



TRAITEMENT DU SIGNAL FILTRAGE FRÉQUENTIEL

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FILTRAGE FRÉQUENTIEL

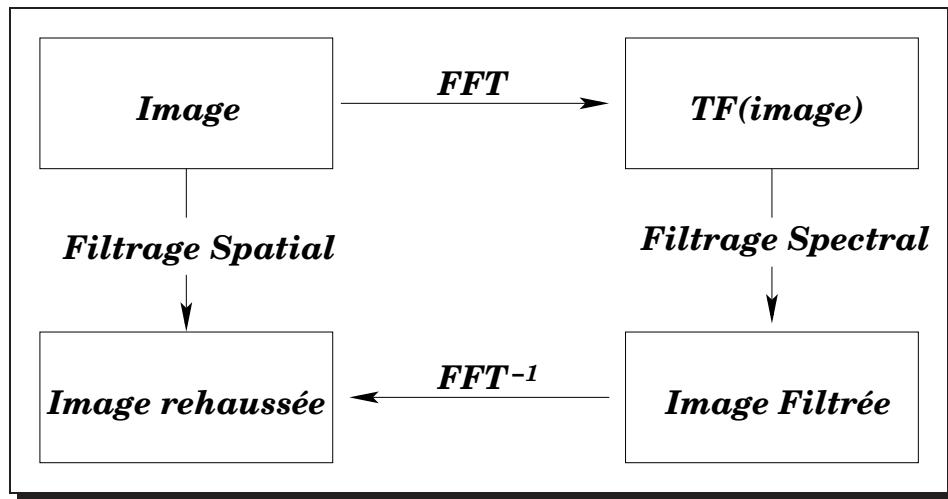
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FILTRAGE FRÉQUENTIEL

INTRODUCTION

Principe du Filtrage Spatial/Fréquentiel



Théorème de Convolution -Rappel-

$$\begin{aligned} f(x, y) * g(x, y) &\stackrel{\mathcal{F}}{=} F(u, \nu) \cdot G(u, \nu) \\ f(x, y) \cdot g(x, y) &\stackrel{\mathcal{F}}{=} F(u, \nu) * G(u, \nu) \end{aligned}$$

donc, si $f(x, y)$ est l'image à filtrer et $G(u, \nu)$, le filtre fréquentiel

$$f(x, y) * g(x, y) = \mathcal{F}^{-1}\{F(u, \nu) \cdot G(u, \nu)\}$$

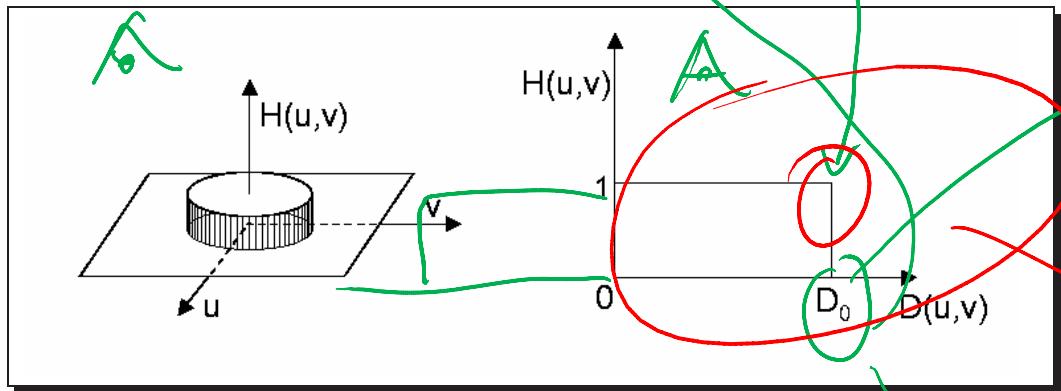
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (1)

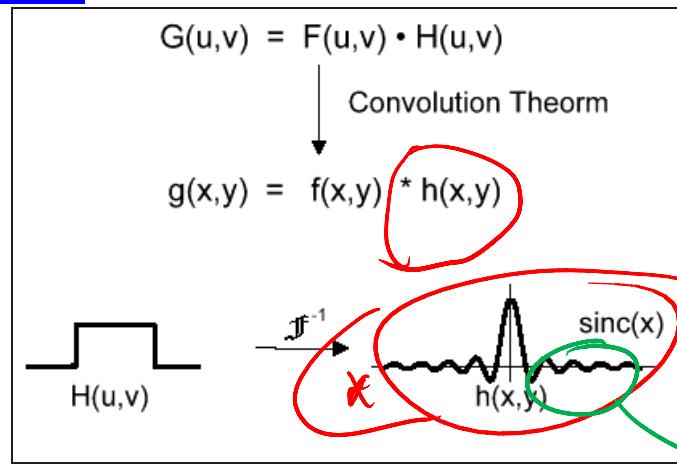
$$H(u, \nu) = \begin{cases} 1 & D(u, \nu) \leq D_0 \\ 0 & D(u, \nu) > D_0 \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupe



Problème

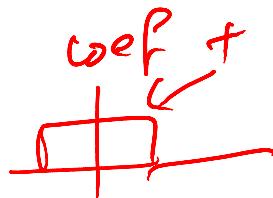


$\uparrow D_0 \rightarrow \downarrow$ Rayons des ondulations (-flou)

