

Precise Variable-Q Filter Design

“DSP Tips and Tricks” introduces practical design and implementation signal processing algorithms that you may wish to incorporate into your designs. We welcome readers to submit their contributions to the Associate Editors Rick Lyons (R.Lyons@ieee.org) or Britt Rorabaugh (dspboss@aol.com).

The quality factor Q of a bandpass filter is the ratio of the filter’s center frequency over its bandwidth. As such, the value Q is a measure of the sharpness of a bandpass filter. Variable-Q filters are filters whose Q can easily be controlled, and these filters have applications in instrumentation, adaptive filtering, audio bandpass and notch filtering, and sonar and ultrasonic systems.

In this article, we describe two ways to implement a variable-Q digital filter. First we present a simple variable-Q filter and then illustrate why its frequency response is less than ideal. Next, we show a trick to modify that filter, making its performance nearly ideal. As always, the choice of which filter to use is made based on the user’s application.

A SIMPLE VARIABLE-Q FILTER

The block diagram of a simple variable-Q narrow bandpass filter is given in Figure 1(a) where the feedback path contains a notch filter. The frequency responses of several bandpass filters having various Q values are illustrated in Figure 1(b).

The simplest (real) notch filter with a notch frequency of F_0 Hz is the filter whose transfer function is given by

$$H_N(z) = 1 - 2 \cos(2\pi F_0/F_s)z^{-1} + z^{-2}, \quad (1)$$

where F_s is the input signal sample rate in hertz. This notch filter attenuates signals whose frequencies are in the vicinity of F_0 Hz and allows other frequencies to pass. Using the notch filter in the feedback path, the frequency response of the Figure 1(b) variable-Q bandpass filter can be represented by

$$H_{BP}(f) = \frac{1}{1 + K \cdot H_N(f)}. \quad (2)$$

For large values of K , it is clear that when the frequency magnitude response of the notch filter is near zero, the frequency response of the bandpass filter will be very near one. At frequencies where the notch filter response is not near zero, the frequency response of the bandpass filter should be less than one. The bandpass filter in Figure 1(b) will tend to pass a narrow band of frequencies in the neighborhood of F_0 Hz. The larger K is, the higher the Q and the narrower the filter’s passband will be.

The transfer function of the simple variable-Q bandpass filter in Figure 1(b) is given by

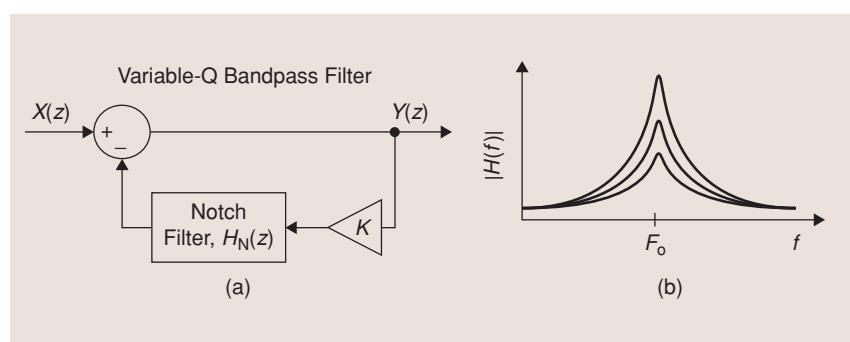
$$H_{BP}(z) = \frac{\alpha}{1 - \alpha K [2 \cos(2\pi F_0/F_s)z^{-1} + \alpha K z^{-2}]}, \quad (3)$$

where $\alpha = 1/(K + 1)$. In implementing the $H_{BP}(z)$ filter, one needs only to store the previous two values of the output. This is a very simple formula, and it can be implemented using a simple microcontroller. The Q of this bandpass filter is proportional to K , and the filter is stable for all positive values of K [1].

