Ordonnancement optimal des pages proposées par un moteur de recherche

Pierre L'Ecuyer











Patrick Maillé, Nicolás Stier-Moses, Bruno Tuffin







Search engines

- Major role in the Internet economy
- most popular way to reach web pages
- ▶ 20 billion requests per month from US home and work computers only

Search engines

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For a given (set of) keyword(s), a search engine returns a ranked list of links: the organic results.

Organic results are supposed to be based on relevance only

Is this true?

Each engine has its own formula to measure (or estimate) relevance. May depend on user (IP address), location, etc.

Best Match ▼

View: II -

Categories

AII

Sporting Goods Bicycles

Condition see all Used (1)

Price

Format see all

see all

see all

 All Listings (1) Auction (0)

to S

Buy It Now (1)

Delivery Options

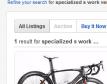
Free international shipping Show only

Returns accepted Completed listings

Sold listings

More refinements...

Recently viewed items



Refine your search for specialized s work venge 52

+ Follow this search

SPECIALIZED S WORKS McLAREN VENGE, DI2, SIZE 52cm, MARK CAVENDISH'S HTC BIKE

\$20.000.00 or Best Offer

From Ireland

More items related to specialized s work venge 52



2012 Specialized Venge Pro SRAM Red 52cm 10 Spd Road Race Bike-Carbon-Nice

\$2,800.00 or Best Offer

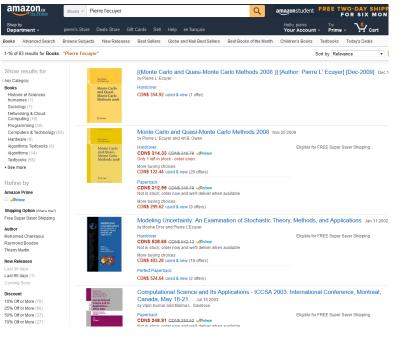
From United States

20 watching

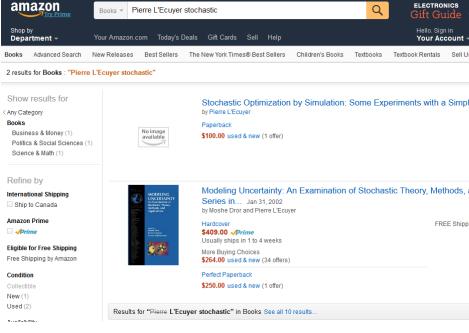


2015 S-Works Venge 52cm Dura Ace Enve w/ Quara

\$6,400.00 From United States or Best Offer



How are items ranked? Relevance vs expected revenue?



How are items ranked? Relevance vs expected revenue?

6



How are items ranked? Relevance vs expected revenue?



How are items ranked? Relevance vs expected revenue?

clearancebooksale

over the past 12 months.

(90.055 total ratings)



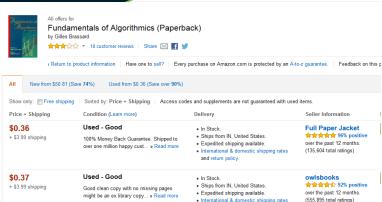
Used - Good

Good condition, wear from reading and use.

All pages are intact. ... » Read more

\$0.37

+ \$3.99 shipping



and return policy.

and return policy.

· Expedited shipping available.

International & domestic shipping rates

In Stock

\$0.41 Used - Good

+ \$3.99 shipping Book has a small amount of wear visible on the binding, cover, pa... ➤ Read more

- Instruct.

- In Stock.

- Ships from MO, United States.

- Ships from MO, Un

How are items ranked? Relevance vs expected revenue?



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About 72,800 results (0.58 seconds)

Baie-Saint-Paul bed - No reservation costs. Great rates www.booking.com/

Bed And Breakfast Baie St Paul Quebec.

Ratings: Selection 9/10 - Customer service 9/10 - Website 9/10 - Travel info 9/10 Booking com has 4 415 061 followers on Google+

Deals Special Offers - We Speak Your Language - Get Instant Confirmation

B&Bs in Baie Saint Paul - trivago.ca

www.trivago.ca/BaieSaintPaul-B&Bs *

B&Bs in Baie Saint Paul from \$60. Choose from All Our Available B&Bs! Save up to 78% - Saves You Time and Money - Impartial Comparison trivago has 79,821 followers on Google+

3* Hotels - Central Hotels (<3km) - Compare 700 000+ Hotels - B&B up to -78%

Baie Saint Paul Bed and Breakfasts

www.bedandbreakfast.com/ ▼

B&B and Guest House from \$29/night. Browse Photos, Guest Reviews & More

Bed and breakfast near baie st paul quebec

□ Sun, Nov 22 □ Mon, Nov 23 Price ▼ Hotel class ▼

À La Chouette

Country-style rooms in a quaint B&B

2 Rue Leblanc



Gite Nature et Pinceaux

\$110

\$115



Arte

Bed Breakfast Baie-St-Paul

www.hotels.com/reservations ▼
+1 866-417-7101
Incredible Offers on Great Hotels.
Red And Breakfast Bale St Paul Quebec.