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3 Rayon

PSF



→ passe bas

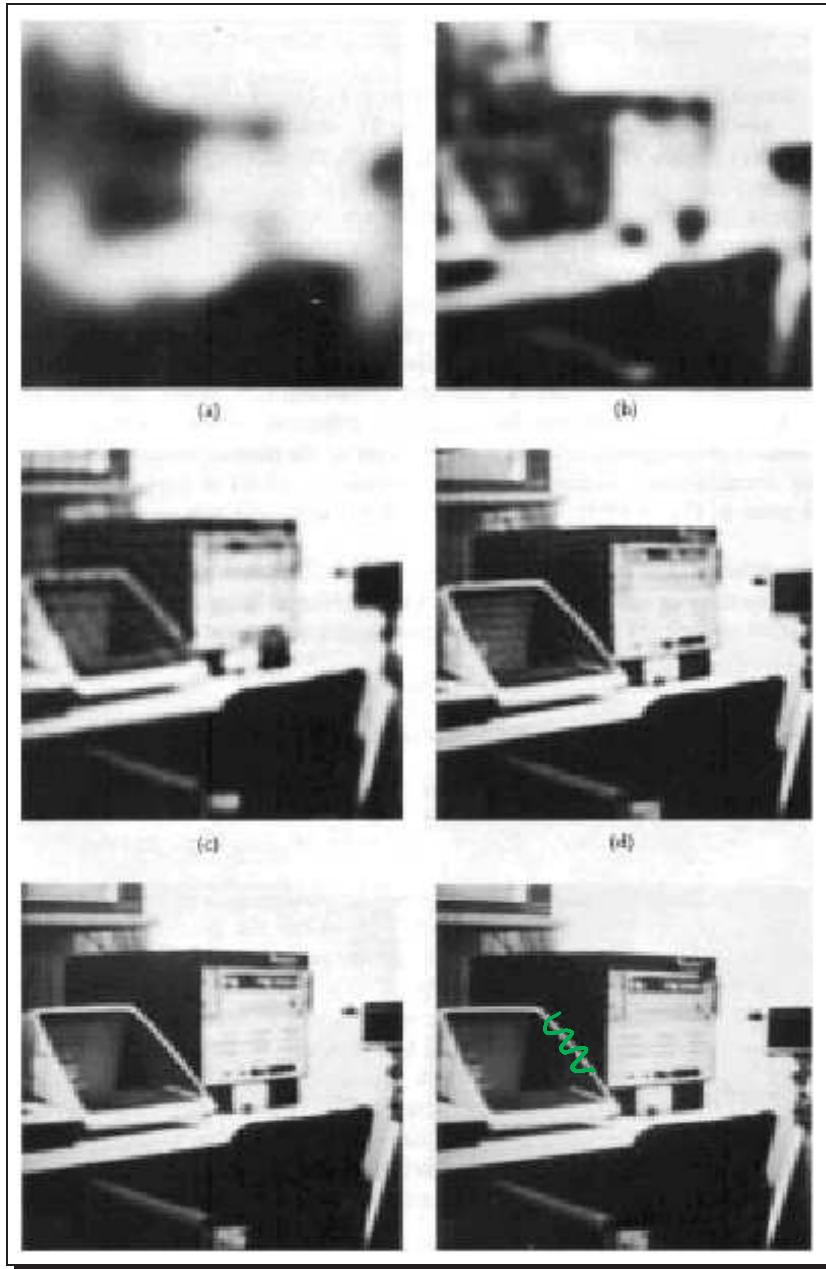


← passe h-

Ring. 2

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (2)



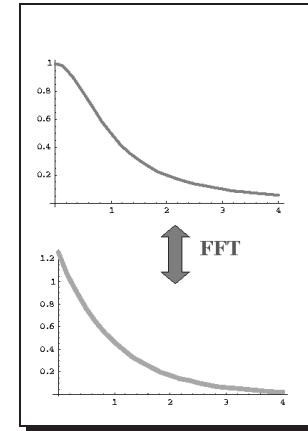
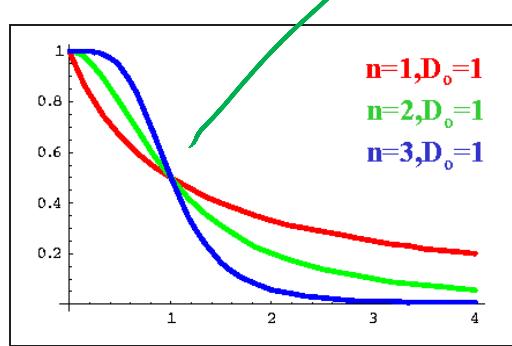
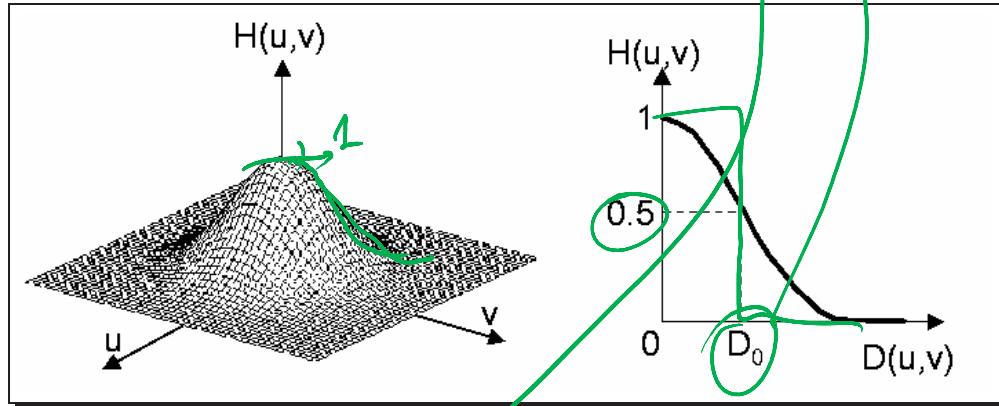
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS DE BUTTERWORTH (1)

$$H(u, \nu) = \frac{1}{1 + (D(u, \nu)/D_0)^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Couverture

