

Can Chimps Go It Alone?

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Abstract. Consider a smart chimpanzee named M from a tribe afflicted with a form of Alzheimer's disease. Think of M as a logspace-bounded Turing machine. M can do simple things like integer arithmetic and matrix multiplication, but M turns sullen and calls for help when asked to perform seemingly equally simple tasks, such as simulating deterministic tree and dag automata.

Is M acting difficult or is she just not smart enough?

Even before the \mathbf{P} versus \mathbf{NP} question, Cook [3] conjectured that no amount of smarts can compensate for Alzheimer's disease¹.

We will review some of the attempts at separating \mathbf{L} from \mathbf{P} inspired by pebbling arguments. Emphasis will be placed on branching programs for the tree evaluation problem, recently studied anew [2]. The problem consists of determining the value that percolates to the root of a (binary) tree when a value from a domain D is prescribed at each tree leaf and an explicit function $f : D \times D \rightarrow D$ is prescribed at each internal node. In a nutshell, lower bounds for restricted branching programs can be proved, but approaches to attack the general model strangely come up against the same barrier that Nečiporuk encountered in a two-page note 50 years ago and that still stands today.

Tree evaluation naturally extends to tree generation [1], where the functions $f : D \times D \rightarrow D$ at internal tree nodes are replaced with functions $f : D \times D \rightarrow \{S : S \subseteq D\}$. This is interpreted as allowing to pick, as the D -value of a node labelled f with left child ℓ and right child r , any value from $f(D\text{-value of } \ell, D\text{-value of } r)$. Tree generation can then be turned into a monotone boolean function. Strong lower bounds for this function have been derived from pebbling intuition [4,1] and we will further discuss some of these.

For a suitable bibliography please consult [2,4,1].

References

1. Chan, S.M.: Just a pebble game. Electronic Colloquium on Computational Complexity (ECCC) 20, 42 (2013)
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3. Cook, S.A.: Characterizations of pushdown machines in terms of time-bounded computers. J. ACM 18, 4–18 (1971)
4. Chan, S.M., Potechin, A.: Tight bounds for monotone switching networks via fourier analysis. Electronic Colloquium on Computational Complexity (ECCC) 19, 185 (2012)

¹ Steve said this in a different language, thankfully.