Quebec Bed & Breakfast

www.expedia.ca/Quebec_Bed_Breakfast ▼ Expedia Guarantees the Best Price. Book a Bed & Breakfast in Quebec

Bed Breakfast Saint Paul

saintpaul.tripadvisor.ca/ ▼ 17 hotels in Saint Paul. Read Hotel Reviews for Saint Paul!

Auberge Le Cormoran

www.belleplage.ca/ ▼
Au coeur de Baie-Saint-Paul.
directement sur le bord du fleuve

Bed Breakfast Baie Saint Paul www.hotelscombined.ca/Baie-Saint-Pa ▼

Save Up to 80% on Accommodation.

One Search, Best Deals, Book Nowl.

Baie-St-Paul Bed & Breakfasts

www.venere.com/hotels ▼ Book Now and Save up to 50%! Bed And Breakfast Baie St Paul Quebec

Affordable bed & breakfast

www.ouvriersdepaix.org/touristic-stay *Bed & breakfast, spa, trails
In the heart of the province

Sun Nov 22

Mon Nov 23

Price +

Back to web results for bed and breakfast baie st-paul quebec

À La Chouette 2 reviews - 4-star hotel

\$115

Cozy B&B for country-style rooms with hardwood floors & WiFi, plus a studio with a kitchen



2 Rue Leblanc

Gîte Les Colibris B & B \$98

No reviews - 4-star hotel



80 SAINTE-ANNE RUE

Gite Nature et Pinceaux

\$110 No reviews - 4-star hotel French country style in an elegant rural inn, with free WiFi & gardens with a fire pit &



hammocks 33 rue du Nordet

Gîte la Chambre des Maît... 2 reviews - 4-star hotel

Casual rooms in a relaxed B&B with a seasonal outdoor pool, plus a quest lounge with a fireplace.



À La Chouette

4-star hotel

Address: 2 Rue Leblanc, Baie-Saint-Paul, QC G37 1W9

Phone: (418) 435-3217

A 5-minute walk from the town centre, this relaxed B&B in a quaint clapboard cottage is a 6-minute walk from the Musée D'Art Contemporain De Baie St-Paul modern art museum and 1.8 km from ... More

Rook a room

Check in B. Booking.com

33 Sun. Nov 22

Read Real Guest Reviews - Get Instant Confirmation

Check out Mon. Nov 23

Website

Book from \$115

View more rates







\$115

per night

Directions



Dufour

La Mar

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About 993,000 results (0.31 seconds)

Justin Trudeau vs. Patrick Brazeau Boxing Match p1 ...



www.youtube.com/watch?v=tb-sqWJyWqA * Mar 31, 2012 - Uploaded by CIReport ca Skinny Indian? What part of Brazeau is skinny? Secondly please ▶12:03 take a good look at Patrick Brazeau, if he is ...

Justin Trudeau -- Patrick Brazeau Charity Boxing Match ... www.youtube.com/watch?v=XuSpZ3 5pTc *

Apr 1, 2012 - Uploaded by FactPointVideo



Justin Trudeau -- Patrick Brazeau Charity Boxing Match --▶ 16:14 English ... they went under. I love watching the ...

Trudeau vs Brazeau BOXING FOR CANCER (Full fight ...



https://www.youtube.com/watch?v=fYIWiZMhaLE Apr 1, 2012 - Uploaded by Reasoning Conspiracy ... link http://www.sunnewsnetwork.ca/video/1540728713001 ► 11:26 Watch the ... Patrick Brazeau and Justin ...

Meet Justin Trudeau: The Tattooed, Boxing New Canadian ... www.nbcnews.com/.../video/meet-justin-trudeau--the-t...



Meet Justin Trudeau: The Tattooed, Boxing New Canadian Prime



Justin Trudeau vs Patrick Brazeau Le combat de boxe ...



www.youtube.com/watch?v=go7nf6xwfyA * Mar 31, 2012 - Uploaded by LP - Mma Nouvelles Justin Trudeau vs Patrick Brazeau Le combat de boxe Justin 1957 Trudeau - Patrick Brazeau Charity Boxing

Justin Trudeau Boxing Match Weigh In With Senator Patrick ...