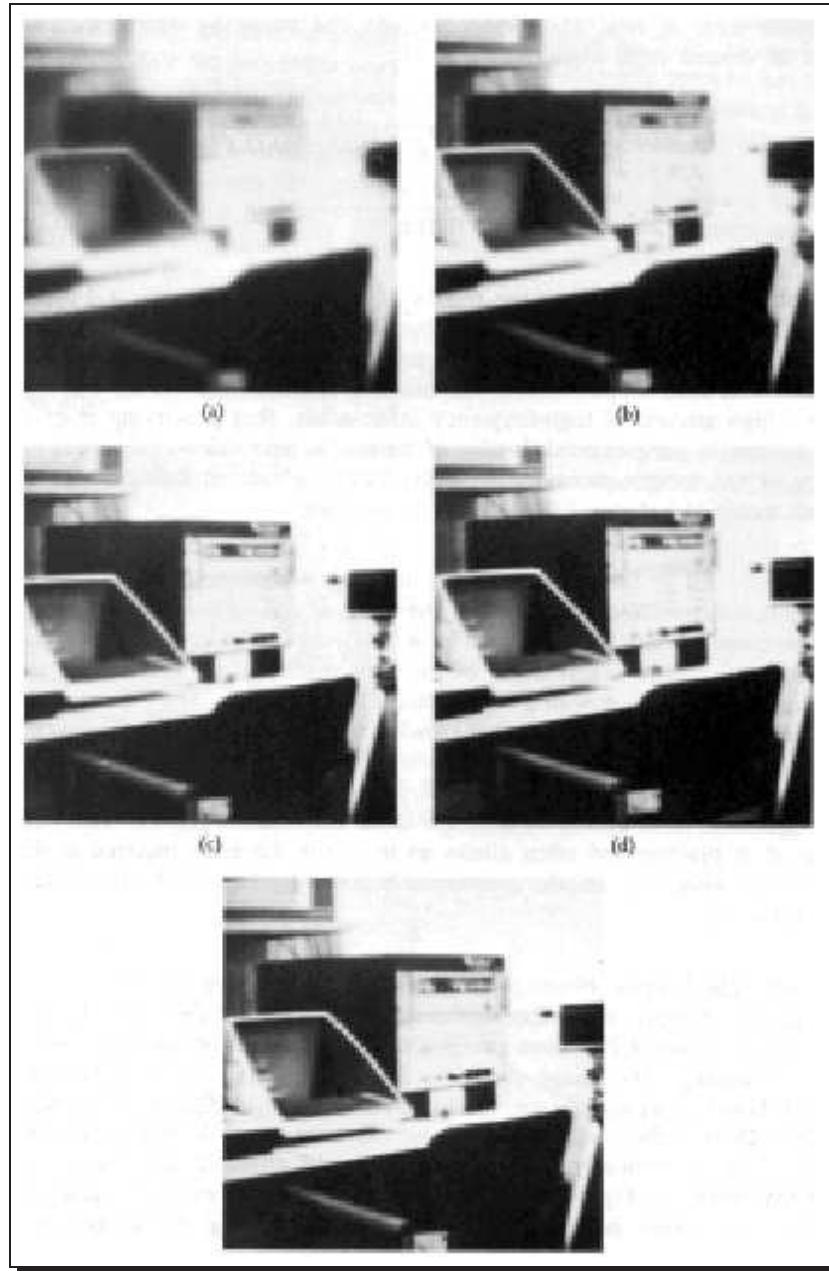


Note

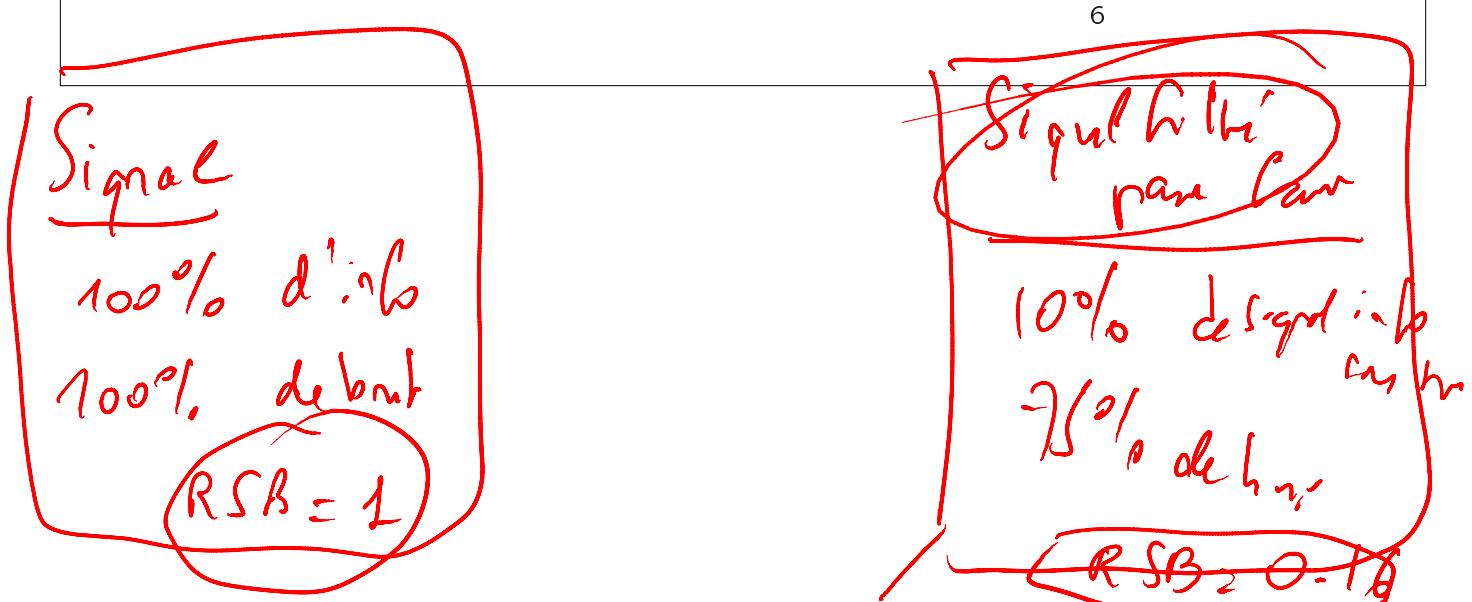
- Flou moins brutal et aucune ondulation -

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS DE BUTTERWORTH (2)



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FILTRAGE FREQUENTIEL

FILTRE PASSE-HAUT IDÉAL

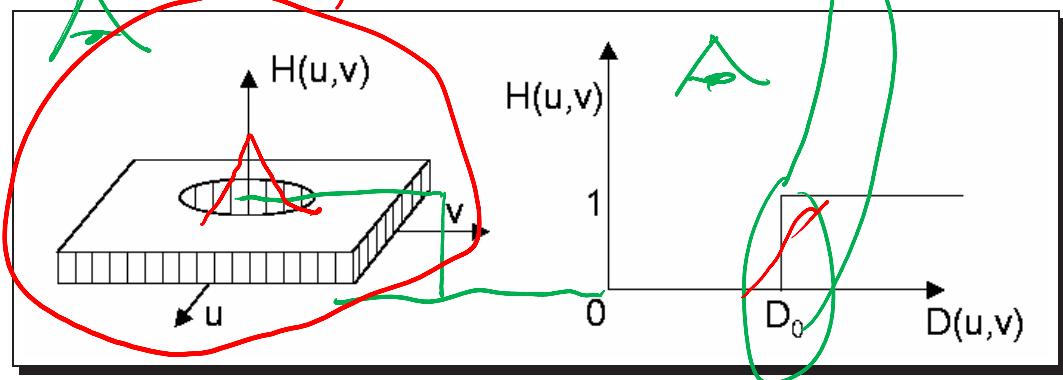
