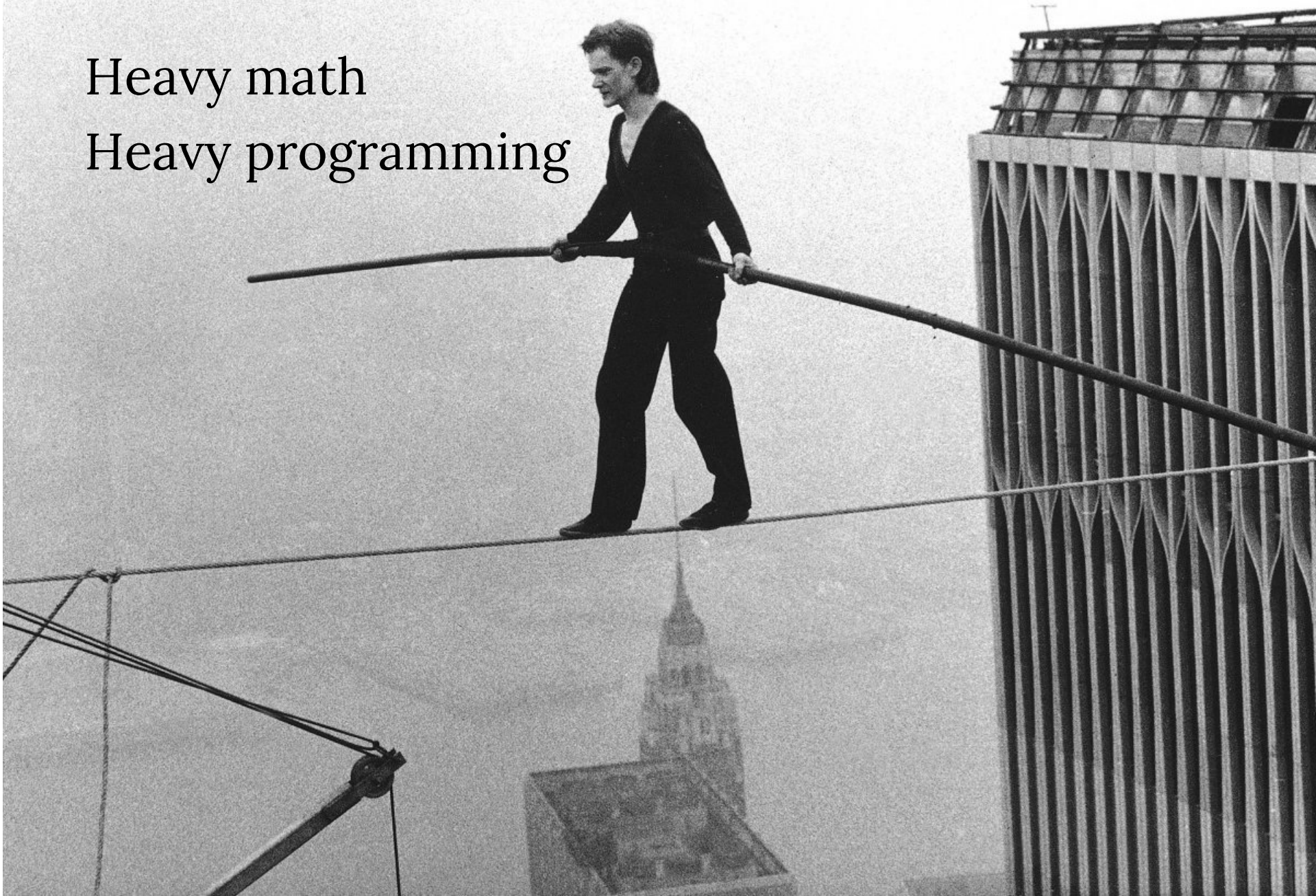
# PREREQS

- For sure:
  - Calculus
  - Linear algebra
- Bonus (but not required):
  - Differential Geometry
  - Partial Differential Equations

# THIS IS NOT AN EASY COURSE

Heavy math

Heavy programming



# **BUT INTERESTING**

I will help  
and be generous with grading

# GRADING

Assignments (33%)		100%
Paper Presentation (25%)		
Participation (2%)		
Final Project (40%)		

# ASSIGNMENTS (33%)

- Coding + math
- ~ 2 weeks long
- C++ (libigl) || matlab || Python



<http://libigl.github.io/libigl/>

# FACE TO FACE GRADING

- Understand each line of your code
- Plagiarism policy

# PAPER PRESENTATION (25%)

- Choose a SIGGRAPH paper on a related topic
  - Give a talk!
  - Others: ask questions
- 
- Choose a paper (and post about it in Piazza) by **Oct 29th**
  - Present on **Nov 23rd**



# **PARTICIPATION (2%)**

- Classroom
- Questions/discussion at paper and project presentations

# FINAL PROJECT (40%)

1. Choose a recent SIGGRAPH paper
2. Propose an improvement/modification
3. Implement
4. Present!

Important dates:

Project proposal: Oct 20<sup>th</sup>, by midnight

Final Presentation: Dec 7<sup>th</sup>, in class

**NO EXAMS**

# WHAT YOU WILL LEARN

## ***Fundamentals:***

Mesh data structures  
Discrete Differential Geometry  
Discrete Laplacians  
Vector Fields

## ***Applications:***

Shape Deformations  
Surface Reconstruction  
Mesh Parameterization  
Remeshing  
Animation/Sketch-based modeling/...  
Learning geometry

# TODAY

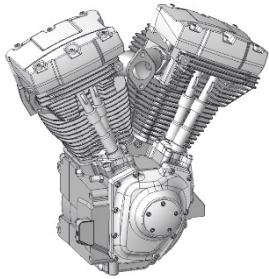
1. Geometric modeling?
2. Shape analysis?
3. Studying shapes: what we need
4. Applications

# TODAY

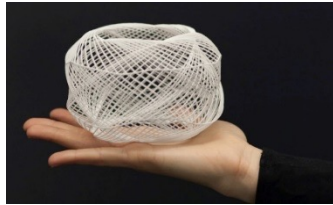
- 1. *Geometric modeling?***
- 2. *Shape analysis?***
3. Studying shapes: what we need
4. Applications

# **WHAT IS GEOMETRIC MODELING?**





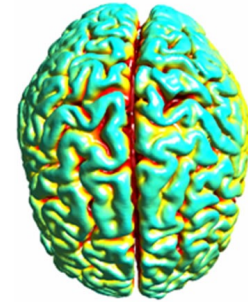
Mechanical  
Engineering



Fabrication



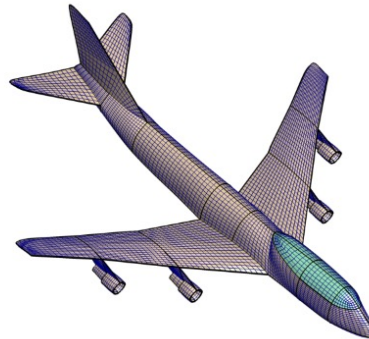
Architecture  
and civil  
engineering



Medical



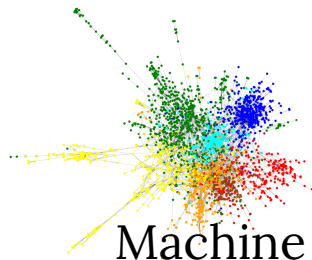
Computer Vision



Geometric  
Modeling



Computer  
Graphics



Machine  
Learning



Augmented/Virtual Reality



Human-Computer Interaction (HCI)

$$\begin{aligned} \min & f(x) \\ \text{s.t. } & g(x) \leq 0 \\ & h(x) = 0 \end{aligned}$$