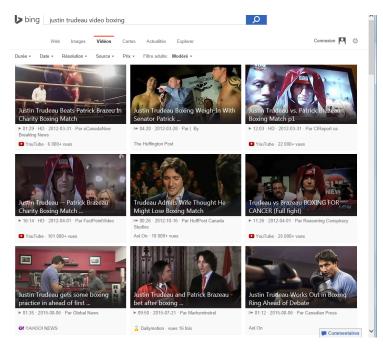
www.youtube.com/watch?v=M0e9D2rBN9w * Mar 28. 2012 - Uploaded by althiaraj Liberal MP Justin Trudeau meets his competitor Conservative

Senator Patrick Brazeau for a weigh in before ..

Justin Trudeau Boxing - YouTube



.youtube.com/watch?v=JLQWA35er3g Aug 6 2015 - Unloaded by Alex Guibord





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NOT A MULCAIR IN THE WORLD

"There's not a country in the world that would find 170 billion barrels of oil under the ground and leave them there. There is not a province in this country that would find 170 billion barrels of oil and leave it in the ground." - J.T. 03/10/12 National Post



Reposter



par Hardtofindhatch

Date de publication : 2015-09-Durée - 03-02

Catégorie : News

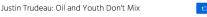
|≈ |P

Utilisez le repost pour partager une vidéo que vous aimez avec vos abonnés. En savoir plus

I Don't Remember, I Don't Recall - Justin par pushinguproses



Eva Simons - I Don't



21 vues

Exporter Aiouter à

Justin Trudeau: Oil and Youth Don't Mix

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Video: Evo Morales caught kneeing player in groin during ... www.telegraph.co.uk > ... > South America > Bolivia ▼



Oct 5, 2010

Evo Morales, the Bolivian president, has been caught on camera kneeing an opposing player in the groin ...

Bolivian president Morales plays football for UN - YouTube

www.youtube.com/watch?v=uHIT3T9b86l Sep 27, 2013 - Uploaded by AFP news agency

> President of Bolivia Evo Morales during a soccer match at a park near the UN on the sidelines of the 68th ...

Bolivian President kicks man in bollocks (in football match ... https://www.youtube.com/watch?v=6XkbNUlx5Bo



Oct 6, 2010 - Uploaded by TrashTVChannel Bolivian president Evo Morales caught on camera knee-ing opponent (Daniel Gustavo Cartagenam) in the ...

Ahmadinejad plays football with Bolivian President Evo ...



www.youtube.com/watch?v=ragkl2EPqxw * Oct 27, 2010 - Uploaded by sherlock72 During a visit of Bolivian President Evo Morales to Iran , he played a friendly football game with Iranian .

Bolivian president Morales plays football for UN - Video ...





Bolivia: Watch Evo Morales play against ex-Real Madrid ...



https://www.youtube.com/watch?v=v9oz0vvD2U4 Oct 8, 2014 - Uploaded by Ruptly TV Video ID: 20141008-057 M/S Evo Morales and former Real ▶ 0:59 Madrid ... Bolivian President Evo Morales ...

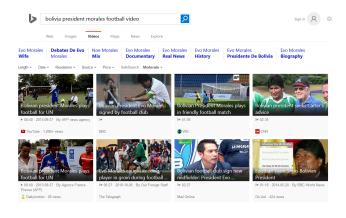
Bolivian President Morales plays in friendly football match ... www.youtube.com/watch?v=PUxWjVVGLOI Jul 31, 2015 - Uploaded by AP Archive

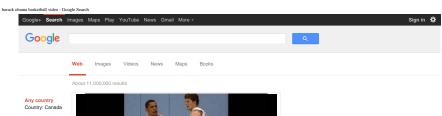


Bolivian President Evo Morales walking onto pitch in Cadiz stadium 2. ... Bolivian President Morales plays ...









Any time Past hour Past 24 hours Past week Past month Past year

All results Verbatim



Barack Obama playing Basketball Game. AMAZING FOOTAGE

https://www.youtube.com/watch?v=0OIDdGQQ0L8

Images for barack obama basketball video









Barack Obama's basketball fail - YouTube

https://www.voutube.com/watch?v=gmTfKPx1Cug 1 Apr 2013 - 1 min - Uploaded by The Telegraph



putin judo video

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About 54,600 results (0.24 seconds)

All YouTube:

'Judo Knight' Putin shows off martial arts skills in wrestling ...



https://www.youtube.com/watch?v=0lxMglj8LuM Dec 23, 2010 - Uploaded by RT

Prime Minister Vladimir Putin has taken another step to boost his 1:36 charisma by ... Bodyguard face-off video ...

Putin's Judo - YouTube



Vladimir Putin has a black belt in the martial art and has ►3:18 competed at a high ... One day.....a dream face off ...



https://www.youtube.com/watch?v=f62mvM2iPiE



Nov 7, 2012 - Uploaded by Putinery

Russian President Vladimir Putin shares his favourite judo moves. ... Let's Learn Judo with Vladimir ...