$$H(u, v) = \begin{cases} 1 & D(u, v) \geq D_0 \\ 0 & D(u, v) < D_0 \end{cases}$$

$$D(u, v) = \sqrt{u^2 + v^2}$$

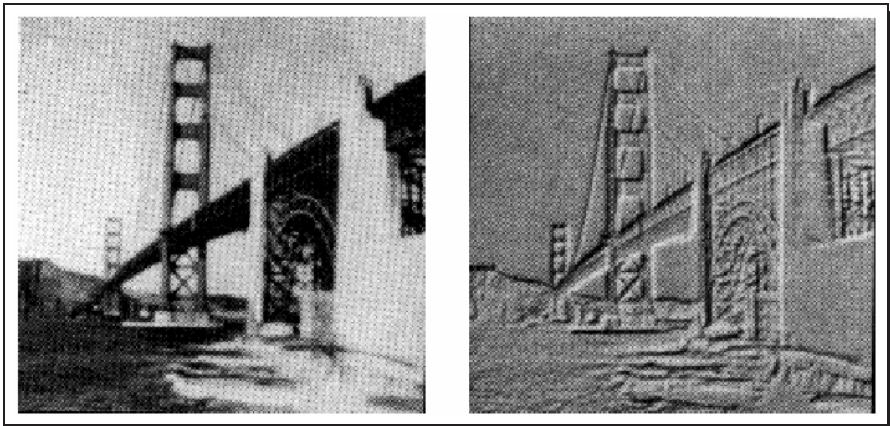
D_0 : Fréquence de Coupe

debut
N° 2

$|F(u)|$



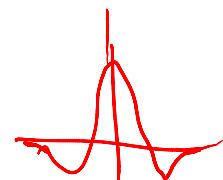
Exemple



$i(x)$

* $\boxed{-1 \ 1}$

* $\boxed{-1 \ 2 \ 1}$



7
image filtre
par huit
+ erreurs
- 128

ϵ_0

↑

Recal(0 255)

