# Intelligence: a matter of (re)copying ? 

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## Intelligence versus G

- no global agreement
- no clear definition
- G-factor, (G)eneral intelligence, (G)eneral mental ability, (G)..., etc.
- due to Charles Spearman

Nevertheless...

- numerous attempts to measure intelligence or $G$ for an individual
- IQ tests
- large varieties of tests


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Choose your answer:


D

## IQ tests examples

Vocabulary based:

1) Which same three-letter word can be placed in front of the following words to make a new word?

## SIGN, DONE, DUCT, FOUND, FIRM, TRACT, DENSE

2) $A B C A B D I I J J K K$ ?

Numerical tests:

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Numerical tests:

| $126 / 9=3^{2}+5$ | true $\circ$ false |
| :--- | :--- |
| $111^{3}=1367631$ | true $\circ$ false |
| $3619 / 235=14,4$ | true false |
| $43+27-13+7+5-21+4-9=33$ | true $\quad$ false |
| $3888 / 360=10,8$ | true $\quad$ false |

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| Which number fits instead of a question mark? | Choose your answer: |  |  |
| :---: | :---: | :---: | :---: |
|  | 451 | 125 | 154 |
| 52257777 ? 451605 | 104 | 54 | 87 |

## From IQ tests to Raven matrices




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"Exotic tests"

- www.test-my-iq.co.uk
- Brad Pitt 126
- First: Hong Kong (110) Last: Zambia (60)


## From IQ tests to Raven matrices

Graphic-based tests:
If you disassemble the block, which piece would be missing?

"Exotic tests"

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Lessons learned ...

- avoid cultural bias $\rightarrow$ no vocabulary-based or numerical tests
- avoid color $\rightarrow$ black and white tests
- something is missing !


## Raven matrices

General pattern to be followed:

- a sequence to be completed
- the solution has to be chosen among a set of candidates
- John Raven (1936)
- 3 batteries of visual tests: standard - colored - advanced
- previous rules adopted
- each test $=$ matrice of 8 pictures to be completed with a 9th one
- the solution to be chosen among a set of 8 candidate pictures
- increasing difficulty (Raven's Progressive Matrices)


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Example of Raven test (modified)


## To complete a sequence ?

- how ? LOGICALLY ;-)
- abc abd ijk ?
- ijk ? no way !
- ijl ... better
the main idea:


## ANALOGY MAKING!

- build up an analogical proportion starting from 3 items $a, b, c$
- pb to be solved: "find the 4th element $d$ such that $a: b:: c: d$
- ability to build up/discover analogy: essential skill of human intelligence!


## Analogical proportions: Boolean view

- 4 items $a, b, c, d$
- informally: dissimilarity of $(a, b)=$ dissimilarity of $(c, d)$.
- Boolean view: dissimilarity indicators: $a \wedge \bar{b}$ and $\bar{a} \wedge b$

$$
a: b:: c: d
$$

iff

$$
a \wedge \bar{b}=c \wedge \bar{d} \text { and } \bar{a} \wedge b=\bar{a} \wedge b
$$

- Standard properties :
(1) basic patterns

$$
a: b:: a: b \text { and } a: a: b: b
$$

(2) symmetry $a: b:: c: d \rightarrow c: d:: a: b$
(3) central permutation $a: b:: c: d \rightarrow a: c:: b: d$
(9) transitivity $a: b:: c: d$ and $c: d:: e: f \rightarrow a: b:: e: f$
(5) equivalent to: $a \wedge d=b \wedge c$ and $a \vee d=b \vee c$
(0) and a lot more...

## Analogical proportion: other patterns

- Truth table ( 6 lines (among $2^{4}=16$ ) lead to truth value 1 )

$$
\begin{array}{llll}
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 \\
\mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\
\mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0}
\end{array}
$$

- Equation solving: we have 3 , find the 4 th one (if you can !)
- 110? $\rightarrow 0, \quad 010 ? \rightarrow \mathbf{1}, \quad 100 ? \rightarrow$ no solution
- Boolean vector extension:

$$
\vec{a}: \vec{b}:: \vec{c}: \vec{d} \text { iff } \forall i \in[1, n], a_{i}: b_{i}: c_{i}: d_{i}
$$

- Functional extension : $a: f(a):: b: f(b)$


## Raven test: a Boolean analysis

Raven matrix: 8 known pictures - 1 unknown picture pic $[3,3]$

| $\operatorname{pic}[1,1]$ | $\operatorname{pic}[1,2]$ | $\operatorname{pic}[1,3]$ |
| :---: | :---: | :---: |
| $\operatorname{pic}[2,1]$ | $\operatorname{pic}[2,2]$ | $\operatorname{pic}[2,3]$ |
| $\operatorname{pic}[3,1]$ | $\operatorname{pic}[3,2]$ | $\operatorname{pic}[1,3]$ |

(1) Working hypothesis 1: rows/columns = samples of (unknown) functional dependencies

- $\forall i \in[1,2], \exists f$ tel que $p i c[i, 3]=f($ pic $[i, 1]$, pic $[i, 2])$ (row)
- $\forall j \in[1,2], \exists g$ tel que $p i c[3, j]=g(p i c[1, j]$, pic $[2, j])$ (column)
(2) Working hypothesis 2 : extended analogical proportion to solve (a,b):f(a,b)::(c,d):f(c,d) for row
(a,b):g(a,b)::(c,d):g(c,d) for column
(pic[1, 1], pic $[1,2]): \operatorname{pic}[1,3]::(\operatorname{pic}[2,1], \operatorname{pic}[2,2]): \operatorname{pic}[2,3])$
(pic[2, 1], $\operatorname{pic}[2,2]): \operatorname{pic}[2,3])::(\operatorname{pic}[3,1], \operatorname{pic}[3,2]): \operatorname{pic}[3,3])$


## A complete example

(1) Boolean coding

Raven test


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## Features



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Raven test
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|  |  | Coding |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\otimes \square$ | - Black dot | 1111 | 0011 | 1100 |
| $\otimes \nabla \triangle$ | - STriangle | 0110 | 0010 | 0100 |
| - $\bigcirc$ | - RTriangle | 1001 | 0001 | ???? |
|  | Circle |  |  |  |

(2) Equation solving process solution 1st feature $\rightarrow$

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## Algorithm...

- Immediate translation of the previous ideas
- Equation solving process
- With suitable Boolean encoding 32 / 36 automatically solved
- Solution build up from scratch ...

Drawbacks:

- We fail to solve 4 tests ... Why ?


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- Manual encoding !


## Iphone encoding?

- Can we do it automatically? Yes we can ...
- How? Using a mobile phone ;-) i.e. a camera
- Picture $=$ matrix of pixel $=$ BMP file
- Matrix of pixel = Boolean encoding! (very, ... very low level...)
- Algorithm still valid: input: 8 BMP files - output: the 9th BMP file !
- 16 tests solved (among the 32 previously solved)!
- Constraints: same size - B\&W - no compression


## Concluding remarks

- Simple, homogeneous and robust process to solve 32 / 36 Raven tests (except 4)
- Mainly (re)-copying previously seen solutions (Polya/CBR?)
- Constructive method
- Mix with SMT... and CogSketch to get automatic encoding ?
- Tested in classification problems, gives surprisingly good results !

Thank you !

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