Putin may join Russian Olympic judo team - YouTube https://www.voutube.com/watch?v=2c3wzbtvikw





The Russian Judo Olympic team may have a new member. ... ▶3334 Russia's Got Talent: Extra video of Putin ...

Putin practices judo - No comment - YouTube



https://www.youtube.com/watch?v=V5z5UAtCxgE Dec 21, 2009 - Uploaded by No Comment TV

http://www.euronews.com/nocomment/ Vladimir Putin announcing 1:36 he could help the Russian Judo Team if ...

Putin Judo - YouTube ▶ 0:32



Russian President Vladimir Putin has a black belt in Judo.

Putin & Co to teach you judo - YouTube

https://www.youtube.com/watch?v=C7yjCaCQPO8

Oct 7, 2008 - Uploaded by RT

françois hollande scooter video

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Videos of françois hollande scooter

bing.com/videos



See more videos of françois hollande scooter.

François Hollande's scooter trips to visit mistress spawn ... www.dailymail.co.uk/news/article-2539600

Video embedded · Francois Hollande ridiculed in video game parody of his secret scooter trips to visit mistress Julie Gavet. Online game lampooning French President ...

François hollande sur un scooter! - YouTube

www.youtube.com/watch?v=1zGXiCAkk1A ▼

BY BATTERIRE! 1 MIN 502 VIEWS ADDED 2015-05-27

Video embedded · Ceci est bien évidemment un fake , ce n'est pas françois hollande...

Sarkozy, Hollande and the scooter wars | Watch the video ...

news.yahoo.com/video/sarkozy-hollande-scooter-wars-230001927.html > Watch the video Sarkozy, Hollande and the scooter wars on Yahoo News MEDIAWATCH: We start with ongoing coverage in France of the Air Algérie disaster. ...

Related searches for françois hollande scooter video

François Hollande Julie Gayet Julie Gavet

Valérie Trierweiler President Hollande

François Hollando Jewish I a Hollando

Photographer reveals how he tracked down François Hollande

Related searches

Valório Trionvoilor

François Hollande Jewish

President François Hollande

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French President François Hollande

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Videos

All YouTube:

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Silvio Berlusconi's bunga bunga parties and woman catch ...



https://www.voutube.com/watch?v=rwgwu_N8kXY Jun 25, 2013 - Uploaded by Taiwanese Animators Silvio Berlusconi's bunga bunga parties and woman catch up with him ... Daily videos brought to you ...

Maps

SILVIO Berlusconi's 'bunga bunga' problem - YouTube



www.voutube.com/watch?v=c2UZEvCvP Q Jun 25, 2013 - Uploaded by Lais Souza Silvio Berlusconi waits to learn his fate as bunga-bunga trial comes to a climax ... More videos for Silvio ...

bunga bunga danse Silvio Berlusconi - YouTube



https://www.youtube.com/watch?v=H4IJw1yFAmM Oct 29, 2010 - Uploaded by MrSmokedrink bunga bunga danse Silvio Berlusconi. ... un video sui festini a base di donne di Berlusconi ,mandato ...

Bunga Bunga feat. Silvio Berlusconi - YouTube



www.youtube.com/watch?v=RU0nUvXfhfA Apr 16, 2014 - Uploaded by The Kaka Island Monkeys iTUNES: https://goo.gl/9iEm2w AMAZON: http://goo.gl/c12pbU ▶2458 FACEBOOK liken und teilen: http://goo.gl ...

Silvio Berlusconi: Bunga Bunga - YouTube



https://www.youtube.com/watch?v=2qic5PRH2mM Sep 30, 2011 - Uploaded by Goldfinger200 In seiner Freizeit ist er Regierungschef, Berlusconi erzählt 542 schmutzige Witze. Viele Italiener fühlen sich ...

Ruby Berlusconi e il BUNGA BUNGA - YouTube



https://www.youtube.com/watch?v=8AFFjsYpjw8 Nov 4, 2010 - Uploaded by giovanni cangaceiro Ruby Berlusconi e il BUNGA BUNGA ... IL VIDEO CHE HA FATTO INFURIARE IL PREMIER ALL ...

silvio berlusconi bunga bunga video

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Silvio Berlusconi and Affairs and Controversy

Bunga Bunga Italian

Silvio Berlusconi **Parties**

Silvio Berlusconi

Silvio Berlusconi

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Scandal

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Biography

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Silvio Berlusconi's 'bunc bunga' problem