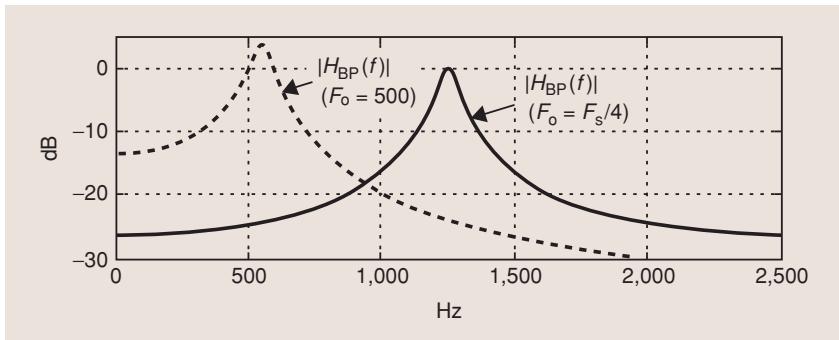
In Figure 2 we plot the frequency response (the solid curve) of the bandpass filter when $F_s = 5,000$ samples per second $K = 10$, and the desired center frequency is $F_0 = F_s/4 = 1.25$ kHz. This filter has an ideal passband gain of unity (0 dB), just as expected.

THE PROBLEM

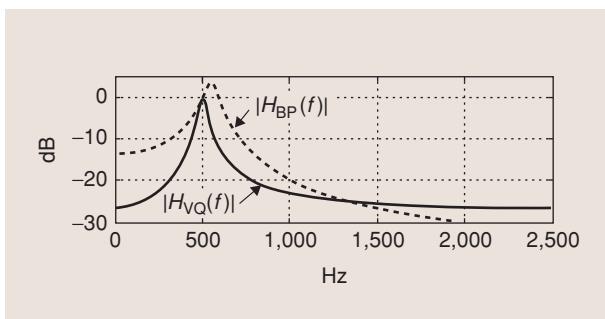
A problem occurs when we set the center frequency of our simple variable-Q filter to a frequency other than $F_s/4$. For example, consider a bandpass filter with the same parameters as earlier save that the desired center frequency is now set to 500 Hz. The magnitude response plot of the filter is shown as the dashed curve



[FIG1] Variable-Q filtering: (a) filter responses for several values of K and (b) block diagram of a simple variable Q-filter.



[FIG2] Magnitude responses of the simple variable-Q filter when $F_o = F_s/4 = 1.25$ kHz (solid line) and when $F_o = 500$ Hz (dashed line).



[FIG3] Magnitude responses of the original ($|H_{BP}(f)|$) and improved ($|H_{VQ}(f)|$) variable-Q filters when $F_o = 500$ Hz.

in Figure 2. There we find a problem: yes, the simple variable-Q bandpass filter's gain is the desired value of unity (0 dB) at 500 Hz, but the filter has a gain of greater than unity (> 0 dB) at a frequency just above 500 Hz. And that gain peak is located, by definition, at the center frequency of the filter. So our simple bandpass filter has a peak gain greater than the desired value of unity, and its center frequency is not the desired 500 Hz.

Interestingly enough, it can be shown that our simple bandpass filter's gain is greater than unity whenever the real part of the notch filter's frequency response is negative, and it is this unpleasant behavior that we wish to correct.

the edge of the passband of a prototype low-pass filter using the transformation, we move the center frequency of our ideal variable-Q bandpass filter from $F_s/4$ to our desired center frequency F_D . The standard low-pass to low-pass transformation requires that the variable z in the transfer function of the prototype filter be replaced by the function

$$\frac{z - a}{1 - az}, \quad (4)$$

where the value of a is given in (5), shown at the bottom of the page. This transformation stretches the response of the prototype filter but does not change its magnitude, thus eliminating the

undesirable peaking seen in the dashed curve of Figure 2.

Automating the calculation of (5) is simple and takes only a few lines of code. Our final variable-Q filter's transfer function has the form shown in (6) (shown at the bottom of the page), where $\beta = 1 + K$, and a satisfies (5).

Implementing this filter requires storing the preceding two values of the filter's output and the preceding two values of the filter's input. We are trading the simplicity of our first design for improved accuracy. Repeating our last example, with $F_s = 5,000$ samples/s, $K = 10$, $F_{old} = F_s/4 = 1.25$ kHz, and the desired center frequency is $F_{new} = 500$ Hz, we find that $a = 0.2065$. Using this design, we find that the frequency response of the improved filter, shown as the solid $|H_{VQ}(f)|$ curve in Figure 3, is nearly ideal. (For ease of comparison, we have included the simple bandpass filter's magnitude response in Figure 3 as the dashed curve.)