Numerical Optimization

**THIS.**



**SENUAS SAGA**  
**HELLBLADE II**



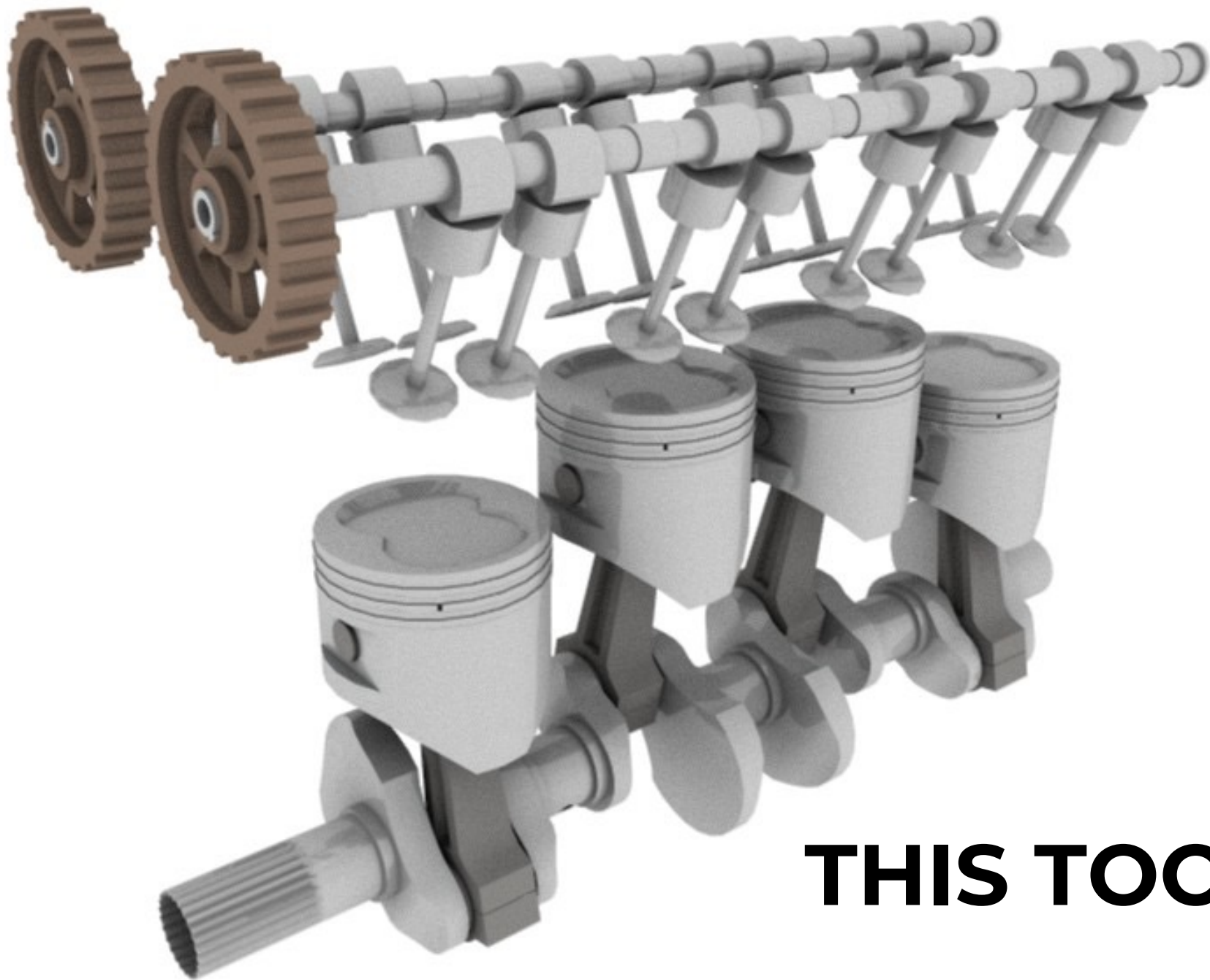
DNEG

# VFX BREAKDOWN CITADEL

**AND THIS.**

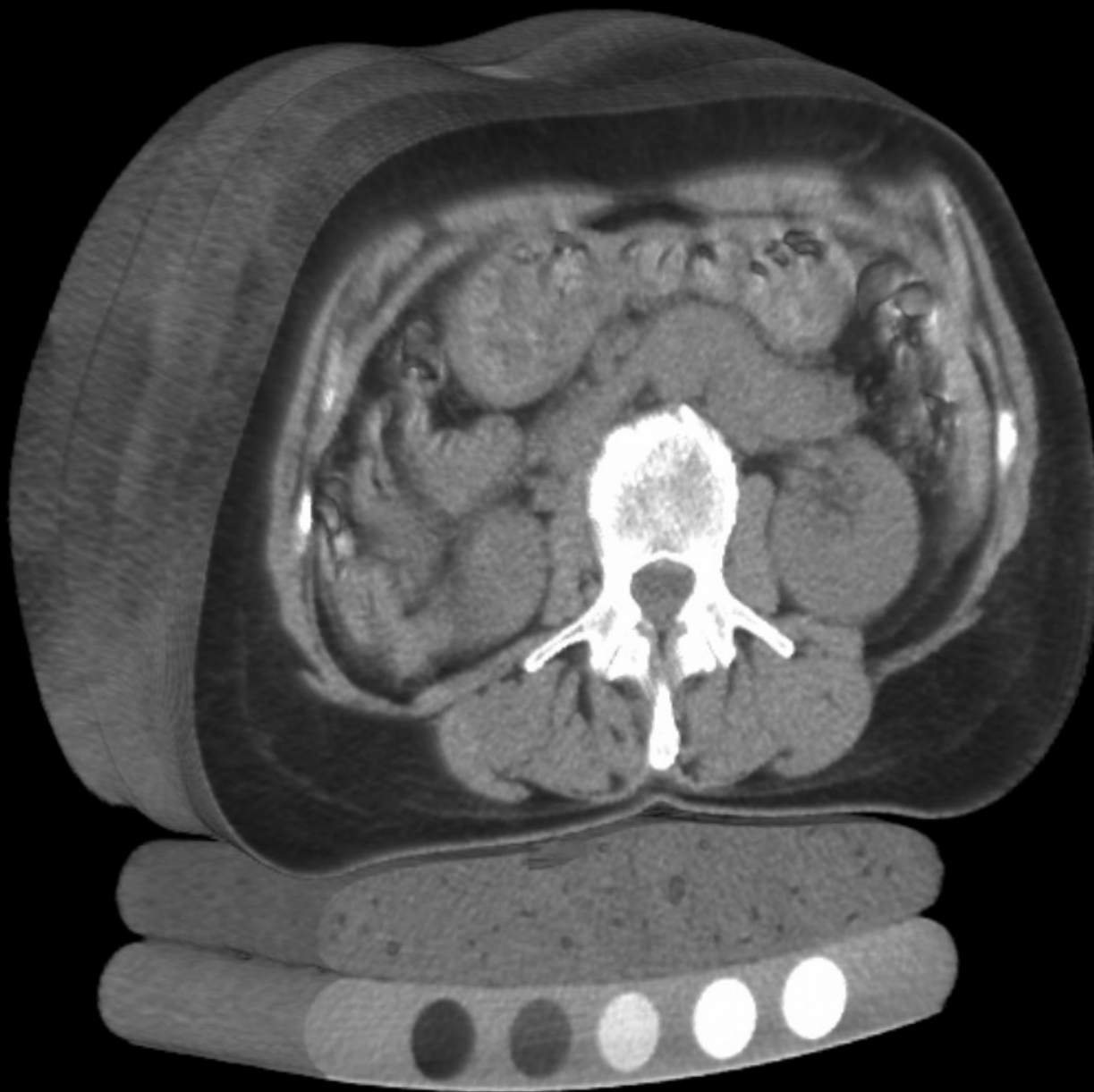






**THIS TOO.**

**AND EVEN THIS.**





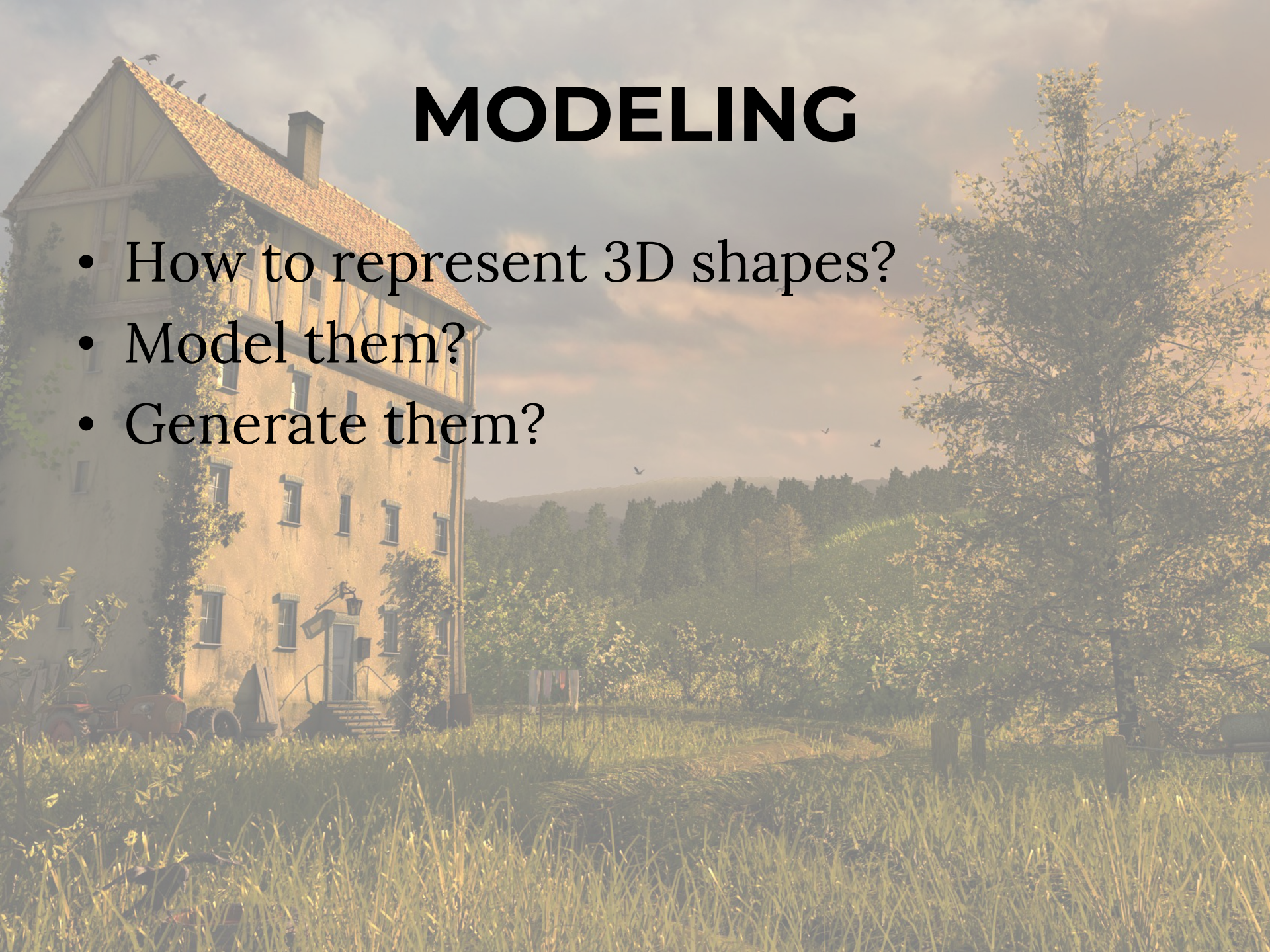
# MODELING





# MODELING

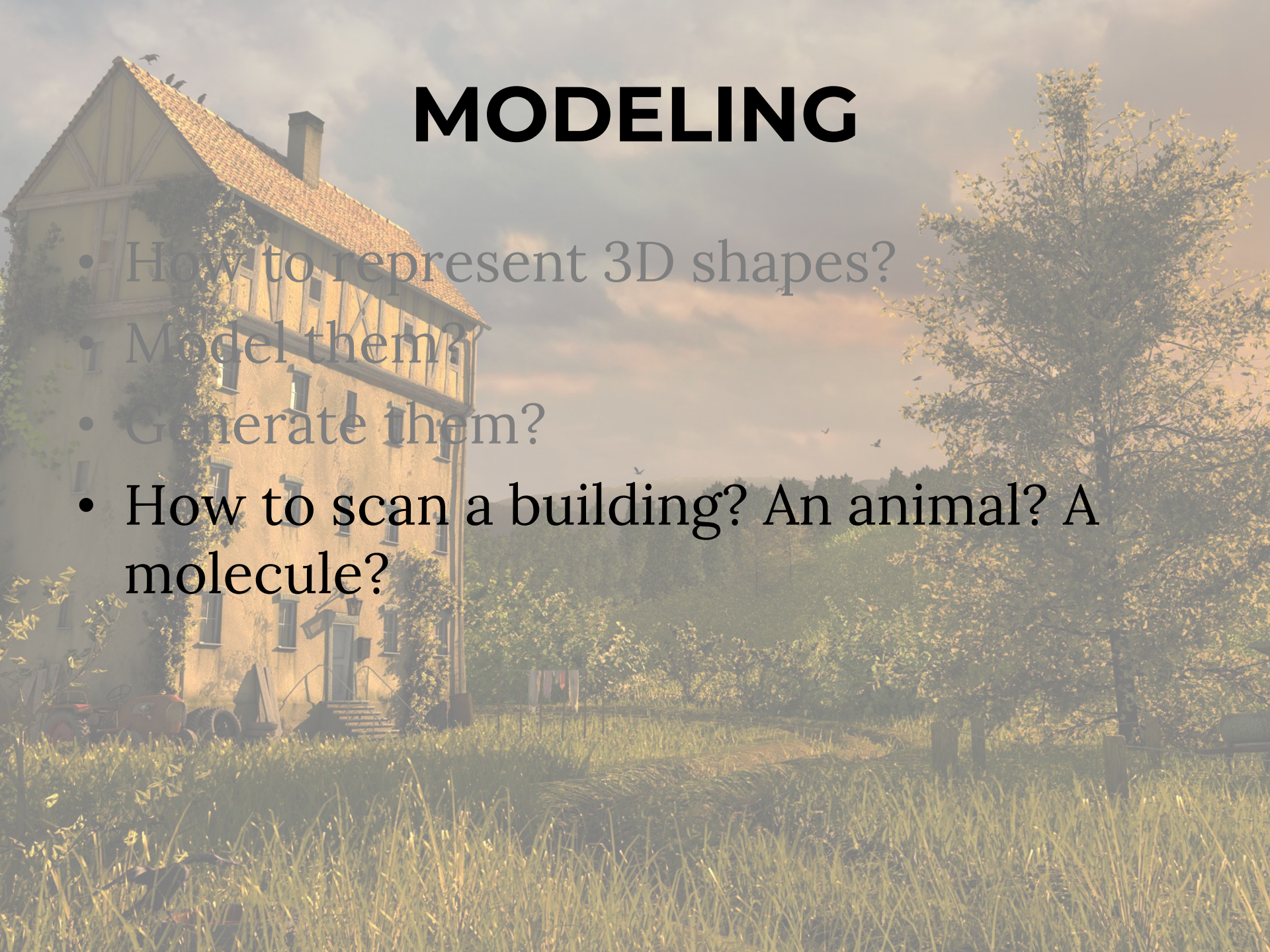
- How to represent 3D shapes?
- Model them?
- Generate them?





# MODELING

- How to represent 3D shapes?
- Model them?
- Generate them?
- How to scan a building? An animal? A molecule?





# MODELING

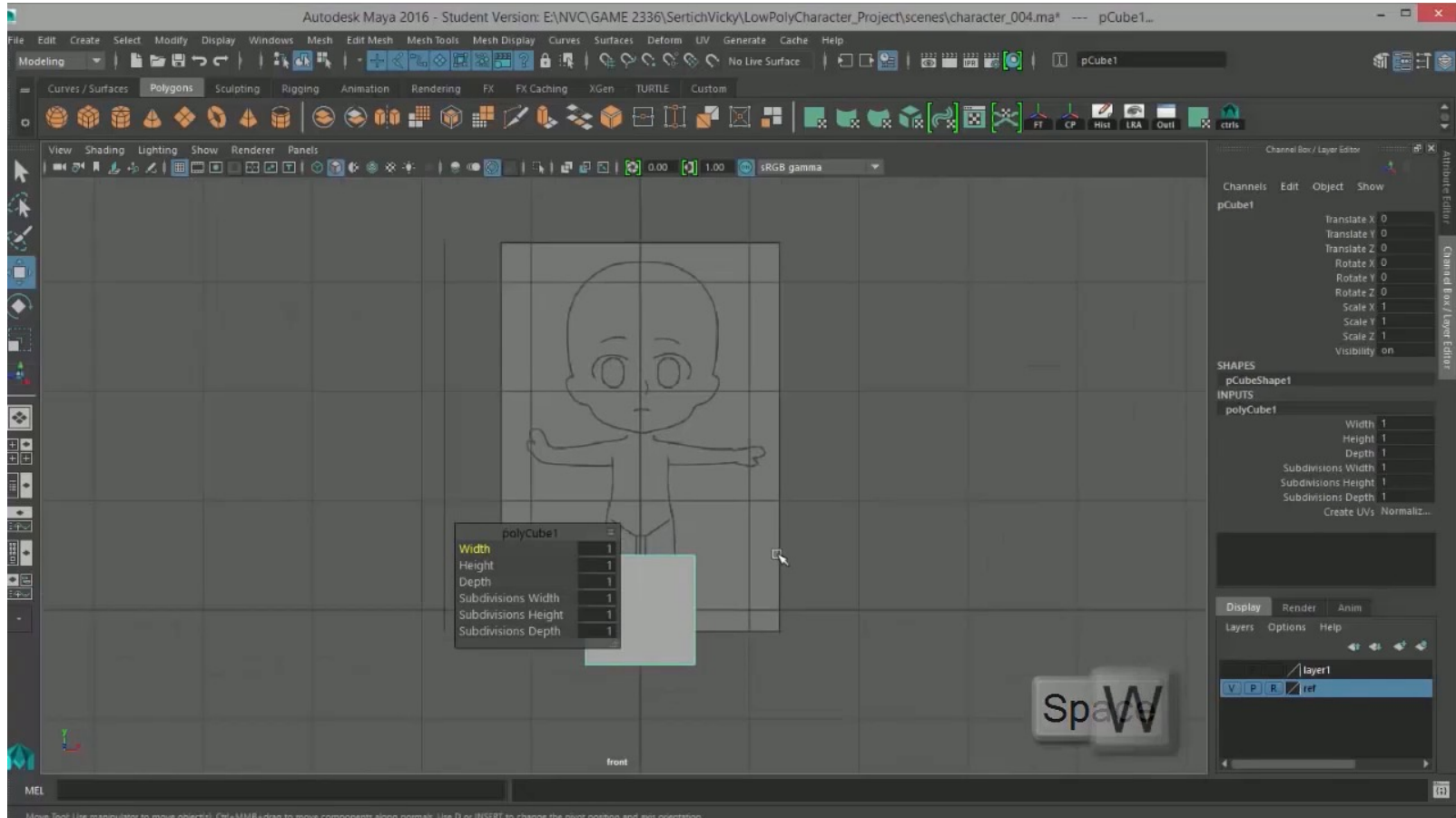
- How to represent 3D shapes?
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- Generate them?
- How to scan a building? An animal? A molecule?
- How to deform a model?



# MODELING

- How to represent 3D shapes?
- Model them?
- Generate them?
- How to scan a building? An animal? A molecule?
- How to deform a model?
- ...

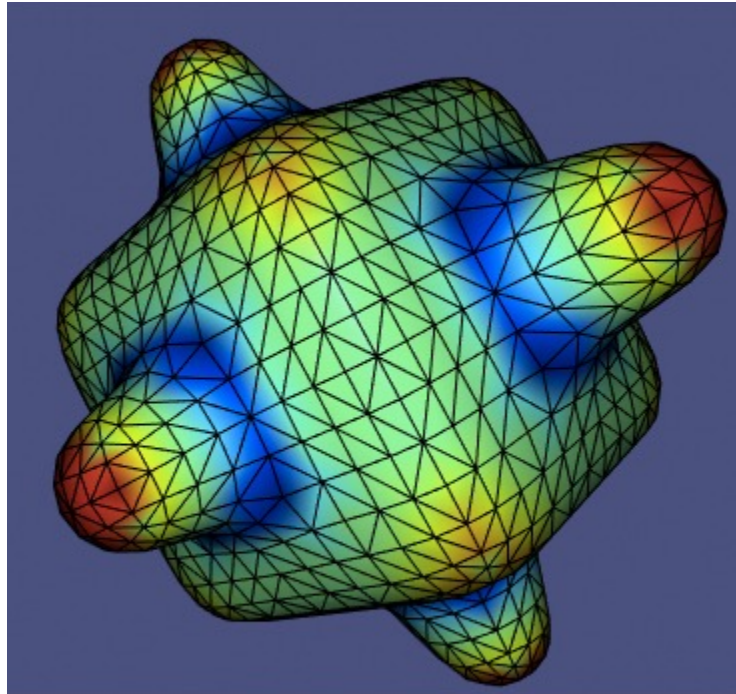
# MODELING SESSION





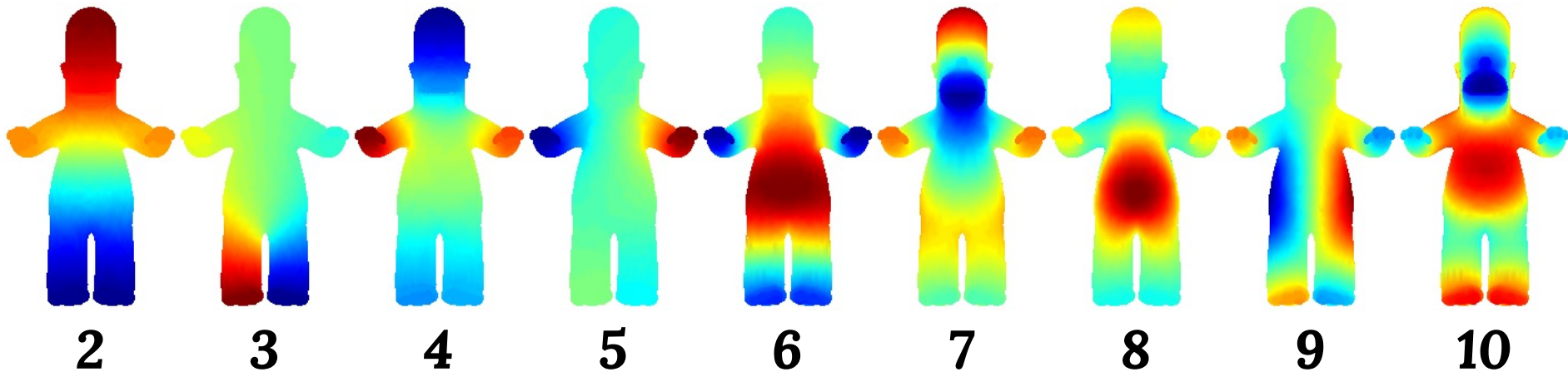
# SHAPE ANALYSIS

Which parts of 3D shape are *interesting*?



# SHAPE ANALYSIS

Can you *hear* the *shape* of a drum?





