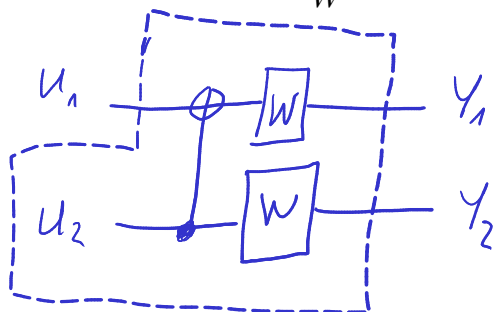
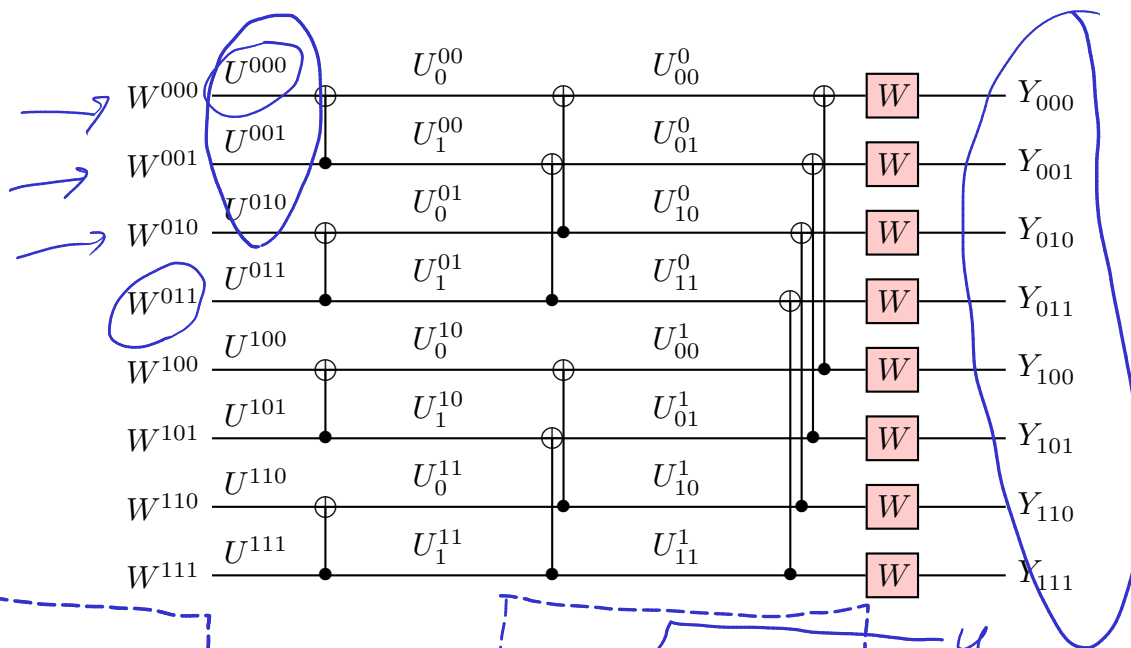


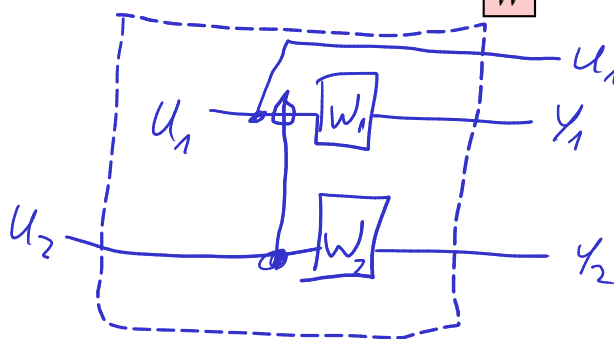
IFT6080 : Cours en direct

Frédéric Dupuis

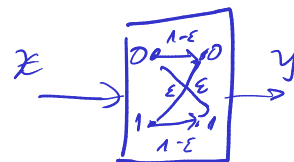
21 mars 2020



Mauvais canal
où U_2 est
considérée comme
du bruit.



