DIRO
IFT 6150

TRAITEMENT D'IMAGES

FILTRAGE FRÉQUENTIEL

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FILTRAGE FRÉQUENTIEL

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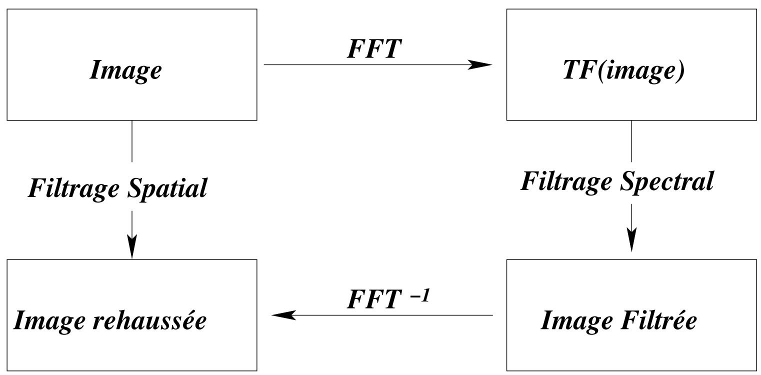
FILTRAGE FRÉQUENTIEL
INTRODUCTION

Rehaussement d'Images par Filtrage Spatial/Fréquentiel



Théorème de Convolution -Rappel-

$$f(x,y) * g(x,y) \quad \blacktriangleleft\!\blacktriangleright \quad F(u,\nu)\ .\ G(u,\nu)$$
$$f(x,y)\ .\ g(x,y) \quad \blacktriangleleft\!\blacktriangleright \quad F(u,\nu) * G(u,\nu)$$

donc, si $f(x,y)$ est l'image à filtrer et $G(u,\nu)$, le filtre fréquentiel

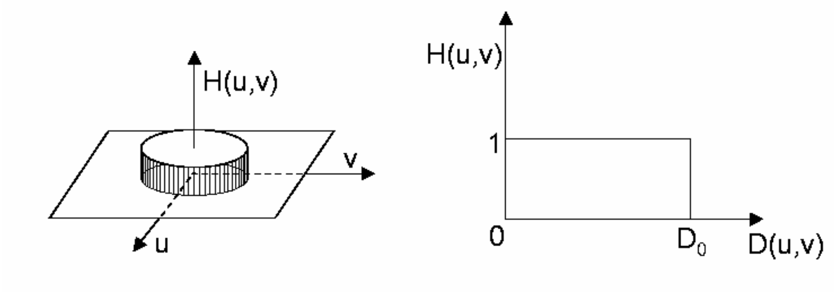
$$\boxed{f(x,y) * g(x,y) = \mathcal{F}^{-1}\Big\{F(u,\nu)\cdot G(u,\nu)\Big\}}$$

FILTRAGE FRÉQUENTIEL
FILTRE PASSE-BAS IDÉAL (1)

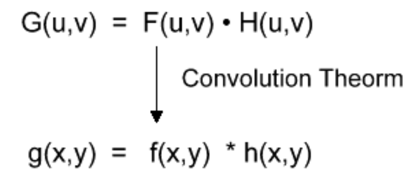
$$H(u,\nu) = \begin{cases} 1 & D(u,\nu) \le D_0 \\ 0 & D(u,\nu) > D_0 \end{cases}$$

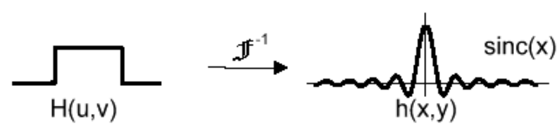
$$D(u,\nu) = \sqrt{u^2+\nu^2}$$

D_0 : Fréquence de Coupure



Problème

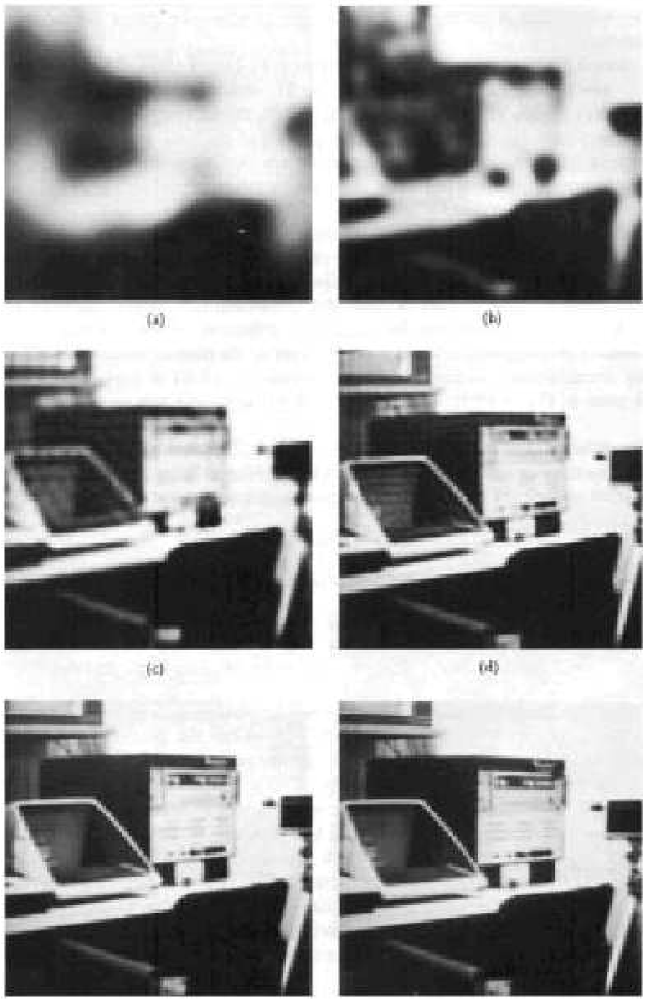




$\uparrow D_0 \longrightarrow \downarrow$Rayons des ondulations (-flou)