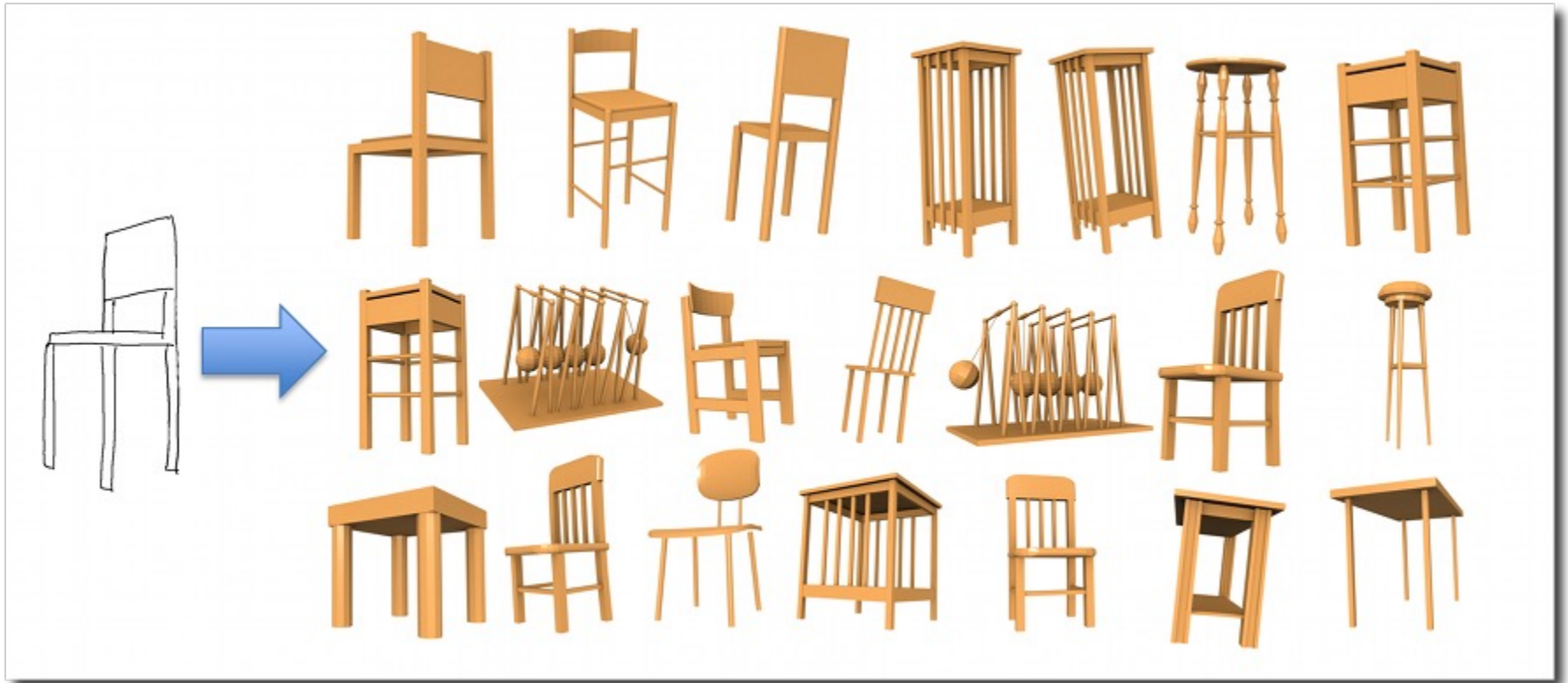
# SHAPE ANALYSIS

How to create vector fields and how to use them?



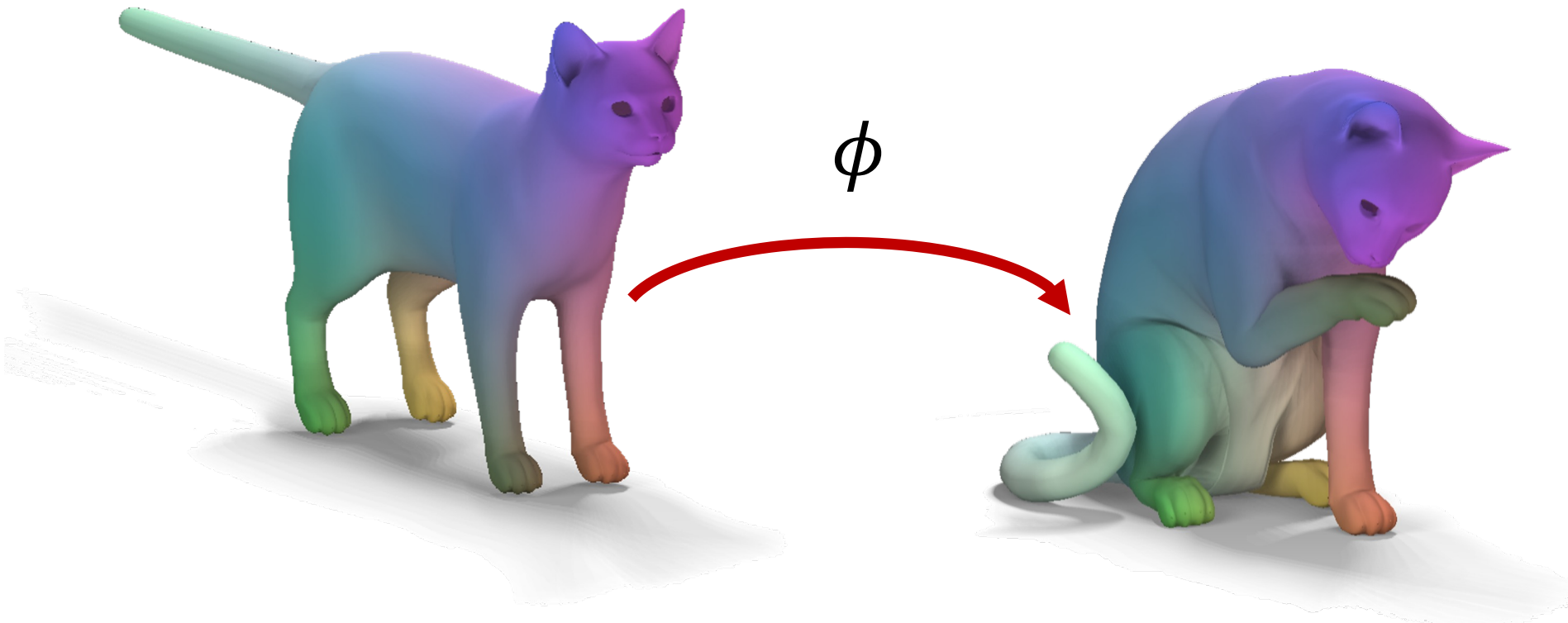
# SHAPE ANALYSIS

How can we *compare* shapes?



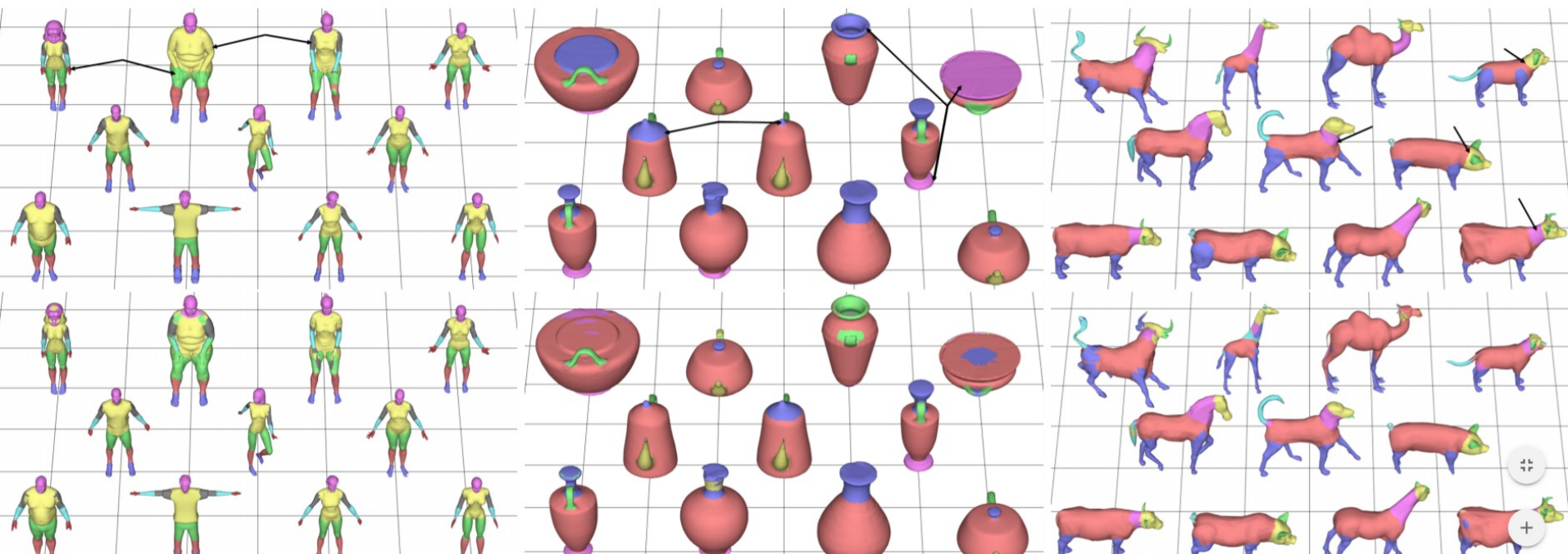
# SHAPE ANALYSIS

- How can we **compare** shapes?
- How to map them onto each other?



# SHAPE ANALYSIS

Segment shapes?



<https://arxiv.org/pdf/1705.11050.pdf>



# ROADMAP

- Data Structures  $\int dx$
- Recap of differential geometry  $\iiint dx dy dz$
- Mesh Simplification & Saliency  $\iint dx dy$
- Vector Fields  $\iiint dx dy dz$
- Mesh Parameterization  $\iint dx dy$
- Shape Deformation  $\iint dx dy$
- 2D modeling  $\int dx$
- Remeshing  $\iint dx dy$
- Learning  $\int dx$



# SOURCES

Based on two courses:  
Many slides are derived from theirs



6.838 (Justin Solomon)

MIT CSAIL

[http://groups.csail.mit.edu/gdpgroup/6838\\_spring\\_2021.html](http://groups.csail.mit.edu/gdpgroup/6838_spring_2021.html)



524 (Alla Sheffer)

UBC CS

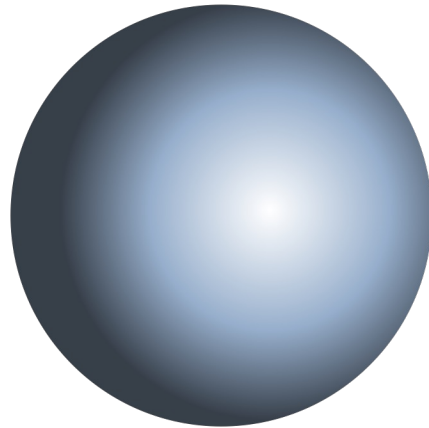
<https://www.cs.ubc.ca/~sheffa/dgp/>

# TODAY

1. Geometric modeling?
2. Shape analysis?
- 3. *Studying shapes: what we need***
4. Applications

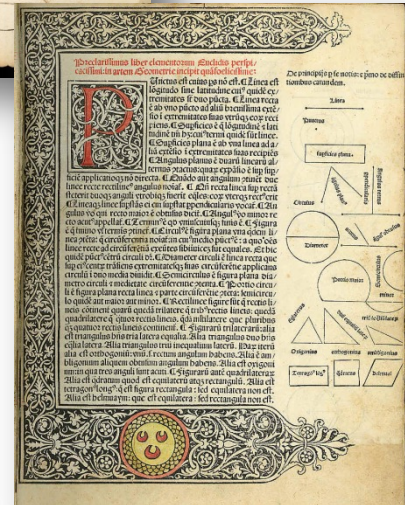
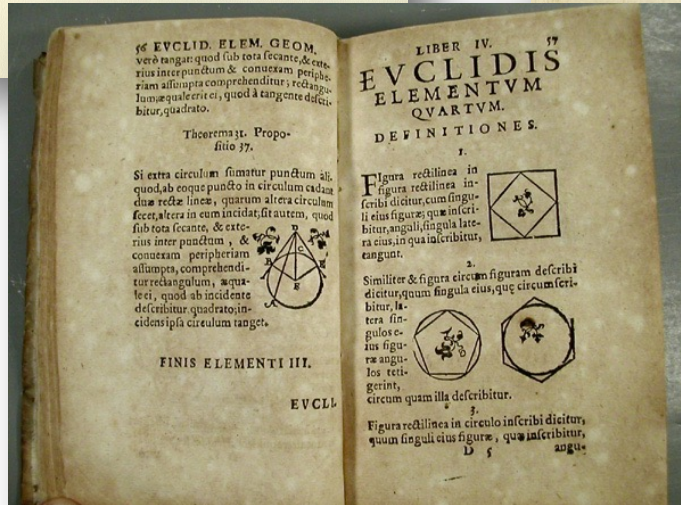
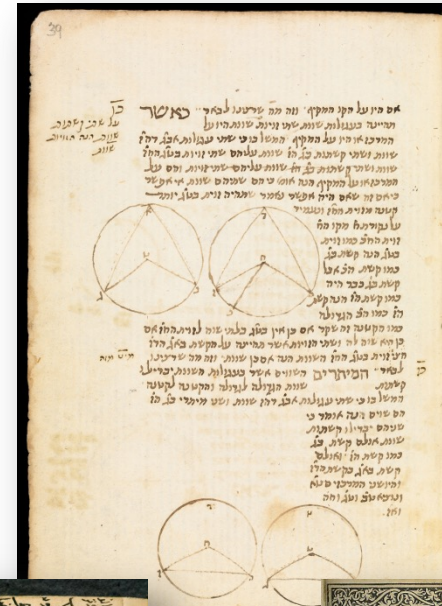
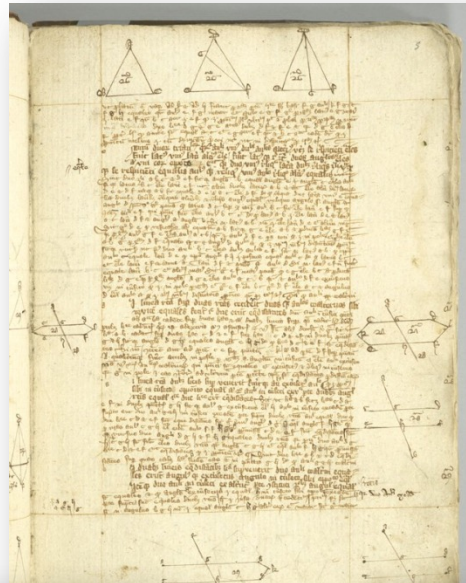
# **WE WANT TO STUDY SHAPES**

What (pure) mathematicians think:





— 10\* —





# EUCLIDEAN GEOMETRY



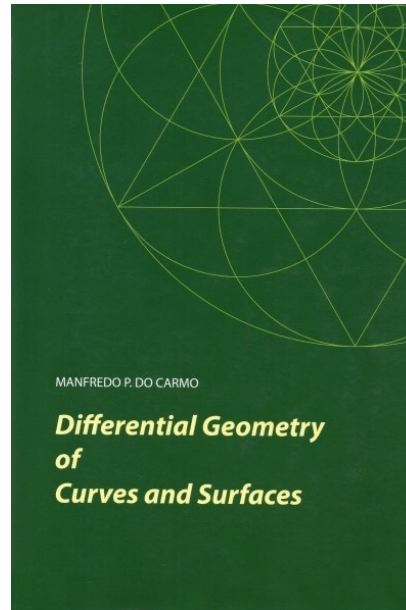


# EUCLIDEAN GEOMETRY

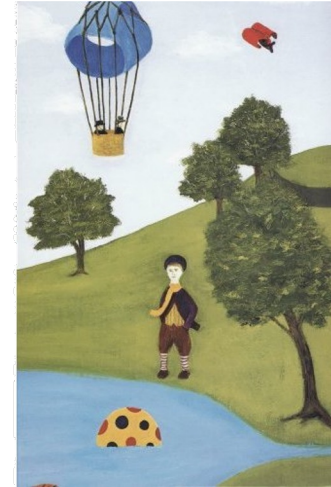
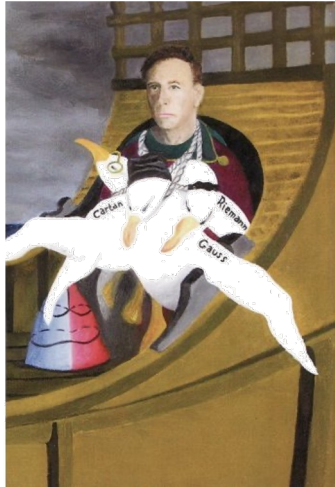
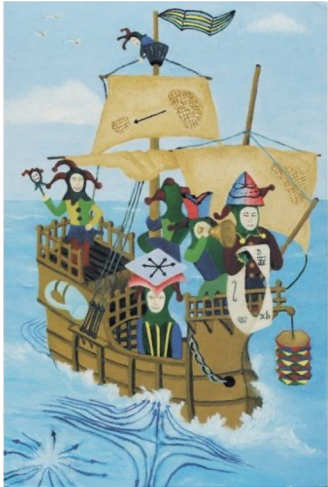


# DIFFERENTIAL GEOMETRY

Easy intro:



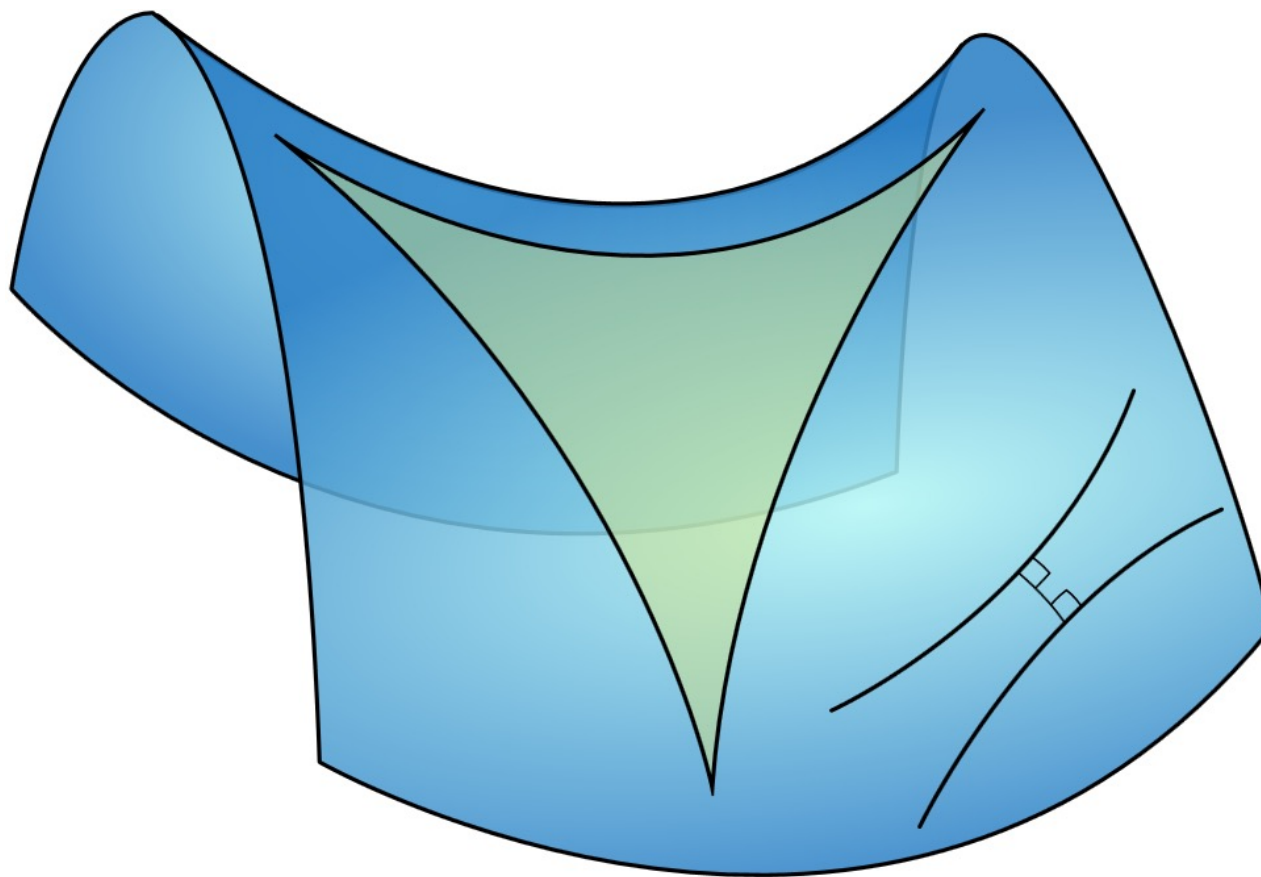
Harder intro (5 volumes):



Spivak: ***A Comprehensive Introduction to Differential Geometry***



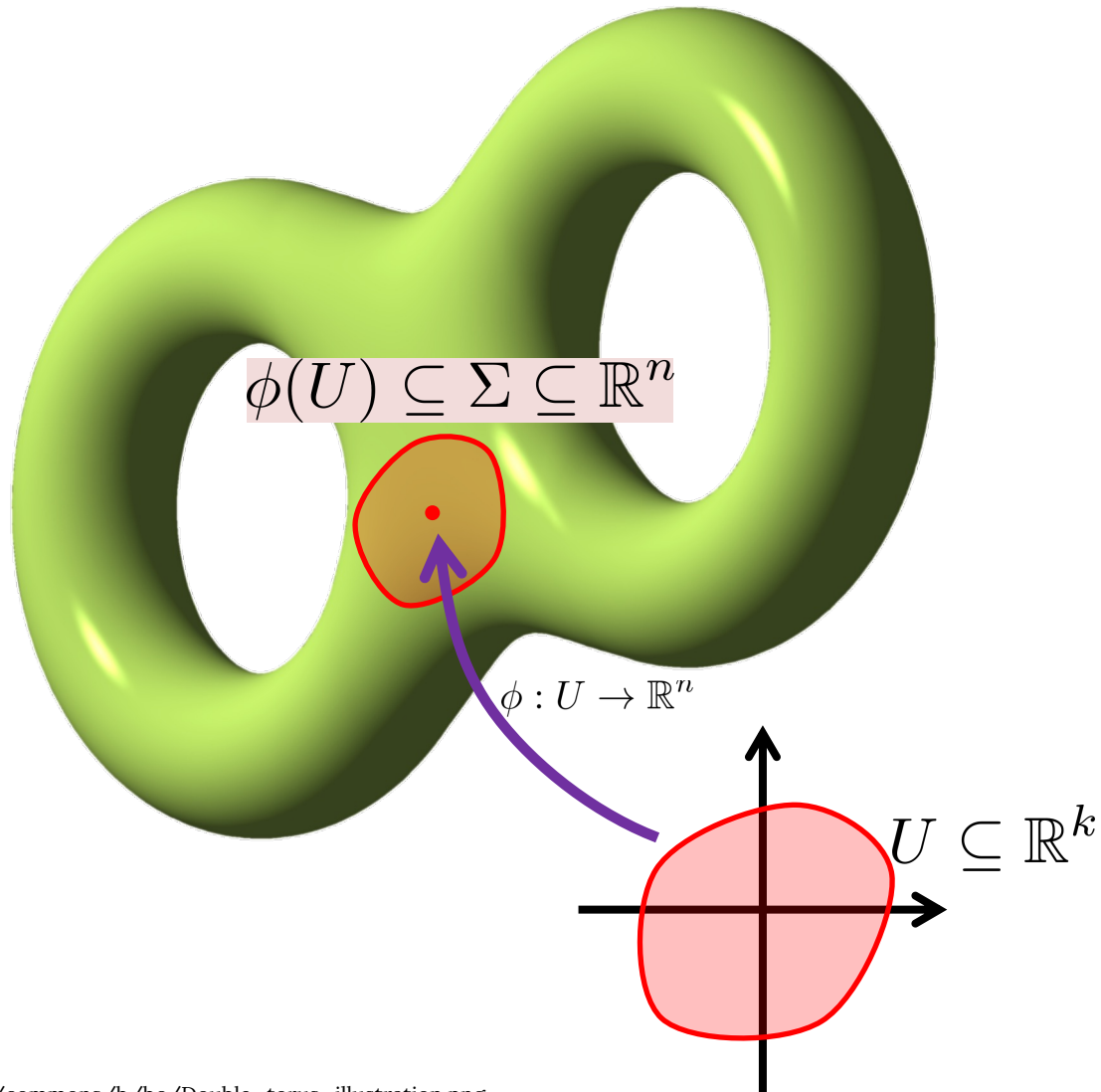
# DIFFERENTIAL GEOMETRY



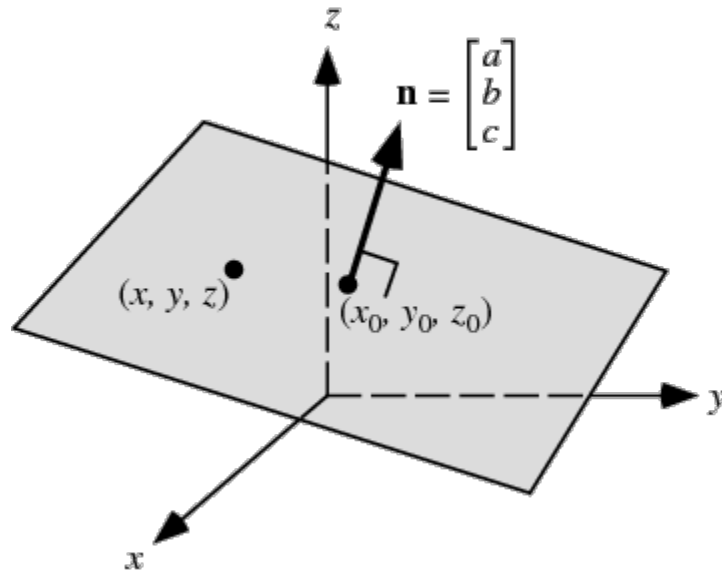
[http://en.wikipedia.org/wiki/Differential\\_geometry](http://en.wikipedia.org/wiki/Differential_geometry)

*Study of smooth manifolds*

# MANIFOLD

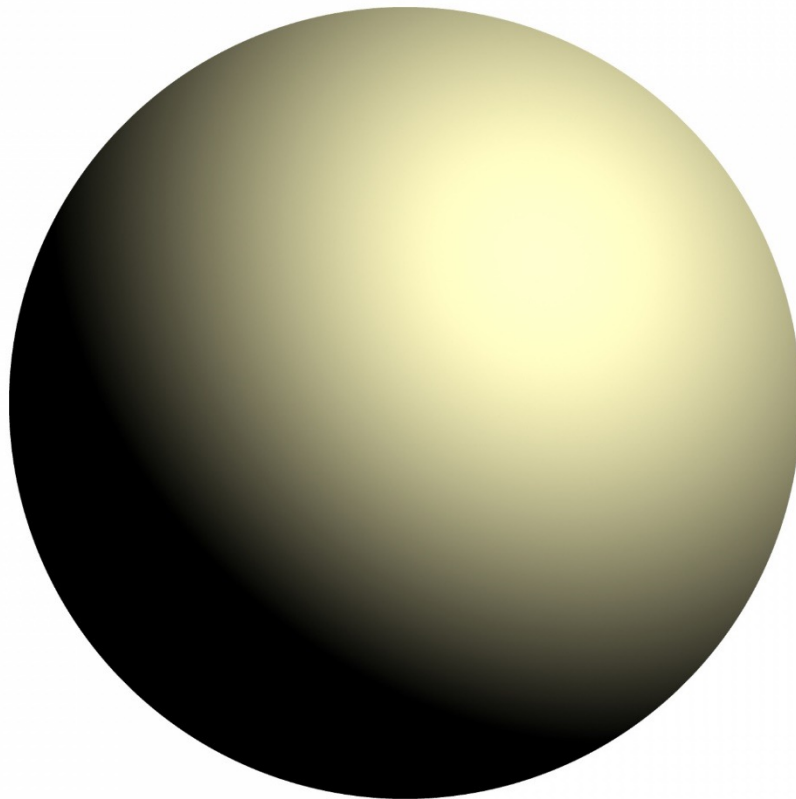


# MANIFOLD?





# MANIFOLD?



# MANIFOLD?

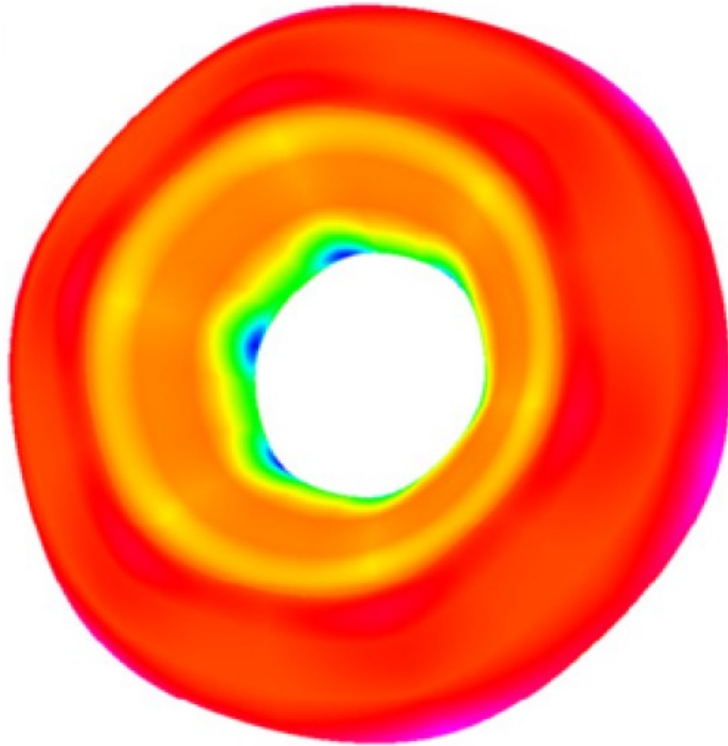


Original model by the Stanford Computer Graphics Laboratory

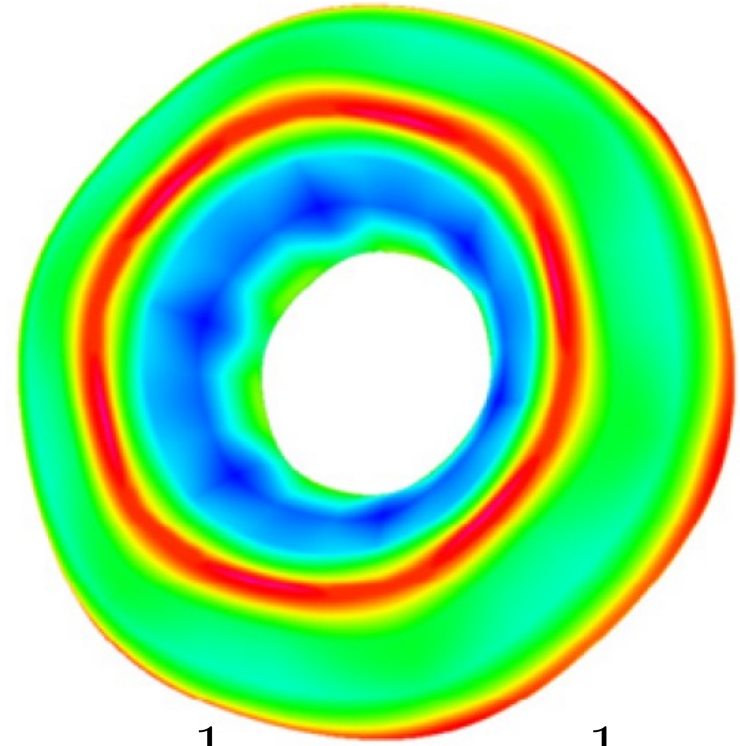
# MANIFOLD?