$$\frac{(02 \pm 2)}{1 \pm 4}$$

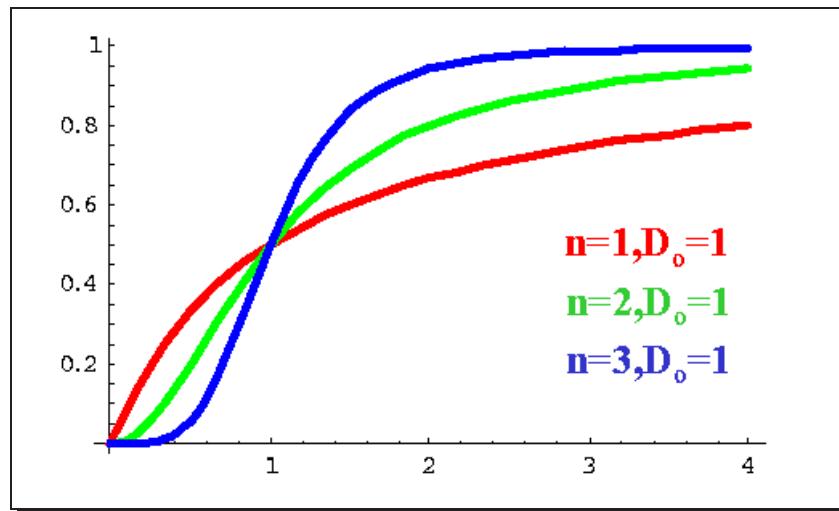
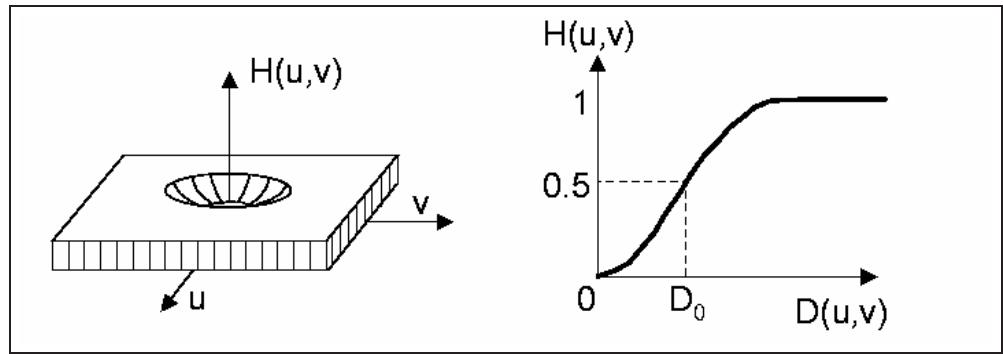
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-HAUT DE BUTTERWORTH

$$H(u, \nu) = \frac{1}{1 + (D_0/D(u, \nu))^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

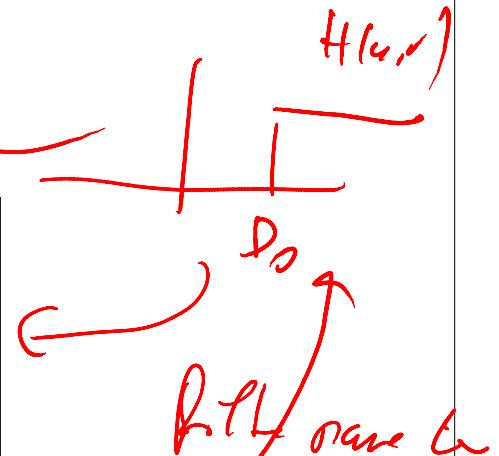
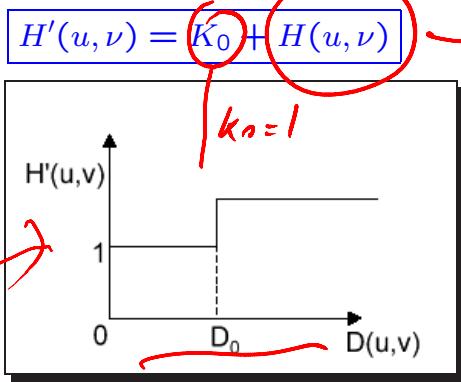
D_0 : Fréquence de Coupure