Using the improved bandpass filter design method, defined by (6), we have

- exact control over the filter's center frequency
- forced the filter's passband gain to always be unity (0 dB)
- improved the filter's frequency response symmetry.

As with all recursive filters implemented using a fixed-point number format, having finite-width coefficient values, the precise locations of the variable-Q filter's poles should be examined to ensure that they reside within the z -domain's unit circle.

CONCLUSIONS

We have shown how to analyze and design simple variable-Q bandpass filters. The simplest variable-Q filter suffers from two shortcomings. Its center frequency is not exactly the planned-for center frequency, and the amplitude at the center frequency is not exactly the planned-for amplitude of unity. We presented a slightly more complicated filter in (6) that meets all the variable-Q filter's desired specifications quite precisely.

(continued on page 119)

$$a = \frac{\tan(\pi/4 - \pi F_D/F_s)}{\sin(2\pi F_D/F_s) + \cos(2\pi F_D/F_s) \cdot \tan(\pi/4 - \pi F_D/F_s)} \quad (5)$$

$$H_{VQ}(z) = \frac{1 - 2az^{-1} + a^2z^{-2}}{(\beta + Ka^2) - 2a(\beta + K)z^{-1} + (a^2\beta + K)z^{-2}} \quad (6)$$

PRODUCTS

There are currently 21 registered PHYs in WiMedia and 19 platforms have achieved WiMedia certification. This means that there exists a base of products over which WUSB and Bluetooth 3.0 products can be built. The first commercially available WUSB products are starting to be deployed in the market. More information about the products is listed in "WiMedia Hardware and Software Products."

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WIMEDIA RESOURCES

Standards

- ECMA – 368, "High Rate Ultra Wideband PHY and MAC Standard," 1st ed., 2005 [Online]. Available: <http://www.ecma-international.org/publications/standards/Ecma-368.htm>
- ECMA – 369, "MAC PHY Interface for ECMA – 368, 1st Edition, 2005 [Online]. Available: <http://www.ecma-international.org/publications/standards/Ecma-369.htm>

Also adopted by ISO/IEC in 2007:

- ISO/IEC 26907:2007—Information technology—Telecommunications and information exchange between systems—High Rate Ultra Wideband PHY and MAC standard.
- ISO/IEC 26908:2007—Information technology—MAC-PHY Interface for ISO/IEC 26907.

Tutorials

- UWB University (www.uwb-u.com). The UWB learning series launched by Staccato Communications is a tool for learning about the technical and marketing aspects of the technology.
- G.R. Hertz, Y. Zang, J. Habetha, and H. Sirin, "Multiband OFDM alliance—The next generation of wireless personal area networks," in *Proc. IEEE Symp. Advances in Wired and Wireless Communication*, pp. 208–214, Apr. 2005.
- R.S. Sherratt, "Design issues toward a cost effective physical layer for multiband OFDM (ECMA-368) in consumer products," *IEEE Trans. Consumer Electron.*, vol. 52, no. 4, pp. 1179–1183, Nov. 2006.

Overviews

- J. Balakrishnan, A. Batra, A. Dabak, R. Aiello, and J. Forester, "Design of multiband OFDM system for realistic UWB channel environments," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 9, pp. 2123–2138, 2004.
- S. Shetty and R. Aiello, "Detect and avoid (DAA) techniques—Enabler for worldwide ultra wideband regulations," in *Ultra Wideband Systems: Technologies and Applications*. Amsterdam, The Netherlands: Elsevier, 2006.

Books

- R. Aiello and A. Batra, *Ultra Wideband Systems: Technologies and Applications*. Amsterdam, The Netherlands: Elsevier, May 2006.

Resources for Further Developments

- WiMedia forum white papers. [Online]. Available: <http://www.wimedia.org/en/resources/index.asp?id=res>
- Wireless USB technical white papers. [Online]. Available: <http://www.usb.org/developers/wusb/docs/whitepapers/>

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REFERENCES

- [1] S. Engelberg, *Digital Signal Processing: An Experimental Approach*. London: Springer-Verlag, 2008, pp. 151–153, 168–170.
- [2] J. Proakis and D. Manolakis, *Digital Signal Processing—Principles, Algorithms, and Applications*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1996, pp. 698–700.
- [3] A. Oppenheim and R. Schafer, *Discrete-Time Signal Processing*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1989, pp. 430–435.