# DIFFERENTIAL GEOMETRY TOOLBOX



$$K := \kappa_1 \kappa_2 = \det \mathbb{I}$$



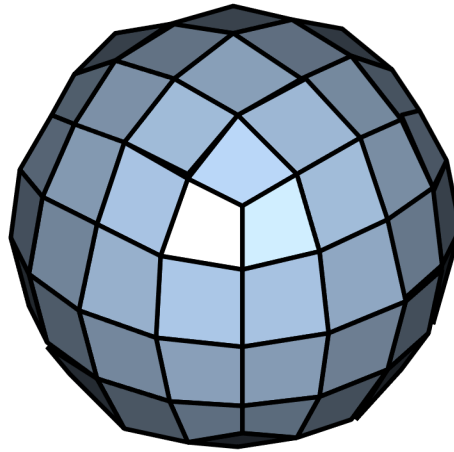
$$H := \frac{1}{2}(\kappa_1 + \kappa_2) = \frac{1}{2}\text{tr } \mathbb{I}$$

<http://www.sciencedirect.com/science/article/pii/S0010448510001983>

## *Curvature and shape properties*

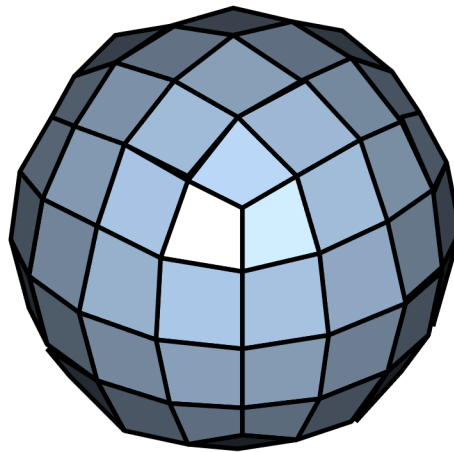
# WE WANT TO STUDY SHAPES

What graphics people think:



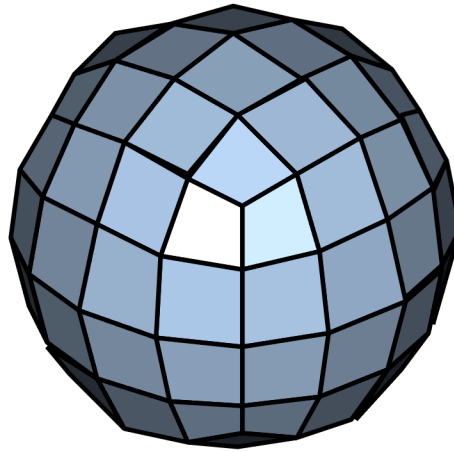
# WE WANT TO STUDY SHAPES

- Collection of flat triangles/quads
- Approximates a smooth surface



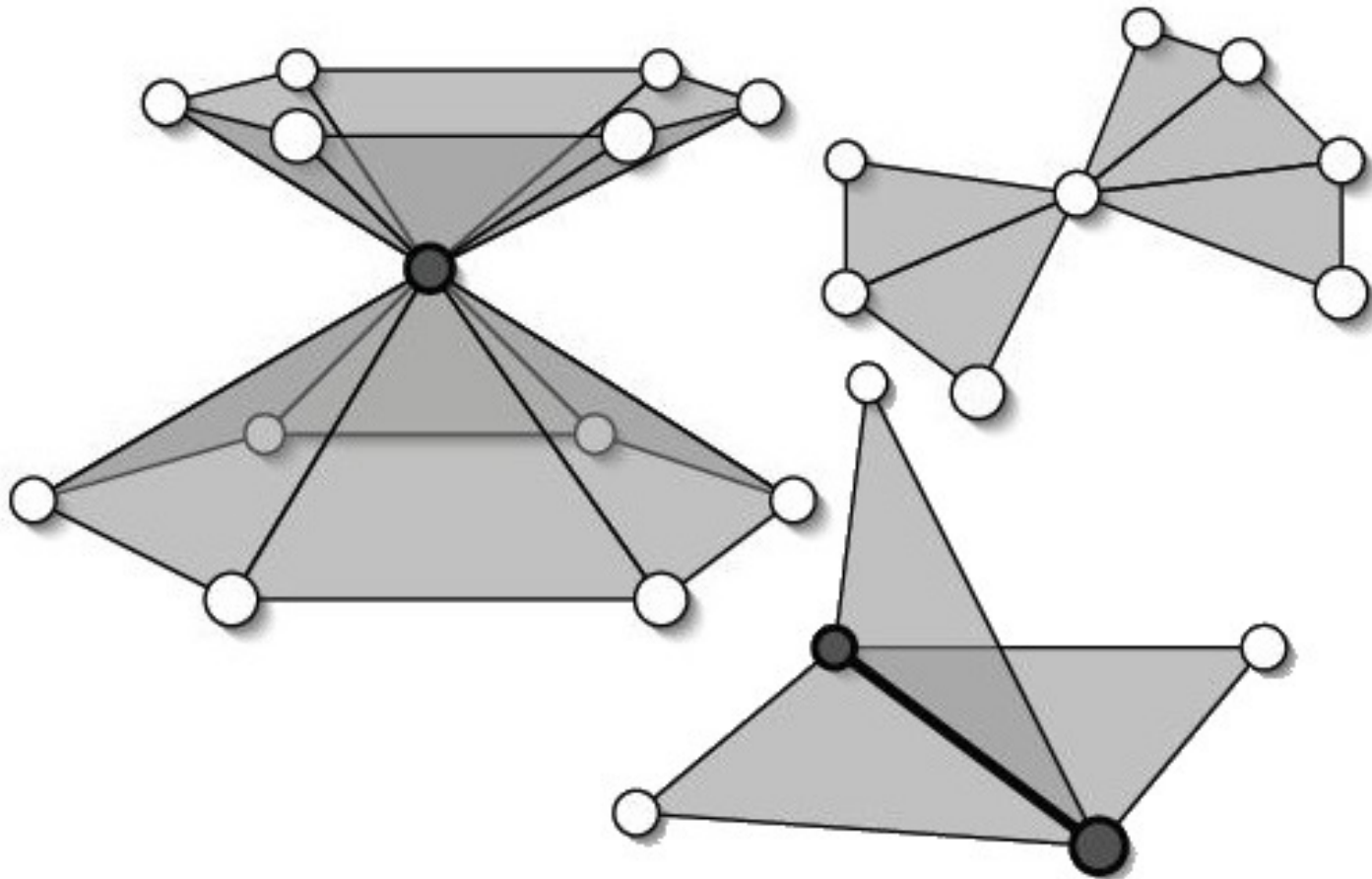
# WE WANT TO STUDY SHAPES

- Can such a mesh have curvature?





# MANIFOLD?

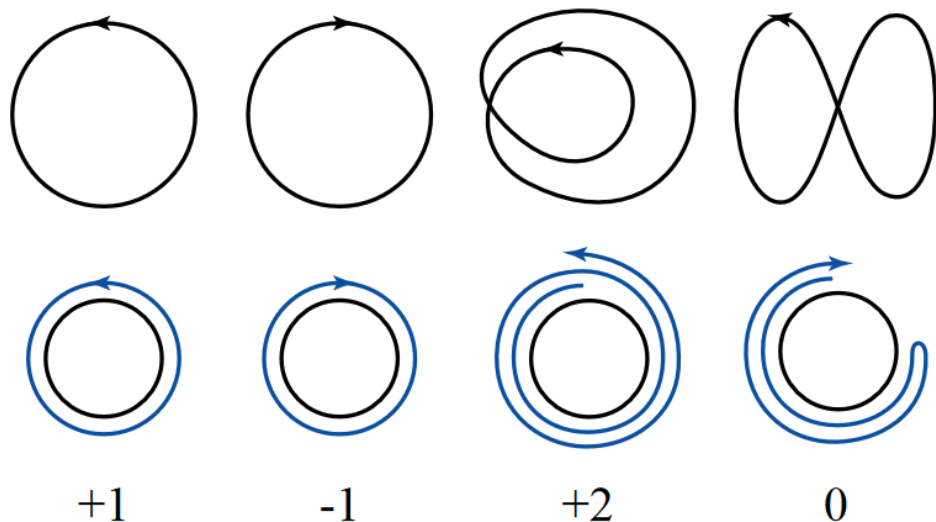


# DISCRETE DIFFERENTIAL GEOMETRY

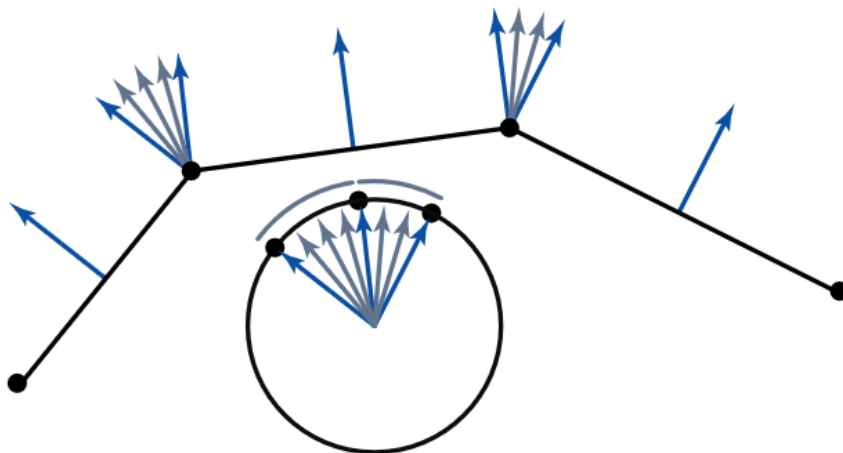
## *Structure preservation:*

- Keeping properties from the continuous abstraction exactly true in a discretization.

# EXAMPLE: TURNING NUMBERS



$$\int_{\Omega} \kappa \, ds = 2\pi k$$



$$\sum_i \alpha_i = 2\pi k$$

# DISCRETE DIFFERENTIAL GEOMETRY

Eurographics Symposium on Geometry Processing (2007)  
Alexander Belyaev, Michael Garland (Editors)

## Discrete Laplace operators: No free lunch

Max Wardetzky<sup>1</sup>

Saurabh Mathur<sup>2</sup>

Felix Kälberer<sup>1</sup>

Eitan Grinspun<sup>2 †</sup>

<sup>1</sup>Freie Universität Berlin, Germany

<sup>2</sup>Columbia University, USA

*the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators.*

*operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators. Finally, we present a family of operators that includes and extends well-known and widely-used operators.*

### 1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tau00, Zha04, FH05, Sor05].

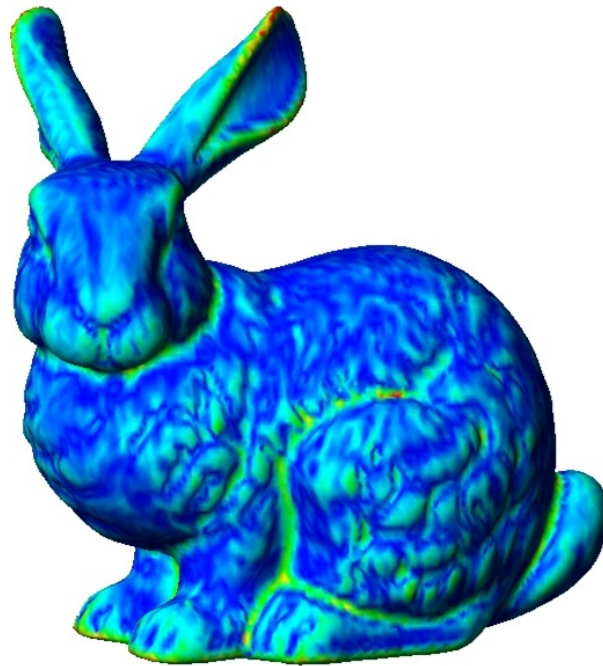
In applications one often requires certain structural prop-

#### 1.1. Properties of smooth Laplacians

Consider a smooth surface  $S$ , possibly with boundary, equipped with a Riemannian metric, *i.e.*, an intrinsic notion of distance. Let the intrinsic  $L^2$  inner product of functions  $u$  and  $v$  on  $S$  be denoted by  $(u, v)_{L^2} = \int_S uv \, dA$ , and let  $\Delta = -\operatorname{div} \operatorname{grad}$  denote the intrinsic smooth Laplace-Beltrami operator [Ros97]. We list salient properties of this operator:

(NULL)  $\Delta u = 0$  whenever  $u$  is constant.

