

Prof. Pierre L'Ecuyer

DEVOIR 8

Devoir à remettre par courriel au correcteur (Mathieu Gascon, matmath.g@hotmail.com) au plus tard le *mardi 7 avril 2020, avant 10h30*.

Les devoirs doivent être faits individuellement: un devoir par étudiant. Il est très important de bien expliquer tout ce que vous faites. Dans la correction, on accordera beaucoup d'importance à la clarté des explications. Attention au plagiat: il n'est pas permis de copier et/ou modifier des solutions venant de quelqu'un d'autre, ou prises sur Internet ou dans des livres.

1. (13 points) Ross (2014), page 346, Exercice 49.

Events occur according to a Poisson process with rate λ . Each time an event occurs, we must decide whether or not to stop, with our objective being to stop at the last event to occur prior to some specified time T , where $T > 1/\lambda$. That is, if an event occurs at time t , $0 \leq t \leq T$, and we decide to stop, then we win if there are no additional events by time T and we lose otherwise. If we do not stop when an event occurs and no additional events occur by time T , then we lose. Also, if no events occur by time T , then we lose. Consider the strategy that stops at the first event to occur after some fixed time s , $0 \leq s \leq T$.

(a) Using this strategy, what is the probability of winning?

(b) What value of s maximizes the probability of winning?

(c) Show that one's probability of winning when using the preceding strategy with the value of s specified in part (b) is $1/e$. (Encore Euler!)

2. (12 points) Ross (2014), page 346, Exercice 50.

Suppose the number of hours between successive bus arrivals at a given bus stop is uniformly distributed on $(0, 1)$. Passengers arrive according to a Poisson process with rate λ per hour. Suppose a bus has just left from the stop. Let X denote the number of people who will get on the next bus. Find $\mathbb{E}[X]$ and $\text{Var}(X)$. How can we compute the entire distribution of X , i.e., $\mathbb{P}[X = x]$ for each integer $x \geq 0$?

Hint: In tables of integral, one can find that

$$\int_0^1 y^n e^{-ay} dy = \frac{n!}{a^{n+1}} \left(1 - e^{-a} \sum_{i=0}^n \frac{a^i}{i!} \right).$$

3. (10 points) Ross (2014), page 348, Exercice 59.

There are two types of claims that are made to an insurance company. Let $N_i(t)$ denote the number of type i claims made by time t , and suppose that $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$

are independent Poisson processes with rates $\lambda_1 = 10$ and $\lambda_2 = 1$. The amounts of successive type 1 claims are independent exponential random variables with mean \$1000 whereas the amounts from type 2 claims are independent exponential random variables with mean \$5000. A claim for \$4000 has just been received; what is the probability that it is a type 1 claim?

4. (15 points)

Supposons que les clients arrivent à un magasin de détail selon un processus de Poisson de taux B de 9 heures à midi, $2B$ de midi à 14 heures, puis de $1.2B$ de 14 heures à 17 heures. Ici, B est une variable aléatoires de loi connue qui ne prend que des valeurs positives et prend la même valeur pour toute la journée. Une plus grande valeur de B correspondra à une journée plus achalandée.

On dispose d'une fonction pour générer B selon la bonne loi, et d'une autre fonction pour générer des (imitations de) variables aléatoires indépendantes uniformes sur $(0, 1)$. Donnez un algorithme permettant de simuler les instants des arrivées successives pour ce processus, pour une journée, en utilisant ces deux ingrédients.