► 03:06 · 2013-06-24 · By CNN

YouTube - 5 000+ views



► 05:42 · 2011-09-30 · By Goldfinger200

YouTube - 21 000+ views



▶ 01:34 · 2013-06-25 · By We Moved! Search TomoNewsUS

YouTube - 15 000+ views

Berlusconi "bunga bunga" sex trial gets underway

▶ 02:33 · HD · 2011-04-06 · By CBS

YouTube - 1 000+ views



▶ 03:42 · 2013-01-01 · By Hans Meiser

YouTube - 1 000+ views



Mail Online



□ 03:05

CNN CNN

Bunga Bunga Silv

Ruby Rubacuori il E → 00:59 · 2010-11-01 · I

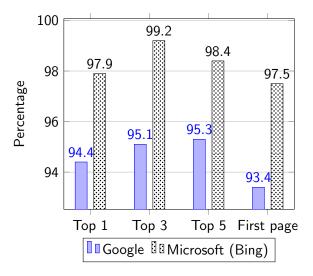
Do search engines return biased results?

Comparison between Google, Bing, and Blekko (Wright, 2012):

- ▶ Microsoft content is 26 times more likely to be displayed on the first page of Bing than on any of the two other search engines
- ► Google content appears 17 times more often on the first page of a Google search than on the other search engines

Search engines do favor their own content

Do search engines return biased results?



Percentage of Google or Bing search results with **own content not** ranked similarly by any rival search engine (Wright, 2012).

Search Neutrality (relevance only)

Some say search engines should be considered as a public utility.

Idea of search neutrality: All content should have the same chance of being displayed if they have equivalent relevance. Content of higher relevance should never be displayed in worst position.

More fair, better for users and for economy, encourages quality, etc.

What is the precise definition of "relevance"? Not addressed here ...

Debate: Should neutrality be imposed by law? Pros and cons.

Regulatory intervention: The European Commission, is progressing toward an antitrust settlement deal with Google.

"Google must be even-handed. It must hold all services, including its own, to exactly the same standards, using exactly the same crawling, indexing, ranking, display, and penalty algorithms."

In general: trade-off in the rankings

From the viewpoint of the SE: Tradeoff between

relevance (long term profit)

versus

expected revenue (short term profit)

Better relevance brings more customers in the long term because it builds reputation.

What if the provider wants to optimize its long-term profit?

Start with a simple stylized model

"To get elegant results and insight, start with simplified model. If too hard, simplify more."

- Newton laws in physics
- Erlang queueing formulas
- Black-Scholes in finance
- Etc.

Model of search requests

```
Request: random vector Y = (M, R_1, G_1, \ldots, R_M, G_M) where M = number of pages that match the request; R_i \in [0,1]: measure of relevance of page i; G_i \in [0,K]: expected revenue (direct or indirect) from page i. Y has a prob. distribution over \Omega = \mathbb{N} \times ([0,1] \times [0,K])^{m_0}. Can be discrete or continuous.
```

```
y = (m, r_1, g_1, \dots, r_m, g_m) denotes a realization of Y.
```

```
c_{i,j}(y) = \mathbb{P}[\text{click page } i \text{ if in position } j] = \text{click-through rate (CTR)}. Assumed \nearrow in r_i and \searrow in j.
```

Model of search requests

Request: random vector $Y = (M, R_1, G_1, \dots, R_M, G_M)$ where

M = number of pages that match the request;

 $R_i \in [0, 1]$: measure of relevance of page i;

 $G_i \in [0, K]$: expected revenue (direct or indirect) from page i.

Y has a prob. distribution over $\Omega = \mathbb{N} \times ([0,1] \times [0,K])^{m_0}$.

Can be discrete or continuous.

 $y = (m, r_1, g_1, \dots, r_m, g_m)$ denotes a realization of Y.

 $c_{i,j}(y) = \mathbb{P}[\text{click page } i \text{ if in position } j] = \text{click-through rate (CTR)}.$ Assumed \nearrow in r_i and \searrow in j.

Decision (ranking) for any request y: Permutation $\pi = (\pi(1), \dots, \pi(m))$ of the m matching pages. $j = \pi(i) = \text{position of } i$.

Local relevance and local revenue for y and π :

$$r(\pi, y) = \sum_{i=1}^{m} c_{i,\pi(i)}(y) r_i,$$
 $g(\pi, y) = \sum_{i=1}^{m} c_{i,\pi(i)}(y) g_i.$

Deterministic stationary ranking policy μ

It assigns a permutation $\pi = \mu(y) \in \Pi_m$ to each $y \in \Omega$.

Long-term expected relevance per request (reputation of the provider) and expected revenue per request (from the organic links), for given μ :

$$r = r(\mu) = \mathbb{E}[r(\mu(Y), Y)],$$

 $g = g(\mu) = \mathbb{E}[g(\mu(Y), Y)].$

Deterministic stationary ranking policy μ

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$$r = r(\mu) = \mathbb{E}[r(\mu(Y), Y)],$$

 $g = g(\mu) = \mathbb{E}[g(\mu(Y), Y)].$

Objective: Maximize long-term utility function $\varphi(r, g)$. Assumption: φ is strictly increasing in both r and g.