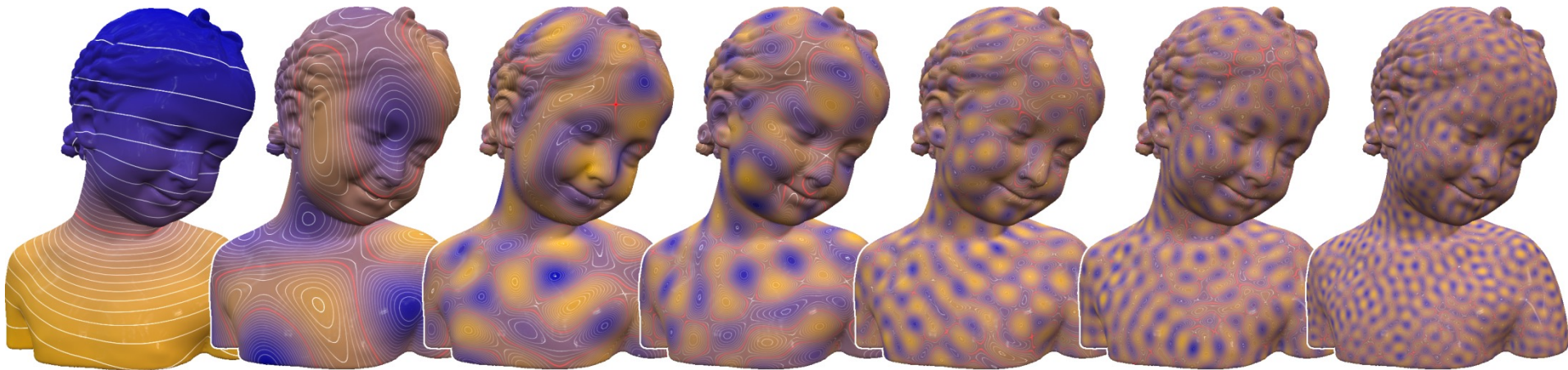
# THEORY: GRAD





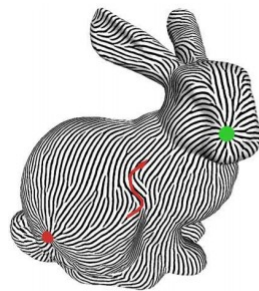
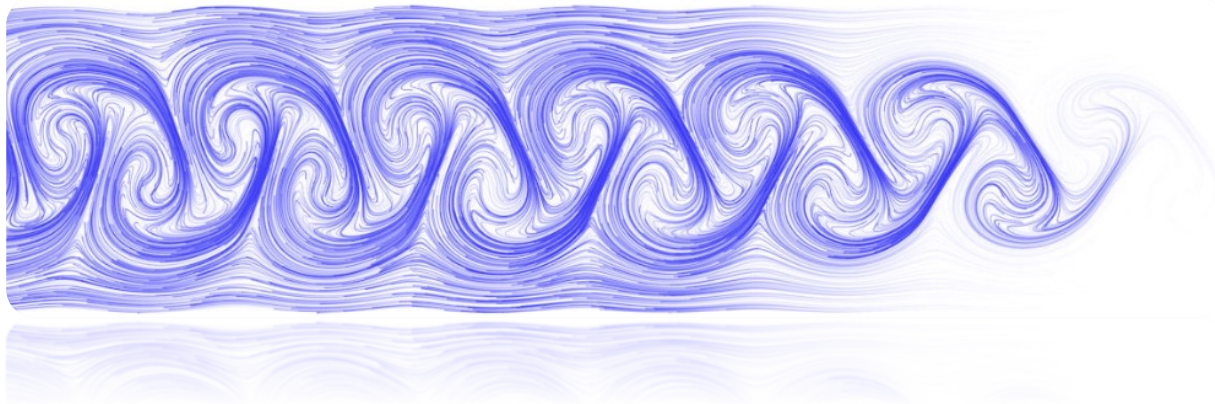
# THEORY: LAPLACIAN

$$\Delta$$



Vallet and Lévy.  
***Spectral Geometry Processing with Manifold Harmonics.***  
EG 2008

# THEORY: VECTOR FIELDS AND FLOWS



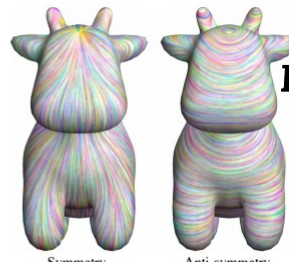
(a) [FSDH07]



(b) [BCBSG10]



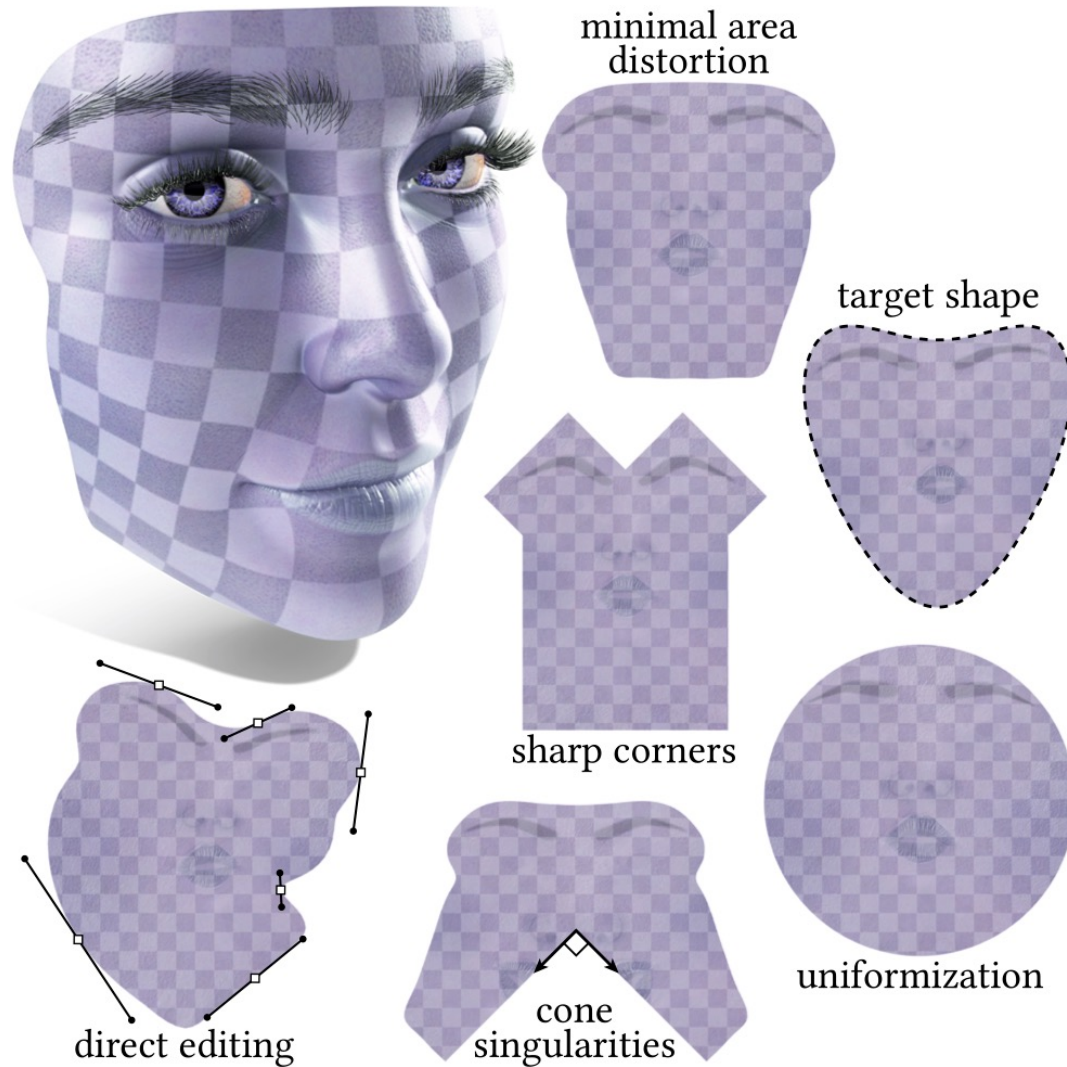
(c) [KCPS13]



(d) [ABCCO13]

Vaxman et al.  
***Directional field synthesis, design, and  
processing.*** EG STAR 2016.

# NUMERICS

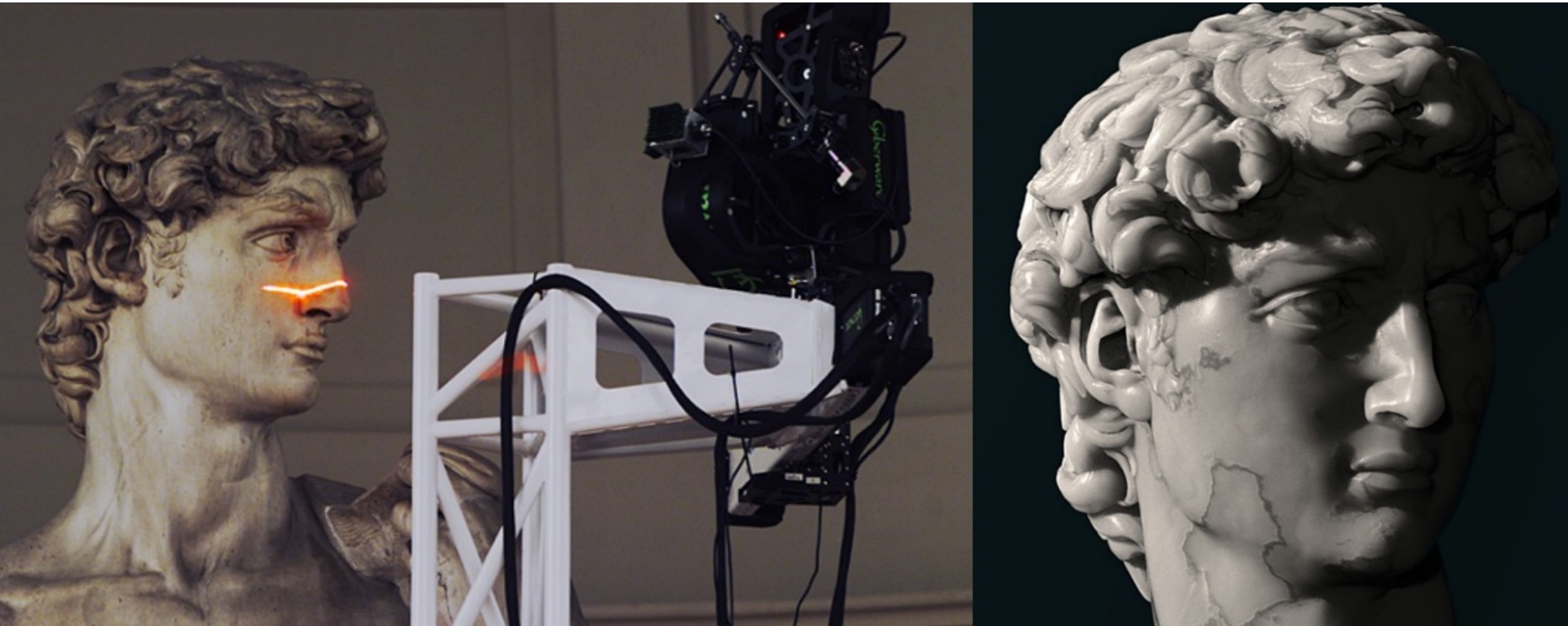


# TODAY

1. Geometric modeling?
2. Shape analysis?
3. Studying shapes: what we need
- 4. *Applications***



# APPLICATION: 3D RECONSTRUCTION



Depth maps  $\xrightarrow{?}$  points  $\xrightarrow{?}$  triangle mesh

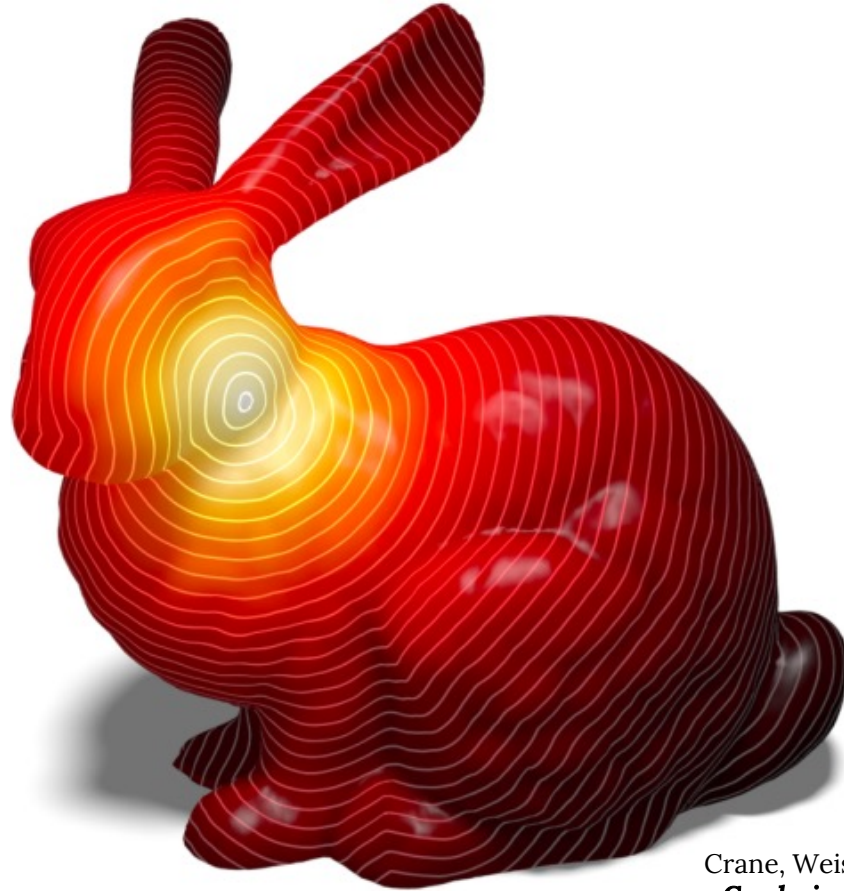


# APPLICATIONS: MESH DEFORMATION



How to propagate deformations to  
*unconstrained parts of shape?*

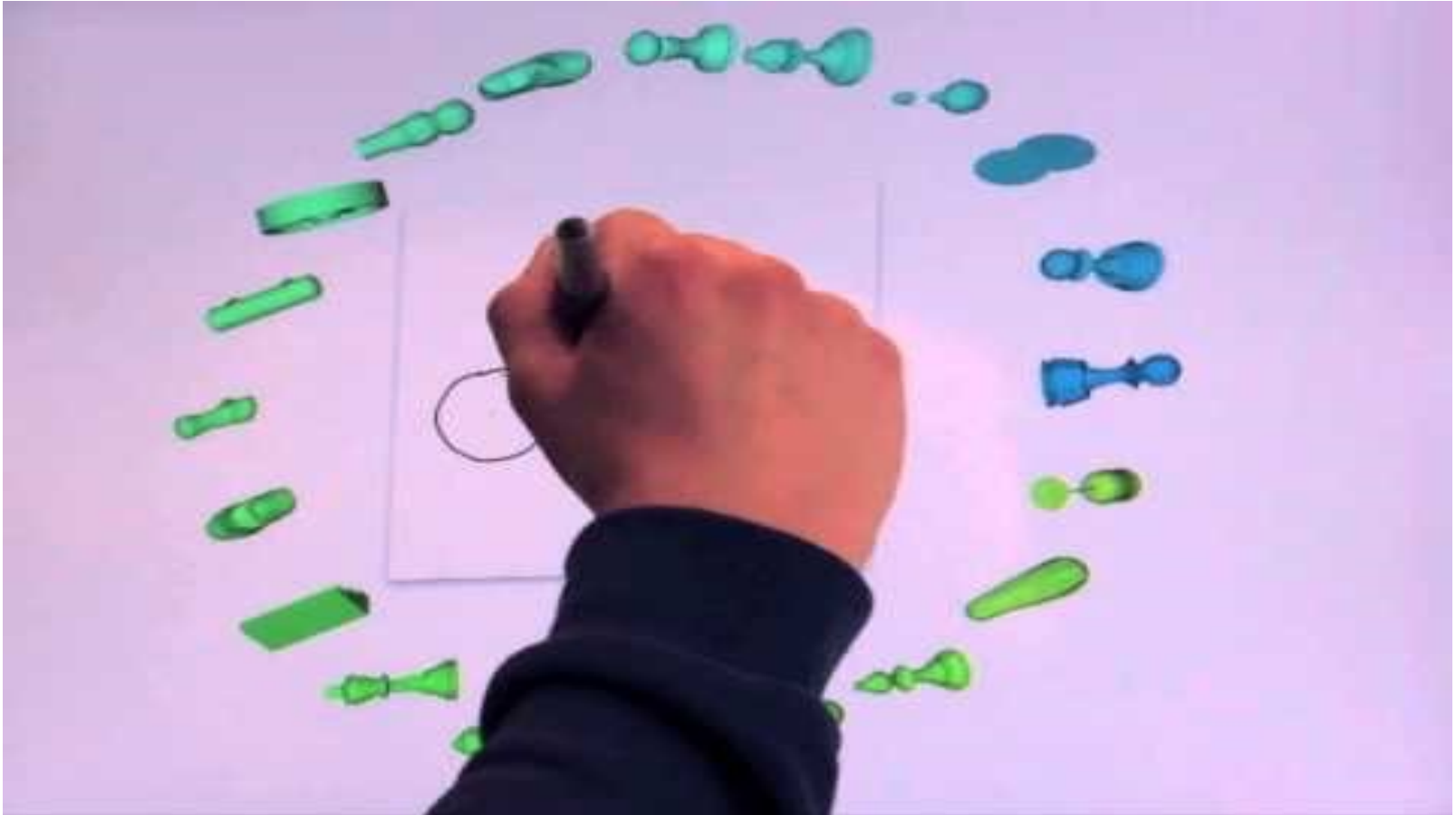
# APPLICATIONS: HEAT AND GEODESICS



Crane, Weischedel, Wardetzky.  
***Geodesics in heat.*** TOG 2013.