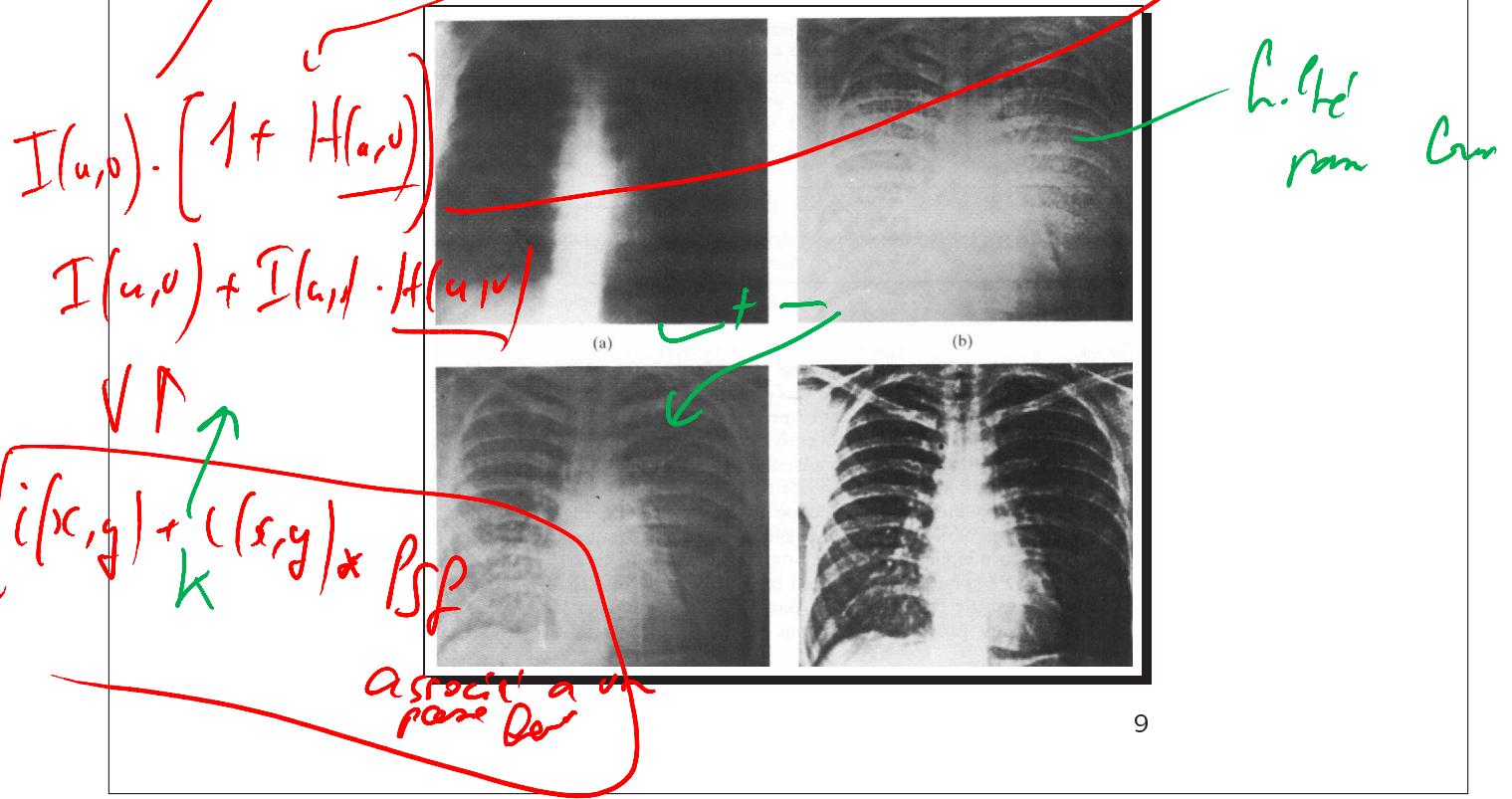
FILTRAGE FRÉQUENTIEL

REHAUSSEMENT DES HAUTES FRÉQUENCES

- Maintient la moyenne et les BF
- Amplifie les HF



Ex. : Filtre PH Butterworth + Rehaussement HF + Égalisation



FILTRAGE FRÉQUENTIEL

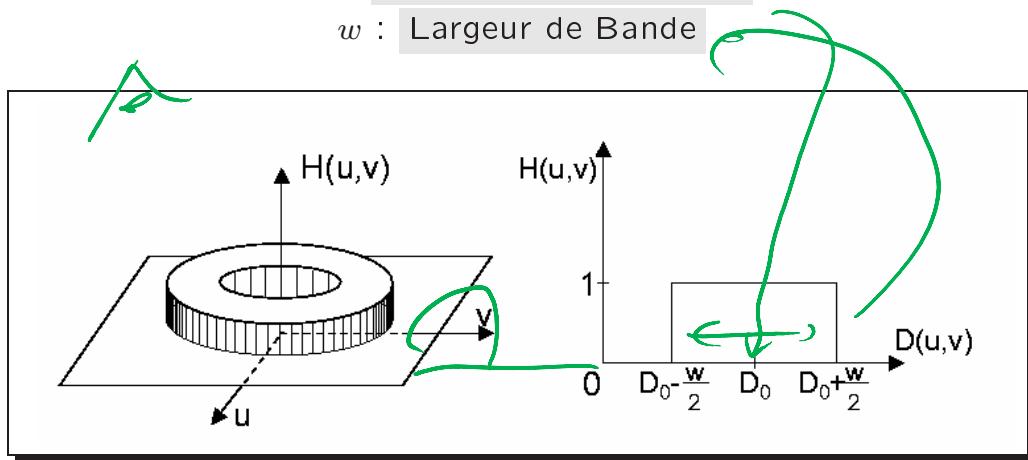
FILTRE PASSE-BANDE

$$H(u, \nu) = \begin{cases} 0 & D(u, \nu) \leq D_0 - \frac{w}{2} \\ 1 & D_0 - \frac{w}{2} < D(u, \nu) < D_0 + \frac{w}{2} \\ 0 & D(u, \nu) \geq D_0 + \frac{w}{2} \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