Example: expected revenue per unit of time

$$\varphi(r,g) = \lambda(r)(\beta + g)$$
, where

 $\lambda(r) = \text{arrival rate}$ of requests, strictly increasing in r; $\beta = \mathbb{E}[\text{revenue per request}]$ from non-organic links (ads on root page); $g = \mathbb{E}[\text{revenue per request}]$ from organic links.

Deterministic stationary ranking policy μ

It assigns a permutation $\pi = \mu(y) \in \Pi_m$ to each $y \in \Omega$.

Q: Is this the most general type of policy?

Long-term expected relevance per request (reputation of the provider) and expected revenue per request (from the organic links), for given μ :

$$r = r(\mu) = \mathbb{E}[r(\mu(Y), Y)],$$

 $g = g(\mu) = \mathbb{E}[g(\mu(Y), Y)].$

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Randomized stationary ranking policy $\tilde{\mu}$

$$\tilde{\mu}(y) = \{q(\pi, y) : \pi \in \Pi_m\}$$

is a probability distribution, for each $y=(m,r_1,g_1,\ldots,r_m,g_m)\in\Omega$. Let $z_{i,j}(y)=\mathbb{P}[\pi(i)=j]$ under $\tilde{\mu}$.

Expected relevance

$$r = r(\tilde{\mu}) = \mathbb{E}\left[\sum_{\pi} q(\pi, Y) \sum_{i=1}^{M} c_{i,\pi(i)}(Y) R_i\right] = \mathbb{E}\left[\sum_{i=1}^{M} \sum_{j=1}^{M} z_{i,j}(Y) c_{i,j}(Y) R_i\right]$$

Expected revenue

$$g = g(\tilde{\mu}) = \mathbb{E}\left[\sum_{\pi} q(\pi, Y) \sum_{i=1}^{M} c_{i,\pi(i)}(Y) G_i\right] = \mathbb{E}\left[\sum_{i=1}^{M} \sum_{j=1}^{M} z_{i,j}(Y) c_{i,j}(Y) G_i\right].$$

In terms of (r,g), we can redefine (simpler) $\tilde{\mu}(y) = \mathbf{Z}(y) = \{z_{i,j}(y) \geq 0 : 1 \leq i,j \leq m\}$ (doubly stochastic matrix).

Q: Here we have a stochastic dynamic programming problem, but the rewards are not additive! Usual DP techniques do not apply.

How can we compute an optimal policy?

Seems very hard in general!

Optimization problem

$$\begin{array}{lll} \max_{\tilde{\mu} \in \tilde{\mathcal{U}}} & \varphi(r,g) \\ & \text{subject to} \\ & r & = & \mathbb{E}\left[\sum_{i=1}^M \sum_{j=1}^M z_{i,j}(Y)c_{i,j}(Y)R_i\right] \\ & g & = & \mathbb{E}\left[\sum_{i=1}^M \sum_{j=1}^M z_{i,j}(Y)c_{i,j}(Y)G_i\right] \\ & \tilde{\mu}(y) & = & \mathbf{Z}(y) = \{z_{i,j}(y): 1 \leq i,j \leq m\} \quad \text{ for all } y \in \Omega. \end{array}$$

Optimization problem

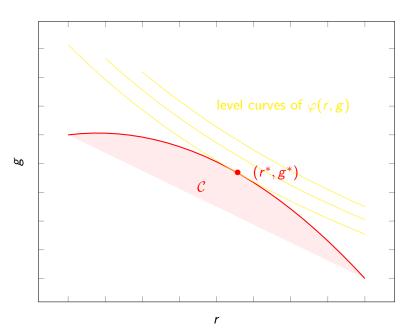
$$\begin{array}{lll} \max_{\tilde{\mu} \in \tilde{\mathcal{U}}} & \varphi(r,g) \\ \text{subject to} \\ & r & = & \mathbb{E}\left[\sum_{i=1}^M \sum_{j=1}^M z_{i,j}(Y)c_{i,j}(Y)R_i\right] \\ & g & = & \mathbb{E}\left[\sum_{i=1}^M \sum_{j=1}^M z_{i,j}(Y)c_{i,j}(Y)G_i\right] \\ & \tilde{\mu}(y) & = & \mathbf{Z}(y) = \{z_{i,j}(y) : 1 \leq i,j \leq m\} \quad \text{ for all } y \in \Omega. \end{array}$$

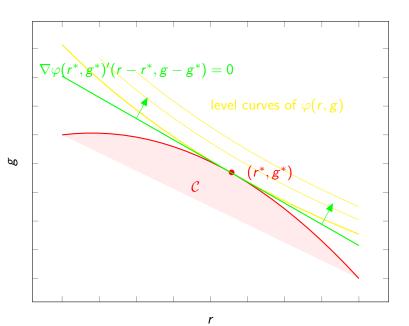
To each $\tilde{\mu}$ corresponds $(r,g)=(r(\tilde{\mu}),g(\tilde{\mu}))$.

