

Preventive Replacement for Multicomponent Systems: An Opportunistic Discrete-Time Dynamic Programming Model

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Key Words — Dynamic programming, Multicomponent system, Preventive replacement, Opportunistic replacement, Stochastic optimization, Control-limit rule.

Readers Aids —

Purpose: Present a derivation

Special math needed for explanations: Probability, Dynamic programming

Special math needed to use results: Same

Results useful to: Maintenance theoreticians

Abstract — We propose a dynamic programming (DP) model for the optimal preventive replacement of elements in a multicomponent system. The model generalizes previous work on the subject by: *i*) allowing for non identical components, *ii*) permitting the computation of an ϵ -optimal strategy for a medium size system (4 to 5 components), *iii*) showing that the control-limit rule does not extend to most of the multicomponent systems, and *iv*) proposing a class of suboptimal strategies to be used when the system is too large for directly implementing the DP algorithm. These features of the model are illustrated in a separately available Supplement via a numerical example inspired from the modular structure of a modern fighter aircraft's engine.

1. INTRODUCTION

Most preventive replacement optimization models assume that the system consists of a single element; but many systems are multicomponent with some form of cost dependency between the components. If economies of scale are possible in the replacement activity, the optimal preventive replacement strategy is usually opportunistic [1].

2. MODEL

Notation

m	number of components in the system
M	$[1, 2, \dots, m]$, the set of components
d	state of a failed component
X	$([0, 1, 2, \dots] \cup \{d\})^m$; vector state space for the space for the system
$J_k(x)$	minimal discounted s -expected cost-to-go for the next k periods if the system is now in state x
$J(x)$	minimal discounted s -expected cost-to-go for an infinite horizon if the system is now in state x

Assumptions

1. The system has m components with non-identical s -independent life-time distributions characterized by discrete non-decreasing failure rates.
2. The state of the system is perfectly observed at discrete times. A strategy tells, for each possible state, which operative components should be replaced (preventive replacement) in addition to the mandatory replacement of failed components.
3. Replacements, if any, are instantaneous and by new components only.
4. The optimality criterion, to be minimized through the choice of an appropriate strategy, is the discounted s -expected cost-to-go over an infinite time horizon. The discount factor per time-step is $\beta < 1$.

At an observation time, the system is in a state $x \in X$ and a set R of components to be replaced is chosen; the set contains at least the set of failed components. The components in the set $W(R)$ must be dismantled, and a cost

$$C(R) \equiv \sum_{i \in R} C_i + \sum_{i \in W(R)} c_i$$

is incurred.

The transition probabilities for the next step can be easily computed from the failure probabilities. A Markov decision process is thus defined. The dynamic programming successive approximation method can be used to obtain an ϵ -optimal strategy and to characterize the functionals J_k and J , as well as the optimal strategy. The following properties are proved in [2-3]:

1. $J(x) = \lim_{k \rightarrow \infty} J_k(x)$ for all x in X .
2. $J_k(x)$ for each k , as well as $J(x)$, are non-decreasing w.r.t. each component of x (d being considered larger than every integer).
3. There exists an optimal stationary strategy $[\theta_*(x), x \in X]$, where $\theta_*(x)$ is the optimal set of components to replace when the system is in state x .
4. Optimality precludes preventive replacements in the absence of failure.
5. The following "control-limit" property holds: $\theta_*(x) = M$ implies $\theta_*(\bar{x}) = M$ for all $\bar{x} \geq x$.

A version of the DP algorithm, allowing approximations and permitting the efficient computation of an ϵ -optimal opportunistic preventive replacement strategy is proposed in the Supplement [3].

A numerical illustration is given in [3], with a 4-component system similar to a modular jet engine. An optimal strategy

(ϵ being negligibly small) has been obtained and happens to be quite complex. It shows, among other things, that the well known, control-limit rule, valid for a 1- or 2-component system, cannot be readily generalized to larger systems. More precisely, $\theta_*(\bar{x})$ can be a strict subset of $\theta_*(x)$ even if $\bar{x} > x$.

3. SIMPLE SUBOPTIMAL STRATEGIES

The optimal strategy is generally very complicated; thus there is an incentive for considering only a restricted class of rules having a fixed predetermined form. Consider, as an example, the following class of rules: for given thresholds $l_i, i = 1, \dots, m$, replace, in addition to the set H of failed components, every component i which is already dismantled (i.e., $i \in W(H)$) and whose age is larger or equal to l_i . To find the optimal l_i values, repeated simulation can be used. For the numerical illustration [3], the best strategy of this form yielded an s -expected cost-to-go for a new system only 3% higher than the optimal value $J(0)$.

This class of suboptimal strategies could also be refined as suggested in [3], in order to take advantage of the increased accessibility to some components when others are dismantled.

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[2] P. L'Ecuyer, "Politiques optimales et sous-optimales pour le remplacement des composantes d'un système (Optimal and suboptimal policies for the replacement of components of a system)." Les Cahiers du GERAD, Report No G-80-05, 1980 Nov. Available from GERAD: Ecole des Hautes Etudes Commerciales: 5255 avenue Decelles; Montréal, Québec H3T 1V6, CANADA.

[3] Supplement: NAPS document No. 04009-B: 9 pages in this Supplement. For current ordering information, see "Information for Readers & Authors" in a current issue. Order NAPS document No. 04009, 31 pages. ASIS-NAPS; Microfiche Publications; PO Box 3513, Grand Central Station; New York, NY 10017 USA.

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