"Bon" canal.



$$\frac{W(0|0)}{W(0|1)} = \frac{1-\varepsilon}{\varepsilon}$$

Def: [Rapport de vraisemblance]: Étant donné un canal $W \in \mathcal{P}(\mathcal{Y}|\mathcal{X}_2)$, et
un $y \in \mathcal{Y}$, $L(W, y) := \frac{W(y|0)}{W(y|1)}$.

Formule récursive pour $L(W^0, y_1, y_2)$ et $L(W^1, u_1, y_1, y_2)$:

$$L(W^0, y_1, y_2) = \frac{W^0(y_1, y_2 | 0)}{W^0(y_1, y_2 | 1)} \quad \left(\text{on veut une expression de : } L(W_1, y_1) \text{ et } L(W_2, y_2) \right)$$

$$= \frac{\frac{1}{2} W_1(y_1 | 0) W_2(y_2 | 0) + \frac{1}{2} W_1(y_1 | 1) W_2(y_2 | 1)}{\frac{1}{2} W_1(y_1 | 1) W_2(y_2 | 0) + \frac{1}{2} W_1(y_1 | 0) W_2(y_2 | 1)}$$

$$= \frac{\frac{W_1(y_1 | 0) W_2(y_2 | 0) + W_1(y_1 | 1) W_2(y_2 | 1)}{W_1(y_1 | 1) W_2(y_2 | 1)}}{\frac{W_1(y_1 | 1) W_2(y_2 | 0) + W_1(y_1 | 0) W_2(y_2 | 1)}{W_1(y_1 | 1) W_2(y_2 | 1)}}$$

$$= \frac{L(W_1, y_1) \cdot L(W_2, y_2) + 1}{L(W_2, y_2) + L(W_1, y_1)}$$

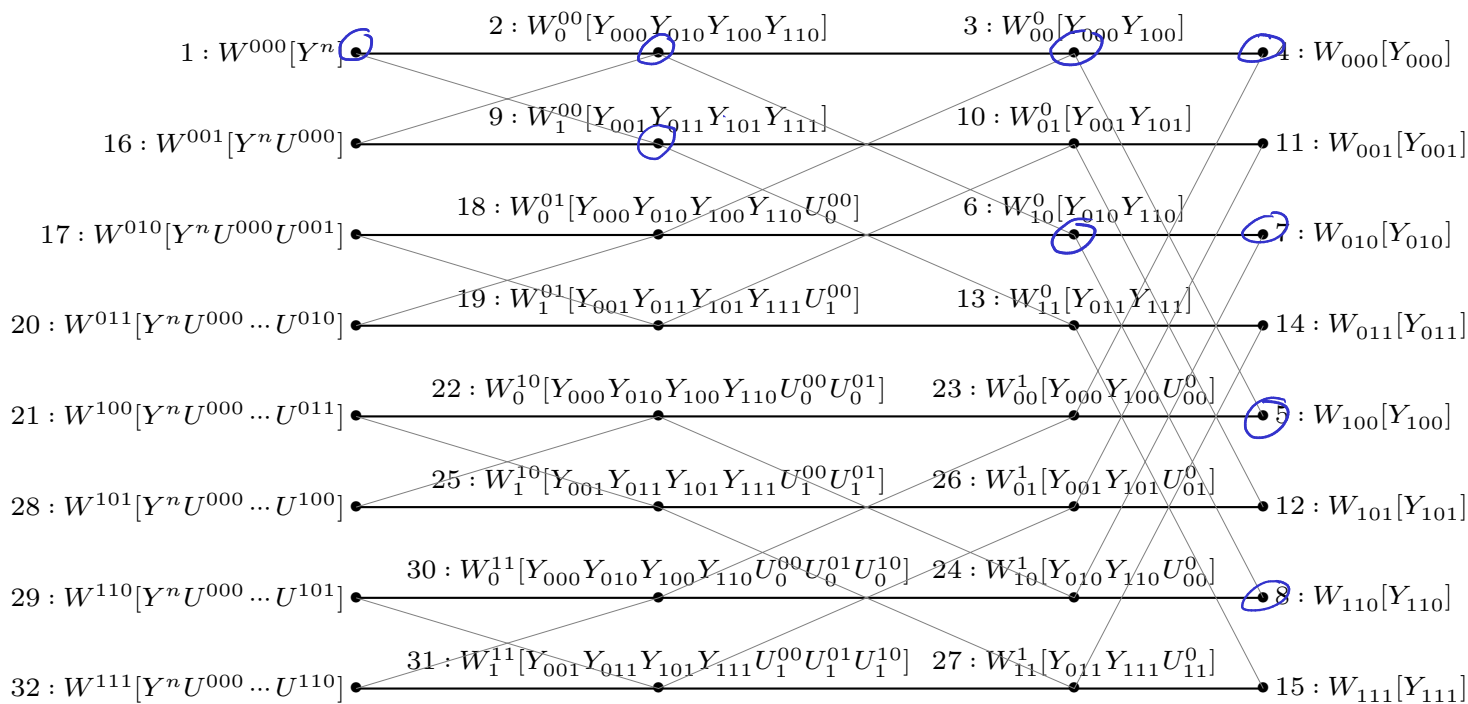
Bon cas :