Proposition: The set $\mathcal{C} = \{(r(\tilde{\mu}), g(\tilde{\mu})) : \tilde{\mu} \in \tilde{\mathcal{U}}\}$ is convex.

Optimal value: $\varphi^* = \max_{(r,g) \in \mathcal{C}} \varphi(r,g) = \varphi(r^*,g^*)$ (optimal pair).

Idea: find (r^*, g^*) and recover an optimal policy from it.





Optimization

$$\nabla \varphi(r^*, g^*)'(r - r^*, g - g^*) = \varphi_r(r^*, g^*)(r - r^*) + \varphi_g(r^*, g^*)(g - g^*) = 0.$$

Let $\rho^* = \varphi_g(r^*, g^*)/\varphi_r(r^*, g^*) = \text{slope of gradient.}$

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$$\max_{(r,g)\in\mathcal{C}} \varphi(r,g) = \max_{(r,g)\in\mathcal{C}} (r-r^* + \rho^*(g-g^*)).$$

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Optimal "solution" satisfies $(r^*, g^*) = \arg \max_{(r,g) \in \mathcal{C}} (r + \rho^* g)$.

The optimal (r^*, g^*) is unique if the contour lines of φ are strictly convex. True for example if $\varphi(r, g) = r^{\alpha}(\beta + g)$ where $\alpha > 0$.

The arg max is unique if and only if green line $\varphi_r(r^*, g^*)(r - r^*) + \varphi_g(r^*, g^*)(g - g^*) = 0$ touches $\mathcal C$ at a single point.

One more assumption

Standard assumption: click-through rate has separable form:

$$c_{i,j}(y) = \theta_j \, \psi(r_i),$$

where $1 \geq \theta_1 \geq \theta_2 \geq \cdots \geq \theta_{m_0} > 0$ (ranking effect) and $\psi: [0,1] \rightarrow [0,1]$ increasing.

Let $\tilde{R}_i := \psi(R_i)R_i$, $\tilde{G}_i := \psi(R_i)G_i$, and similarly for \tilde{r}_i and \tilde{g}_i . Then

$$r = \mathbb{E}\left[\sum_{i=1}^{M}\sum_{j=1}^{M}z_{i,j}(Y)\theta_{j}\tilde{R}_{i}\right]$$

and similarly for g.

Optimality conditions, discrete case

Definition. A linear ordering policy with ratio ρ (LO- ρ policy) is a (randomized) policy that ranks the pages by decreasing order of their value of $\tilde{r}_i + \rho \tilde{g}_i$ with probability 1, for some $\rho > 0$, except perhaps when $\theta_{j'} = \theta_j$ where the order does not matter.

Theorem. Suppose Y has a discrete distribution, with $p(y) = \mathbb{P}[Y = y]$. Then any optimal randomized policy must be an LO- ρ policy.

Idea of proof: by an interchange argument. If for some y with p(y) > 0, page i at position j has lower score $\tilde{r}_i + \rho \tilde{g}_i$ than the page at position j' > j with probability $\delta > 0$, we can gain by exchanging those pages, so this is cannot be optimal.

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Just finding ρ^* appears sufficient to determine an optimal policy

Nice!

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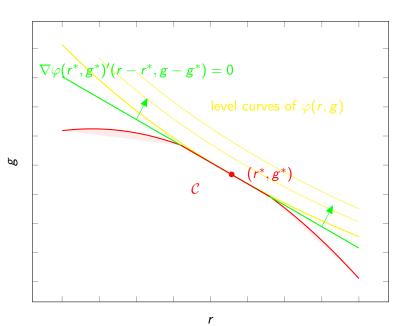
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The answer is **NO**.

Specifying ρ^* is not enough to uniquely characterize an optimal policy when equality can occur with positive probability.



Counter-example.

Single request $Y = y = (m, r_1, g_1, r_2, g_2) = (2, 1, 0, 1/5, 2)$.

$$\psi(r_i) = 1$$
, $(\theta_1, \theta_2) = (1, 1/2)$, $\varphi(r, g) = r(1+g)$.