w : Largeur de Bande



FILTRAGE FRÉQUENTIEL

FILTRE SPECTRAL LOCAL

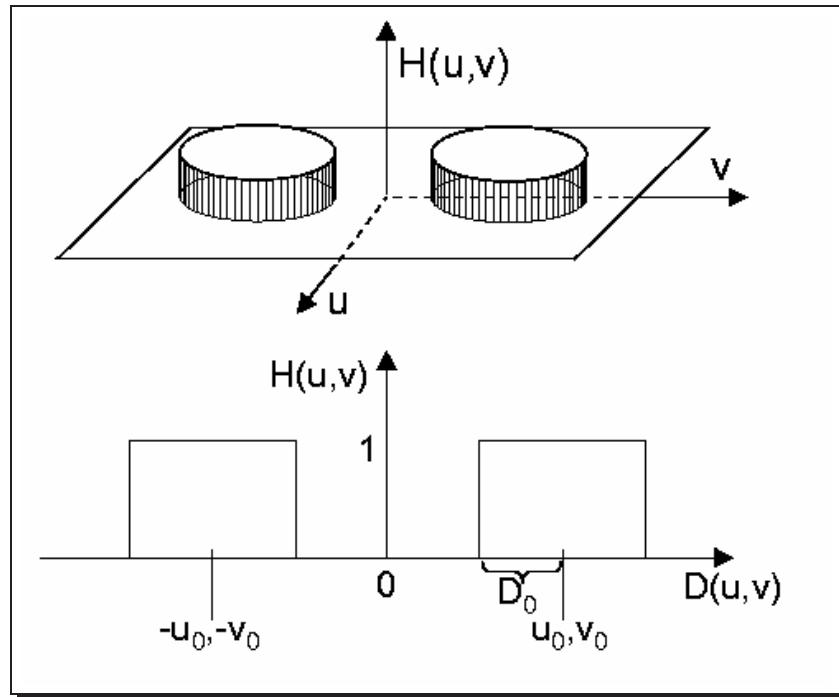
$$H(u, \nu) = \begin{cases} 1 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 0 & \text{sinon} \end{cases}$$

$$D_1(u, \nu) = \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2}$$

$$D_2(u, \nu) = \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2}$$

D_0 : Rayon autour de la fréquence locale

u_0, ν_0 : Coordonnée de la fréquence locale



FILTRAGE FRÉQUENTIEL

FILTRE A REJECTION DE BANDE (1)

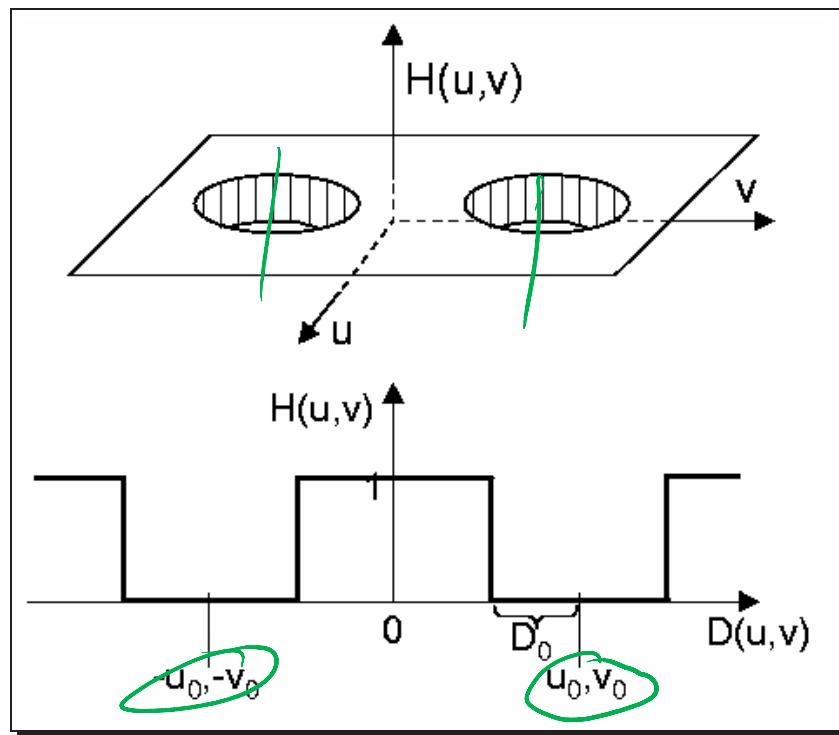
$$H(u, \nu) = \begin{cases} 0 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 1 & \text{sinon} \end{cases}$$