$$L(W^1, u_1, y_1, y_2) = \frac{W^1(u_1, y_1, y_2 | 0)}{W^1(u_1, y_1, y_2 | 1)}$$

$$= \frac{W_1(y_1 | u_1) \cdot W_2(y_2 | 0)}{W_1(y_1 | u_1 + 1) W_2(y_2 | 1)}$$

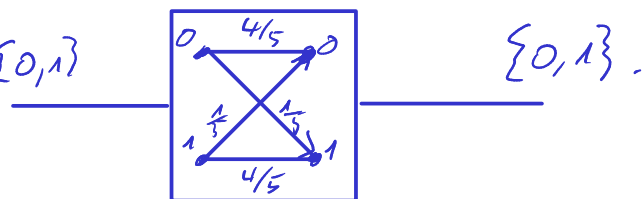
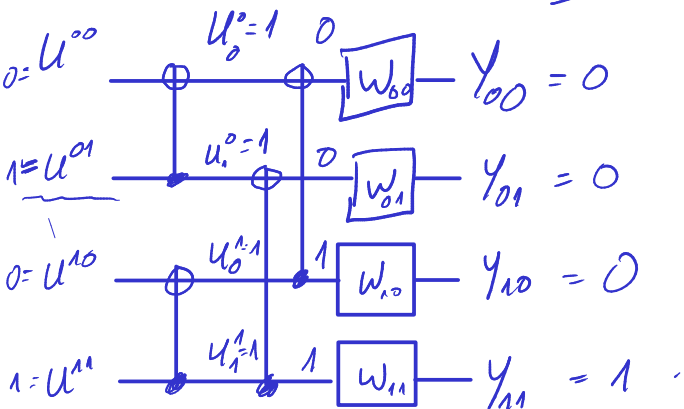
$$= \frac{W_1(y_1 | u_1)}{W_1(y_1 | u_1 + 1)} \cdot L(W_2, y_2)$$

$$\rightarrow \begin{cases} L(W_1, y_1) \cdot L(W_2, y_2) & \text{si } u_1 = 0 \\ \frac{L(W_2, y_2)}{L(W_1, y_1)} & \text{si } u_1 = 1 \end{cases}$$



Complexité de l'algo de décodage:
 $O(\# \text{ de canaux} \cdot \# \text{ de codes}) = O(n \cdot \log n)$.