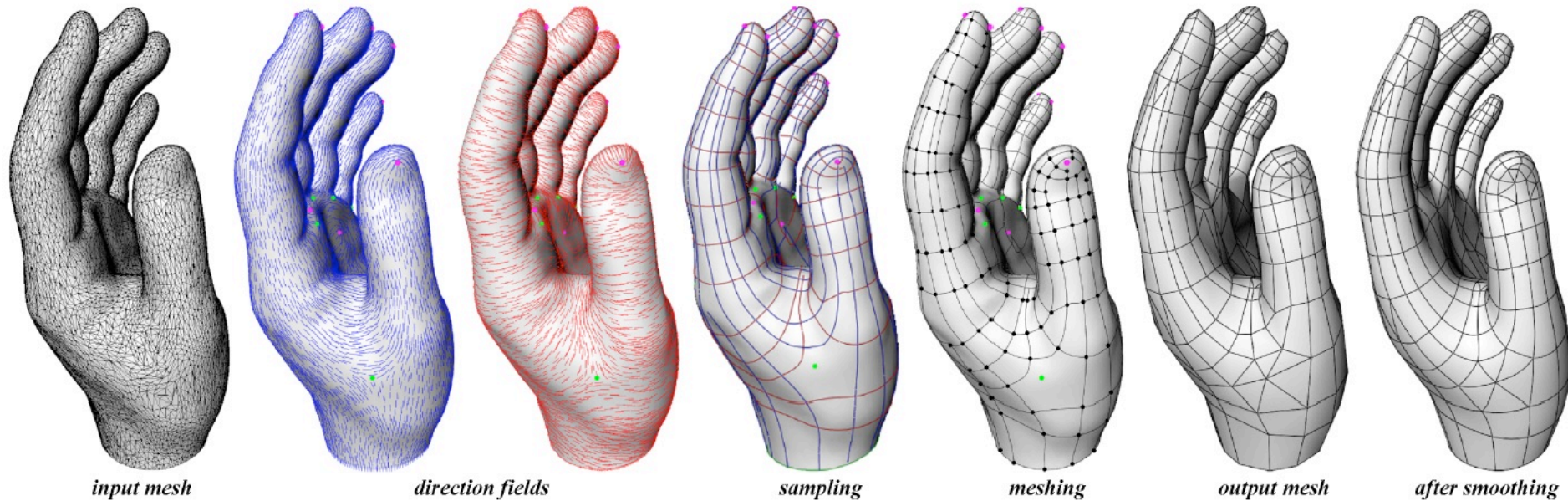
How to compute distances  
between points on a surface?

# APPLICATIONS: RETRIEVAL



How to ***compare*** shapes of  
***different modalities?***

# APPLICATIONS: REMESHING



Pierre Alliez, David Cohen-Steiner, Olivier Devillers, Bruno Lévy, and Mathieu Desbrun. 2003. Anisotropic polygonal remeshing. ACM Trans. Graph. 22, 3 (July 2003)

How to **convert** any mesh  
into a **nice** one?

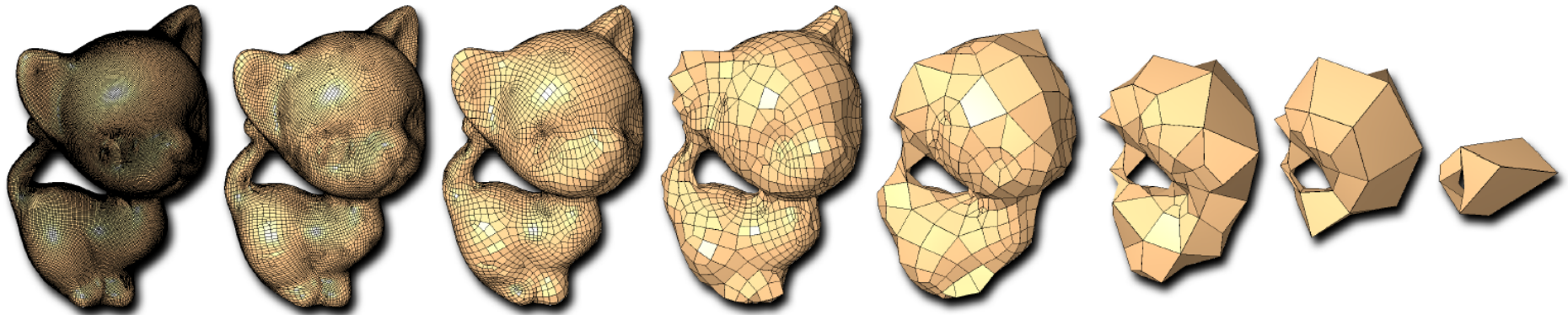
# APPLICATIONS: QUADMESHING



How to make a *mesh* out of *quads*?



# APPLICATIONS: MESH SIMPLIFICATION

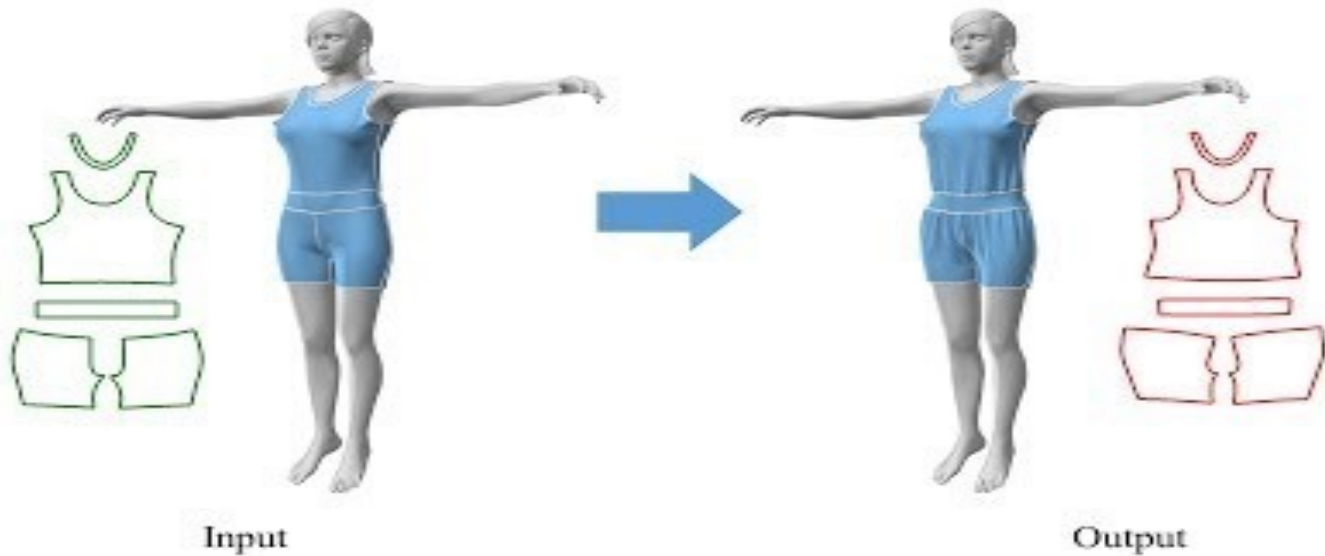


Joel Daniels, Cláudio T. Silva, Jason Shepherd, and Elaine Cohen. 2008. Quadrilateral mesh simplification. ACM Trans. Graph. 27, 5, Article 148 (December 2008)

How to **compress** geometry?

# APPLICATIONS: CLOTH MODELING

## FoldSketch



# APPLICATIONS: SKETCH-BASED MODELING

...perceptual validation

#1 Humans are consistent in perceiving regularity cues in a design sketch.



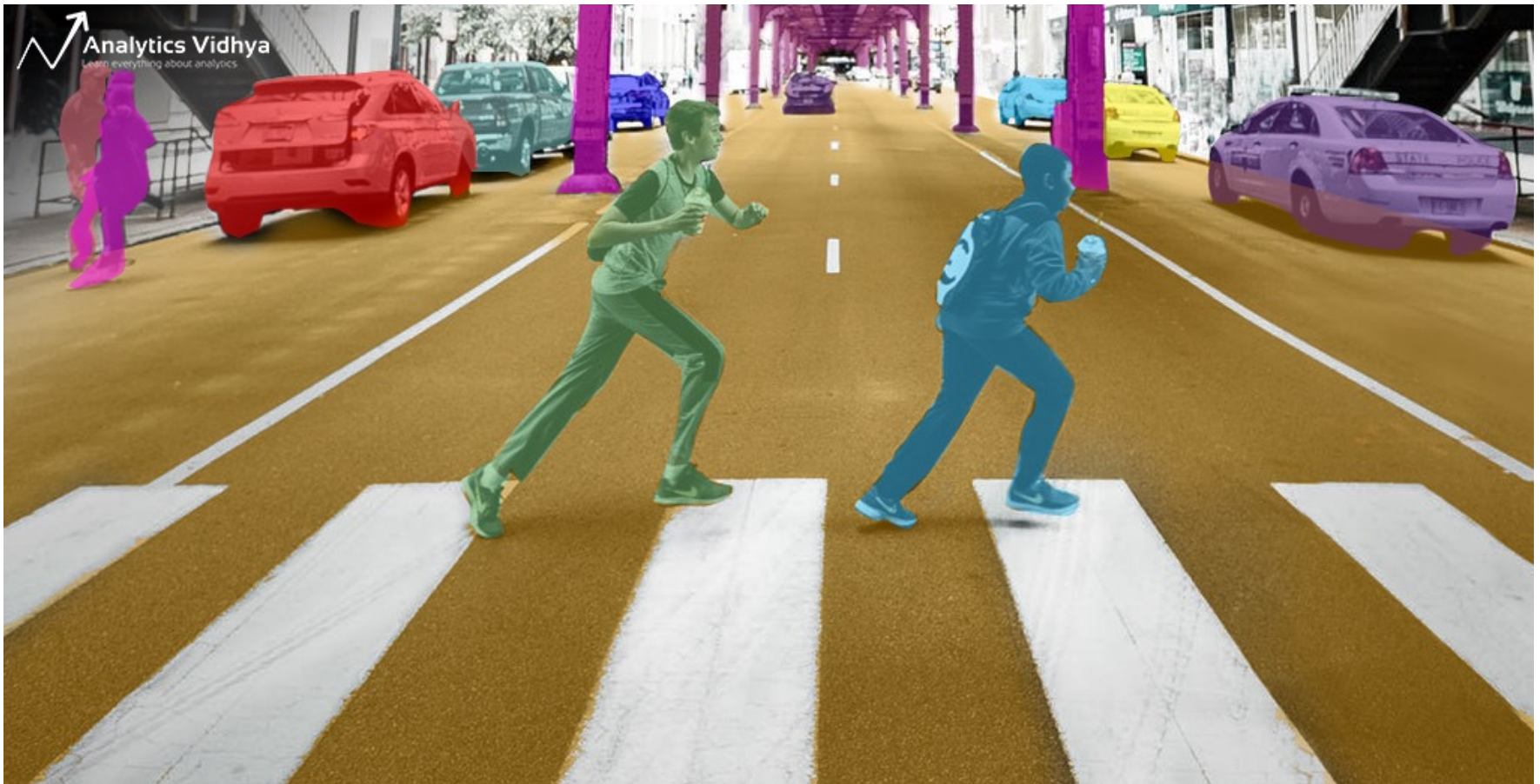
# APPLICATIONS: SKETCH-BASED MODELING AND POSING