$$D_1(u, \nu) = \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2}$$

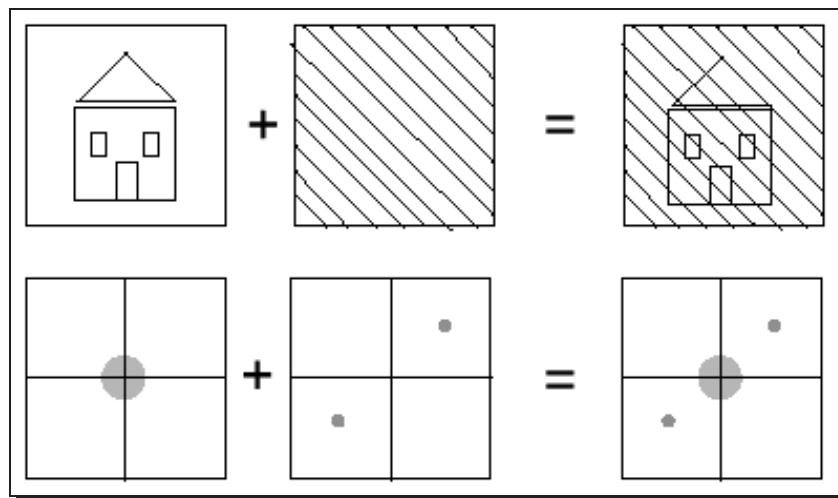
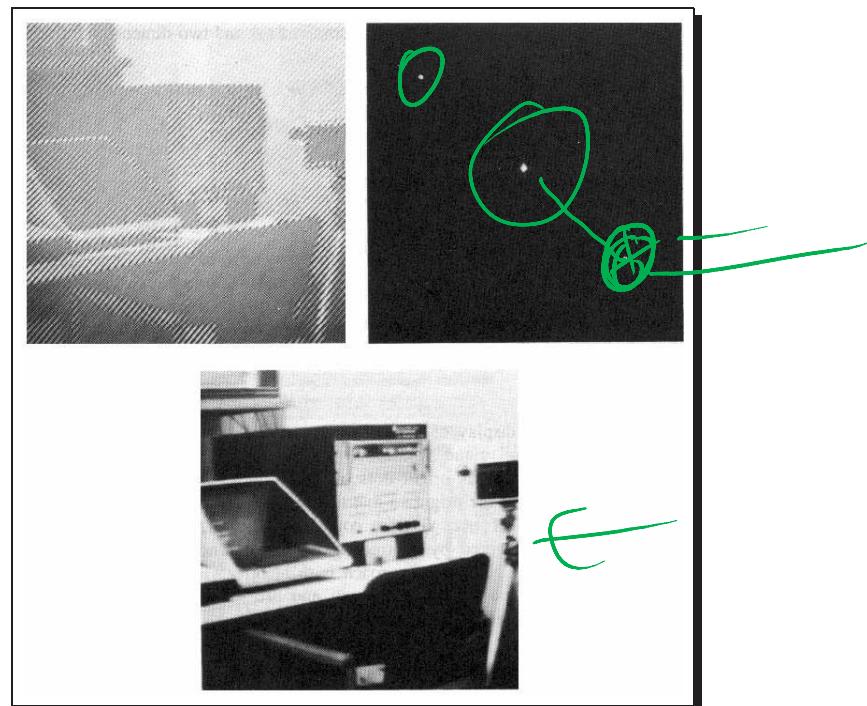
$$D_2(u, \nu) = \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2}$$

D_0 : Rayon autour de la fréquence locale

u_0, ν_0 : Coordonnée de la fréquence locale



FILTRAGE FRÉQUENTIEL FILTRE A REJECTION DE BANDE (2)



FILTRAGE FRÉQUENTIEL

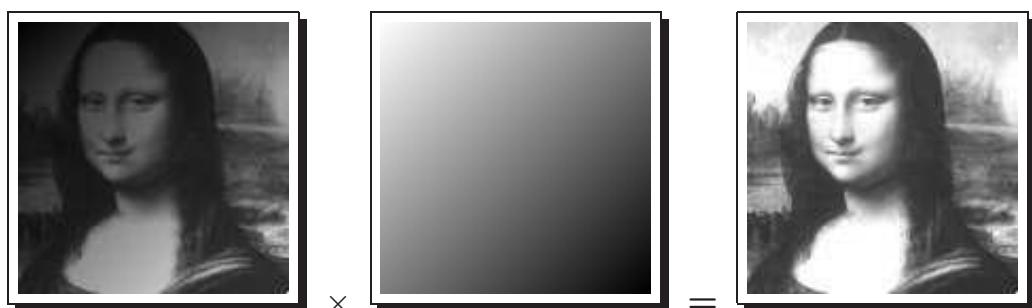
FILTRAGE HOMOMORPHIQUE (1)



$$f(x, y) = i(x, y) \cdot r(x, y)$$

- $i(x, y)$: Illumination ► très basses fréquences
- $r(x, y)$: Réflectance ► plus hautes fréquences

► Opération Ponctuelle



► Filtrage Homomorphique

FILTRAGE FRÉQUENTIEL

FILTRAGE HOMOMORPHIQUE (2)

$$\begin{aligned}
 z(x,y) &= \log(f(x,y)) \\
 &= \log(i(x,y) \cdot (r(x,y))) = \log(i(x,y)) + \log(r(x,y))
 \end{aligned}$$



$$Z(u,v) = I(u,v) + R(u,v)$$

Filtre Passe-haut $H(u,v)$:

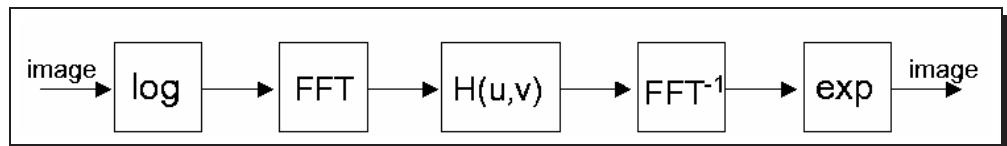
$$S(u,v) = H(u,v) \cdot Z(u,v) = H(u,v) \cdot I(u,v) + H(u,v) \cdot R(u,v)$$



$$s(x,y) = i'(x,y) + r'(x,y)$$

$$\begin{aligned}
 g(x,y) &= \exp(s(x,y)) = \exp(i'(x,y)) \cdot \exp(r'(x,y)) \\
 &= i_0(x,y) \cdot r_0(x,y)
 \end{aligned}$$

Diagramme Bloc



FILTRAGE FRÉQUENTIEL
FILTRAGE HOMOMORPHIQUE (3)