Exemple à 2 codes



$$L(W_{00}, y_{00}) = \frac{w(0|0)}{w(0|1)} = \frac{4/5}{1/5} = 4$$

$$L(W_{01}, y_{01}) = \frac{w(0|0)}{w(0|1)} = 4$$

$$L(W_{10}, y_{10}) = \frac{w(0|0)}{w(0|1)} = 4$$

$$L(W_{11}, y_{11}) = \frac{w(1|0)}{w(1|1)} = \frac{1/5}{4/5} = \frac{1}{4}$$

$$L(W^0, y^n) \approx 1.18$$

$$L(W_0^0, y_{00}, y_{01}) \approx \frac{17}{8}$$

$$L(W_1^0, y_{01}, y_{11}) = \frac{8}{5}$$

$$L(W_{00}, y_{00}) = 4, L(W_{10}, y_{10}) = 4, L(W_{01}, y_{01}) = 4, L(W_{11}, y_{11}) = \frac{1}{4}$$

$$L(W_0^0, y_{00} y_{10}) = \frac{L(W_{00}, y_{00}) \cdot L(W_{10}, y_{10}) + 1}{L(W_{00}, y_{00}) + L(W_{10}, y_{10})} = \frac{17}{8}$$

$$L(W_1^0, y_{01} y_{11}) = \frac{L(W_{01}, y_{01}) \cdot L(W_{11}, y_{11}) + 1}{L(W_{01}, y_{01}) + L(W_{11}, y_{11})} = \frac{2}{\frac{5}{4}} = \frac{8}{5}$$

$$L(W^0, y^n) = \frac{L(W_0^0, y_{00} y_{10}) \cdot L(W_1^0, y_{01} y_{11}) + 1}{L(W_0^0, y_{00} y_{10}) + L(W_1^0, y_{01} y_{11})} = \frac{\frac{17}{8} \cdot \frac{8}{5} + 1}{\frac{17}{8} + \frac{8}{5}} \approx 1.18$$

$$L(W^{01}, y^n)$$

$$L(W_0^1, y_{00} y_{01}) = \frac{17}{8}$$

$$L(W_1^1, y_{01} y_{11}) = \frac{8}{5}$$

$$L(W^{01}, y^n) = L(W_0^1) \cdot L(W_1^0)$$

$$= \frac{17}{8} \cdot \frac{8}{5} = \frac{17}{5} > 1$$

⇒ Décode vers 0

$$\Rightarrow u^{01} = 0$$

$$L(W^{10}, u^{00} u^{01} y^n) = 1$$

$$L(W_0^1, u_0^0 y_{00} y_{10}) = 1$$

$$L(W_1^1, u_1^0 y_{01} y_{11}) = \frac{1}{16}$$

$$L(W_{00}, y_{00}) = 4$$

$$L(W_{10}, y_{10}) = 4$$

$$L(W_{01}, y_{01}) = 4$$

$$L(W_{11}, y_{11}) = \frac{1}{4}$$

$$L(W_0^1) = \frac{L(W_{10}, y_{10})}{L(W_{00}, y_{00})} = \frac{4}{4} = 1$$

$$L(W_1^1) = \frac{L(W_{11}, y_{11})}{L(W_{01}, y_{01})} = \frac{\frac{1}{4}}{4} = \frac{1}{16}$$

$$L(W^{10}) = \frac{L(W_0^1) L(W_1^1) + 1}{L(W_0^1) + L(W_1^1)}$$

$$= \frac{\frac{1}{16} + 1}{\frac{17}{16}} = \frac{1+16}{17} = 1$$

⇒ Décode vers ???

Output du décodeur = 00?1
Input Jet 0101

$$L(w^n, u^{00} u^{01} u^{10} y^n)$$

$$L(w_0^1) = 1$$

$$L(w_1^1) = \frac{1}{16}$$

$$L(w^n) = L(w_0^1) \cdot L(w_1^1)$$

$$= \frac{1}{16} < 1$$

$\Rightarrow$ Décodage vers 1

$$\Rightarrow u^n = 1$$

