

The MaximaList

A site for people who use Maxima to do Mathematics

Tag: Euler's Method

Using Maxima To Do Some Math Described in the Movie “Hidden Figures”



I looked in the NASA archives for the work of Katherine Johnson described so dramatically in the film “Hidden Figures”. I found this [1960 paper](#):



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TECHNICAL NOTE

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DETERMINATION OF AZIMUTH ANGLE AT BURNOUT FOR PLACING A SATELLITE OVER A SELECTED EARTH POSITION

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In particular, I was struck by the film's use of "Euler's Method" as the critical plot point, and sadly couldn't find anything about the use of that method in the archive, nor mention of it in the text of Margot Lee Shetterly's book on which the film "Hidden Figures" is based.

I did however read in the paper about a "iterative procedure" needed to solve for one variable.

Turns out the sequence of equations described in the report, when assembled into an expression in the single variable of interest, defines a contraction mapping and so converges by functional iteration, as is numerically demonstrated in the paper.

At the bottom I've linked a fuller treatment in a .wxmx file. Briefly, the authors describe the problem:

$$\Delta\lambda_{1-2e} = \lambda_{2e} - \lambda_1 + \omega_E [t(\theta_{2e}) - t(\theta_1)] \quad (19)$$

$$\cos(\theta_{2e} - \theta_1) = \sin \phi_2 \sin \phi_1 + \cos \phi_2 \cos \phi_1 \cos \Delta\lambda_{1-2e} \quad (20)$$

Time from perigee is expressed as

$$t(\theta) = \frac{T}{2\pi}(E - e \sin E) \quad (8)$$

Eccentric anomaly (fig. 1(b)) is given by

$$E = 2 \tan^{-1} \left(\sqrt{\frac{1-e}{1+e}} \tan \frac{\theta}{2} \right) \quad (9)$$

In the use of equations (19) and (20) an iterative procedure is required, since the time $t(\theta_{2e})$ from perigee to the equivalent position is not known initially. A satisfactory first approximation is to assume that

$$[t(\theta_{2e}) - t(\theta_1)] \approx \frac{T}{360}(\lambda_{2e} - \lambda_1) \quad (21)$$

This value, substituted into equation (19), yields a first approximation for $\Delta\lambda_{1-2e}$. This approximation substituted into equation (20) yields a first approximation for θ_{2e} from which, by equation (8), a new value of $t(\theta_{2e})$ is obtained. The iteration of these three steps is continued until the value of θ_{2e} converges. For convenience these equations are listed in appendix B.

Here's everything from the paper needed to do that in Maxima:

Satellite velocity at burnout, V_1 , ft/sec	25761.345
Distance of satellite from center of earth at burnout, r_1 , ft	21,637,933
Elevation angle, γ_1 , deg	0.5
Velocity for circular orbit, V_c , ft/sec	25506.28
p/r_1	1.020022269
Semimajor axis, a , ft	22081775.58
Angle between perigee and burnout, θ_1 , deg	± 23.969
Eccentricity, e	0.0219118
Period, T , min	91.585
Time from perigee to burnout, $t(\theta_1)$, min	5.842

```
:i6) a:22081775.58;
th1:23.969;
e:0.0219118;
T:91.585;
R:20903520;
g0:32.174;
```

Case A, eastward launch. - For case A, $\phi_1=28.50^\circ$ N., $\lambda_1=279.45^\circ$ E.; the given launch position is $\phi_2=34.00^\circ$ N., $\lambda_2=241.00^\circ$ E.; the number of orbital passes n is 3; and the equivalent selected longitude λ_{2e} is 309.689° E. The values

```
(%i14) /* Eastward Launch */
phi1:28.5;
lambda1:279.45;
phi2:34.0;
phi20:phi2;
lambda2:241.0;
lambda20:lambda2;
n:3;
lambda2e:309.689;

omega_E      angular velocity of the earth, 0.25068 deg/min (taken as
              0.25 deg/min in this report)

(%i15) omE:.25;
(omE) 0.25

(%i22) cosd(t):=block(float(cos(t*%pi/180)));
sind(t):=block(float(sin(t*%pi/180)));
tand(t):=block(float(tan(t*%pi/180)));
acosd(x):=block(float(180/%pi*acos(x)));
asind(x):=block(float(180/%pi*asin(x)));
atand(x):=block(float(180/%pi*atan(x)));
fpi:float(%pi);
(fpi) 3.141592653589793
```

I put together equations 19,20,8, and 9, together with all those constants defined by the paper and customized trig function that act on angles measured in degrees, into an expression of the form

$$\theta_{2e} = F(\theta_{2e})$$

and applied the built-in Maxima numerical solver `find_root()`. The result agrees reasonably well with the result $\theta_{2e} = 50.806$ quoted in the paper.

```

7 (%i103)  expr:th1+acosd(sind(phi2)*sind(phi1) +
    cosd(phi2)*cosd(phi1)*cosd(l2e-11+omE*((T/2/fpi)*
    (2*atan(sqrt((1-e)/(1+e))*tand(th2e/2))-e*sin(2*atan(sqrt((1-e)/(1+e))*tand(th2e/2))))-tth1)))$

7 (%i104)  find_root(-th2e+expr,th2e,30,60);
(%o104) 51.70462296671066

7 (%i105)  time(%o104);
(%o105) [0.015625]

7 (%i106)  diff(expr,th2e)$

7 (%i107)  subst(th2e=51.7,%);
(%o107) 0.051291416972275

```

In the above, I computed the derivative of the mapping F evaluated at the solution and found that the value is much less than one, showing it is a contraction.

[Here's my full wxMaxima session.](#)

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