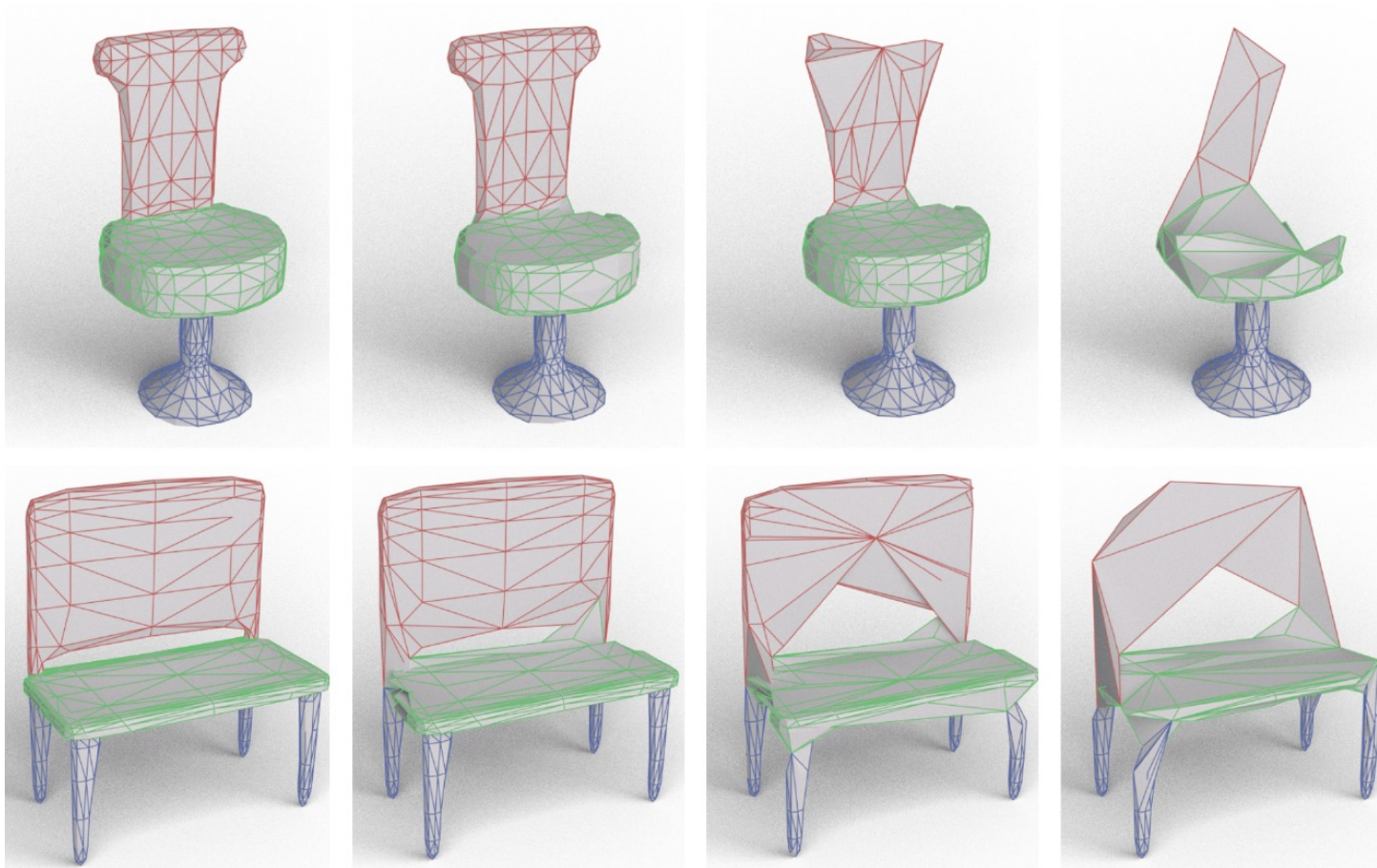
© Olga Posukh, 2022



# APPLICATIONS: LEARNING



# APPLICATIONS: LEARNING

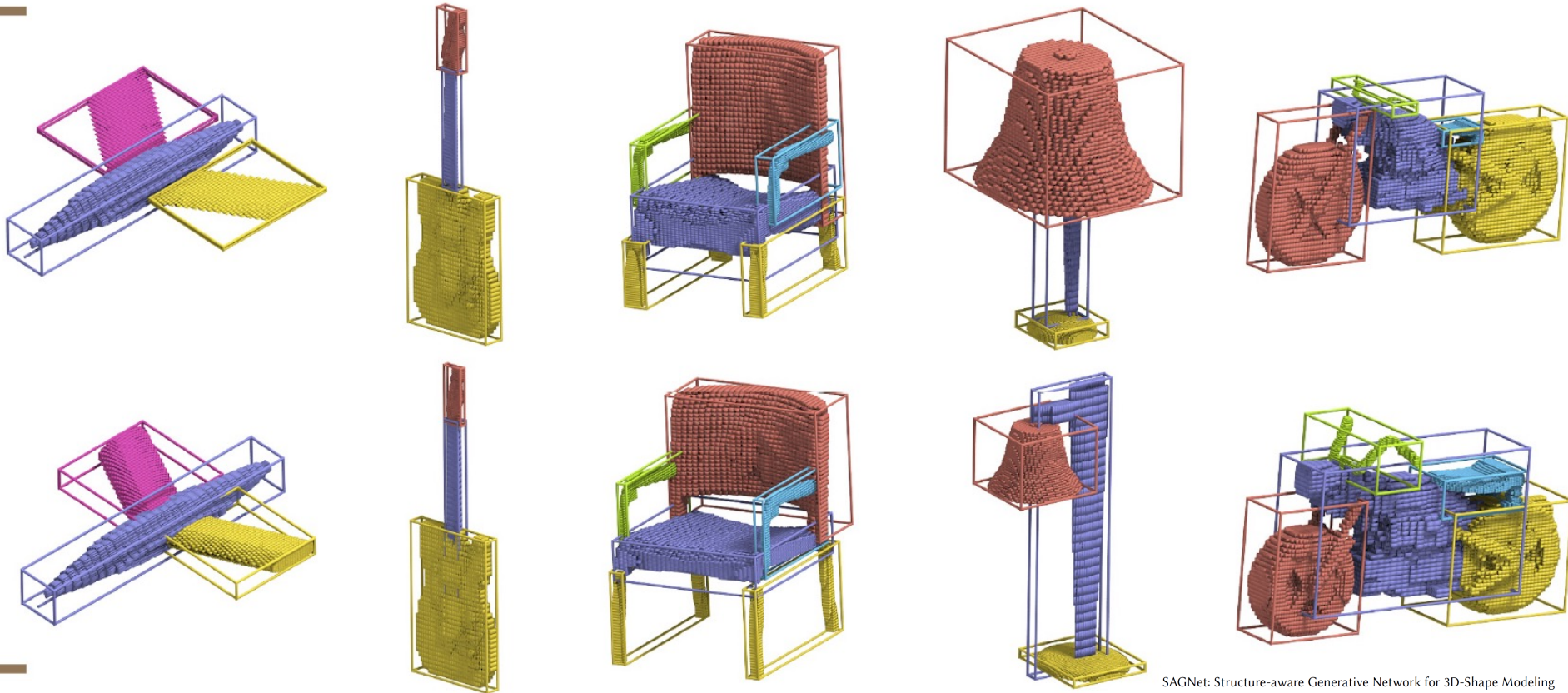


"MeshCNN: A Network with an Edge" by Rana Hanocka, Amir Hertz, Noa Fish, Raja Giryes, Shachar Fleishman, Daniel Cohen-Or, SIGGRAPH 2019

How to bring *learning* approaches  
from images to **3D geometry**?



# APPLICATIONS: LEARNING



SAGNet: Structure-aware Generative Network for 3D-Shape Modeling

ZHIJIE WU, Shenzhen University  
XIANG WANG, Shenzhen University  
DI LIN, Shenzhen University  
DANI LISCHINSKI, The Hebrew University of Jerusalem  
DANIEL COHEN-OR, Shenzhen University  
HUI HUANG\*, Shenzhen University

How to bring *learning* approaches  
from images to **3D geometry**?

# </INTRO>

<http://tiny.cc/ift6113>

Piazza!

Office hour: Fri, 4:30-5:30 pm,  
Zoom/AA3357

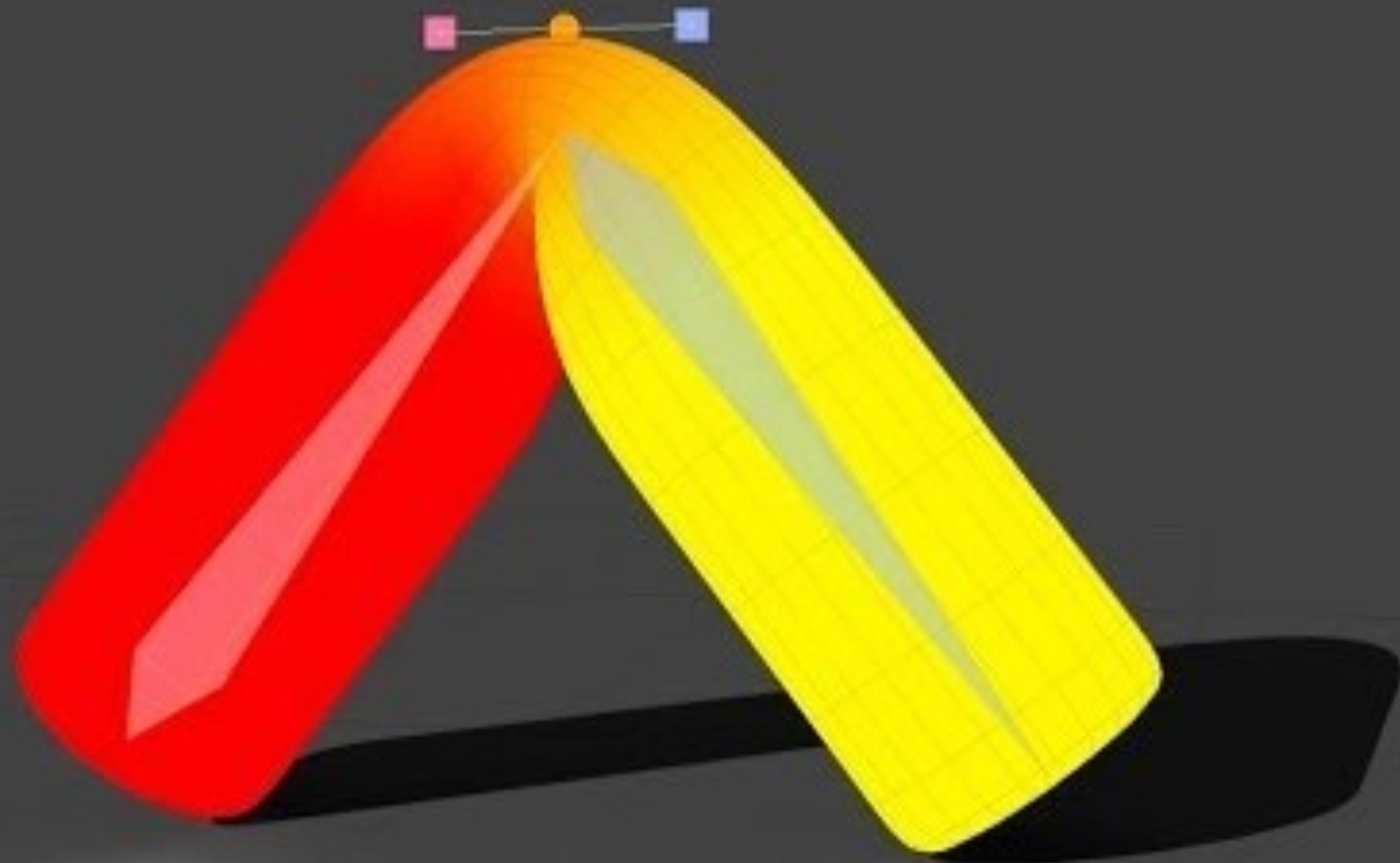




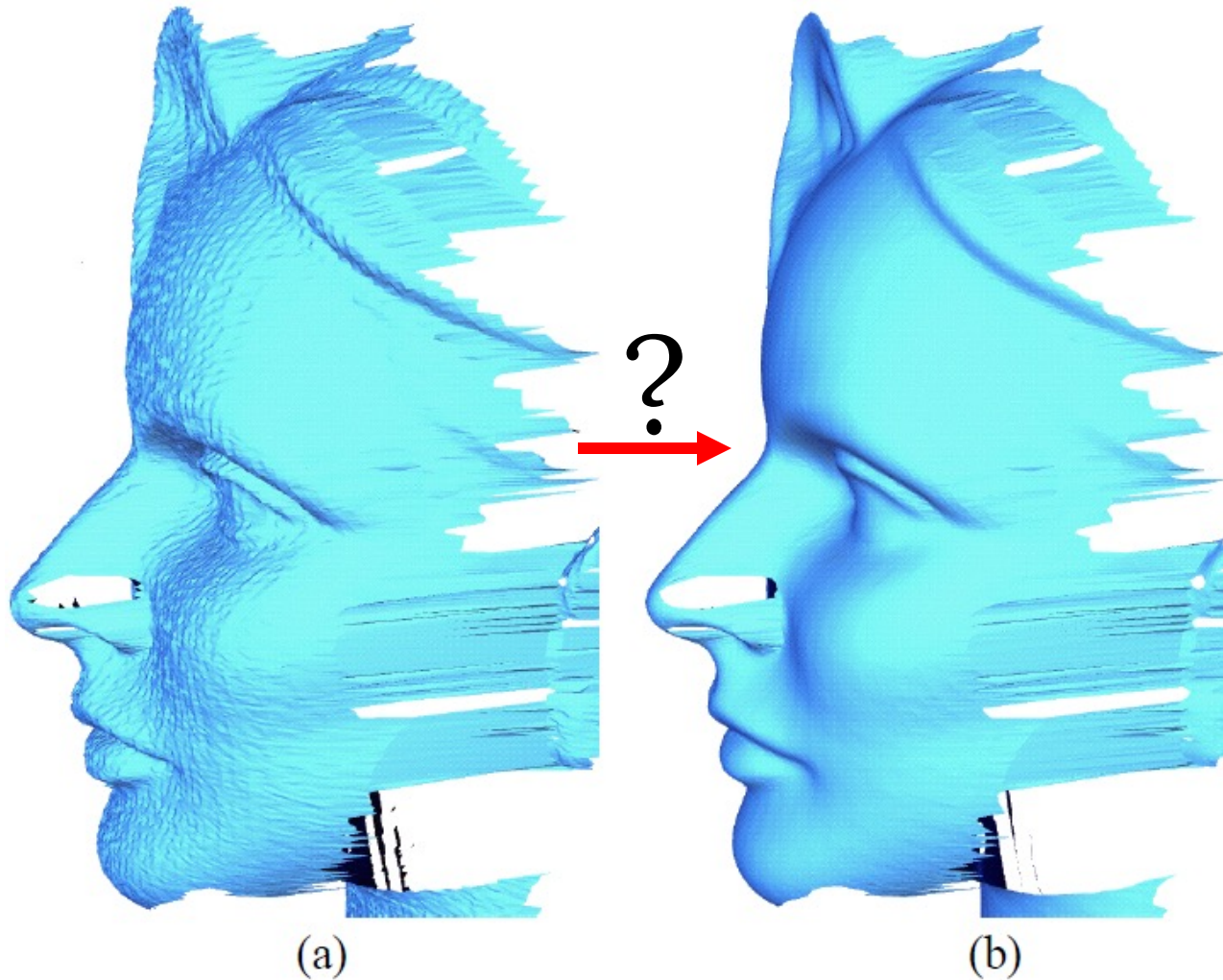
# WARM-UP PROBLEMS



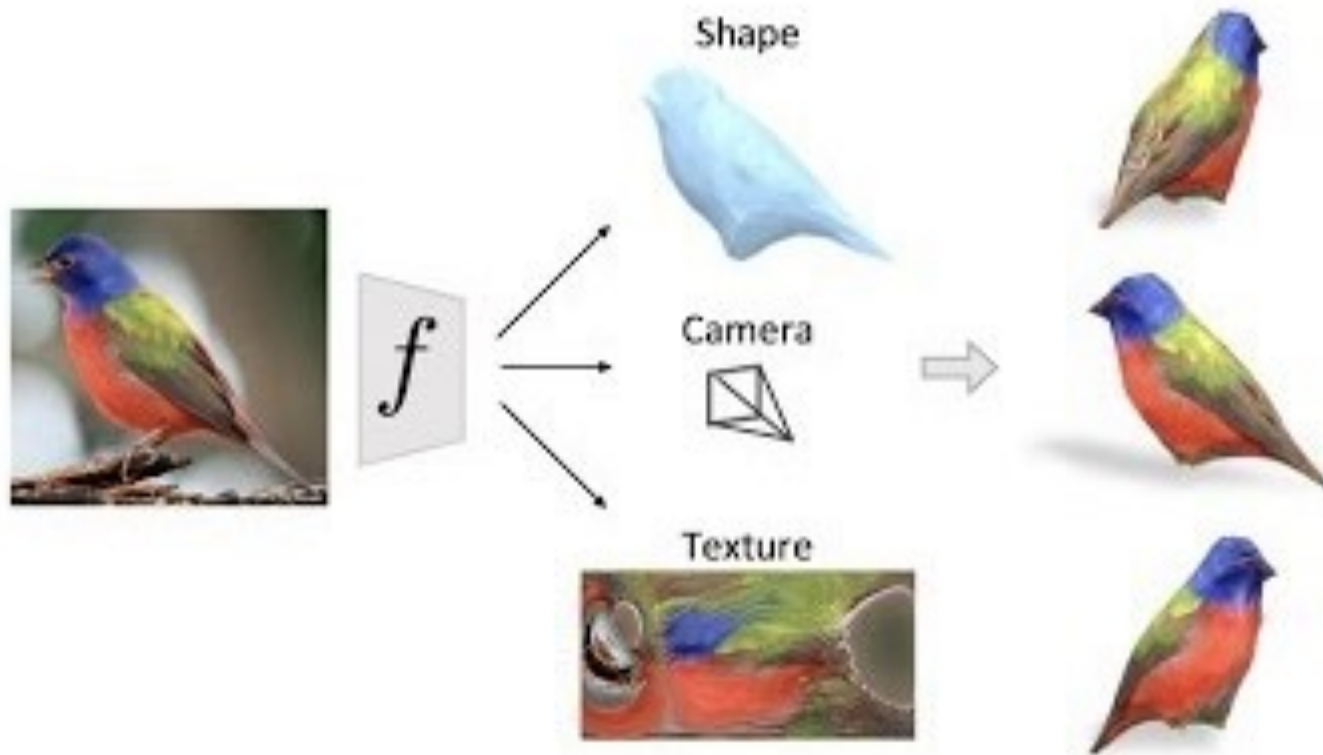
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# WARM-UP PROBLEMS



# WARM-UP PROBLEMS



# **LIBIGL & EIGEN**

- Eigen: a linear algebra library
- libigl: geometry processing library