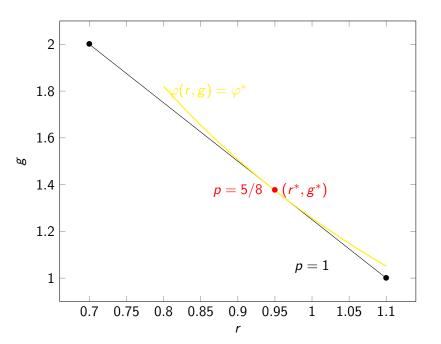
For each request, $\mathbb{P}[\mathsf{ranking}\ (1,2)] = p = 1 - \mathbb{P}[\mathsf{ranking}\ (2,1)].$

One finds that $\varphi(r,g) = (7+4p)(3-p)/10$, maximized at $p^* = 5/8$.

This gives $r^* = 19/20$, $g^* = 11/8$, $\varphi(r^*, g^*) = 361/160$.

$$p=0$$
 gives $\varphi(r,g)=336/160$ and $p=1$ gives $\varphi(r,g)=\varphi^*=352/160$.

No optimal deterministic policy here!



Continuous distribution for Y

Definition. A randomized policy $\tilde{\mu}$ is called an LO- ρ policy if for almost all Y, $\tilde{\mu}$ sorts the pages by decreasing order of $\tilde{R}_i + \rho \tilde{G}_i$, except perhaps at positions j and j' where $\theta_j = \theta_{j'}$, at which the order can be arbitrary.

Theorem (necessary conditions). Any optimal policy must be an LO- ρ policy with $\rho = \rho^*$.

Continuous distribution for Y

Assumption A. For any $\rho \ge 0$ and j > i > 0,

$$\mathbb{P}[M \geq j \text{ and } \tilde{R}_i + \rho \tilde{G}_i = \tilde{R}_j + \rho \tilde{G}_j] = 0.$$

Theorem (sufficient condition). Under Assumption A, for any $\rho \geq 0$, a deterministic LO- ρ policy sorts the pages for Y uniquely with probability 1. For $\rho = \rho^*$, this policy is optimal.

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In this case, it suffices to find ρ^* , which is a root of

$$\rho = \tilde{h}(\rho) := h(r,g) := \varphi_g(r,g)/\varphi_r(r,g).$$

Can be computed by a root-finding technique.

Proposition. (i) If h(r,g) is bounded over $[0,1] \times [0,K]$, then the fixed-point equation $\tilde{h}(\rho) = \rho$ has at least one solution in $[0,\infty)$. (ii) If the derivative $\tilde{h}'(\rho) < 1$ for all $\rho > 0$, then the solution is unique.

Proposition.

Suppose $\varphi(r,g) = \lambda(r)(\beta + g)$.

- (i) If $\lambda(r)/\lambda'(r)$ is bounded for $r \in [0,1]$ and $g(\rho(0)) > 0$, then the fixed point equation has at least one solution in $[0,\infty)$.
- (ii) If $\lambda(r)/\lambda'(r)$ is also non-decreasing in r, then the solution is unique.

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In this continuous case, computing an optimal deterministic policy is relatively easy. It suffices to find the root ρ^* and use the LO- ρ^* policy, which combines optimally relevance and profit.

What to do for the discrete case?

Very often, only a randomized policy can be optimal. But such a policy is very hard to compute and use in general! Not good!

Much simpler and better solution:

In case of equality, add a small noise to R_i for each i involved in an equality.

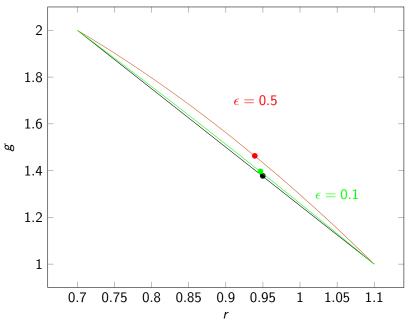
These noises can be taken uniform over $(-\epsilon, \epsilon)$ for some tiny ϵ .

Then the probability of an equality becomes 0.

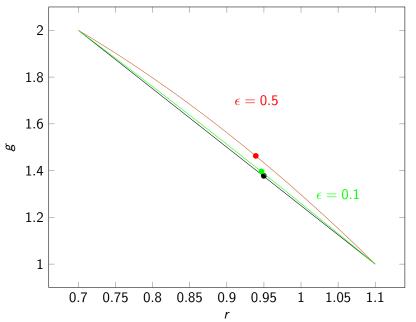
For very small ϵ (e.g., $10^{-10}R_i$), the impact is negligible.

If suffices to compute ρ^* with this "perturbed distribution."

Application to previous example



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More details

P. L'Ecuyer, P. Maille, N. Stier-Moses, and B. Tuffin. Revenue-Maximizing Rankings for Online Platforms with Quality-Sensitive Consumers.

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