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (2)



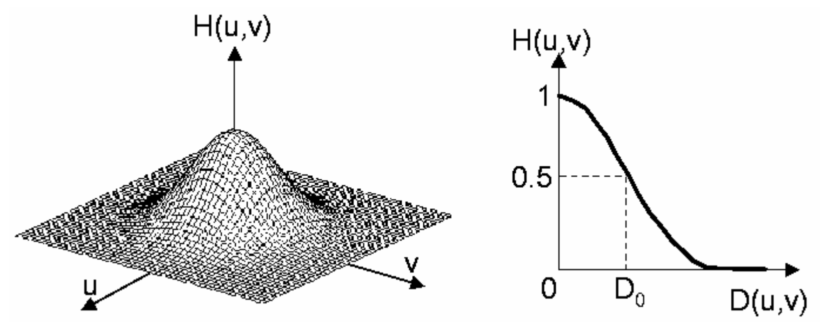
FILTRAGE FRÉQUENTIEL

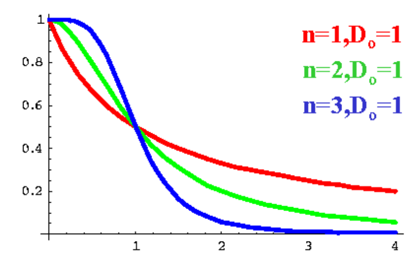
FILTRE PASSE-BAS DE BUTTERWORTH (1)

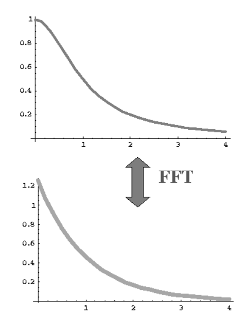
$$H(u,\nu) = \frac{1}{1+\left(D(u,\nu)/D_0\right)^{2n}}$$

$$D(u,\nu) = \sqrt{u^2+\nu^2}$$

D_0 : Fréquence de Coupure







Note

- Flou moins brutal et aucune ondulation -

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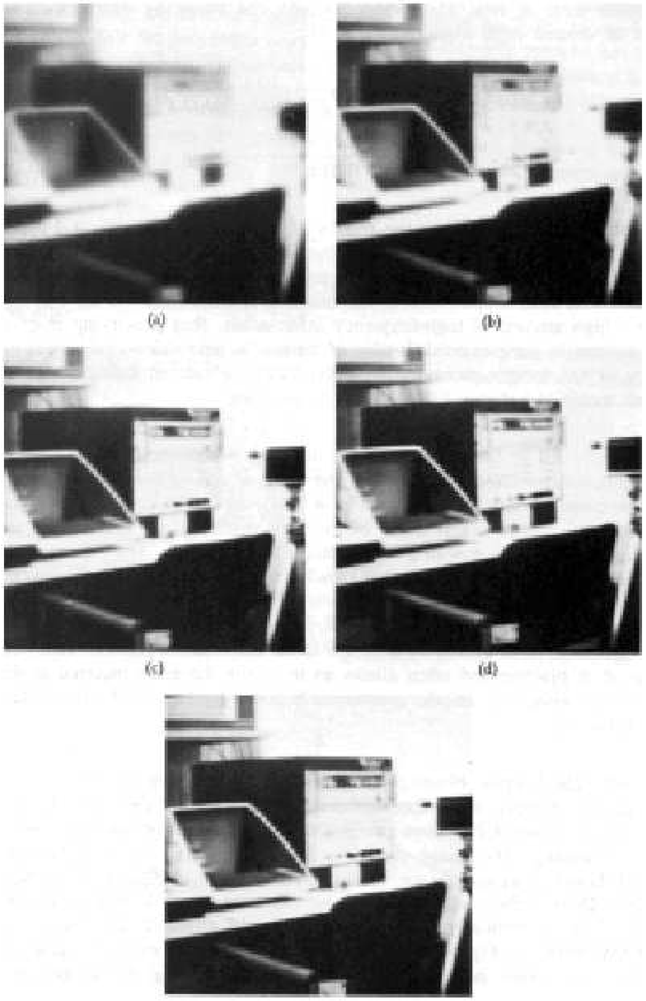
FILTRE PASSE-BAS DE BUTTERWORTH (2)



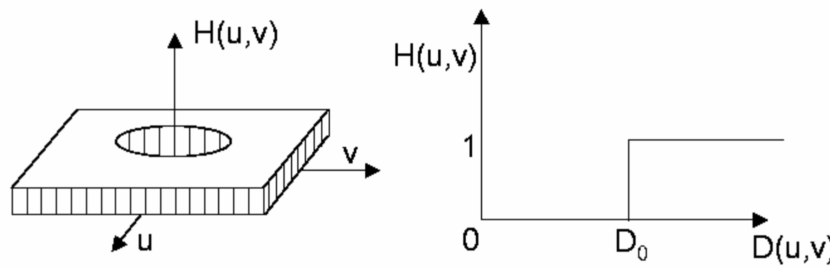
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-HAUT IDÉAL

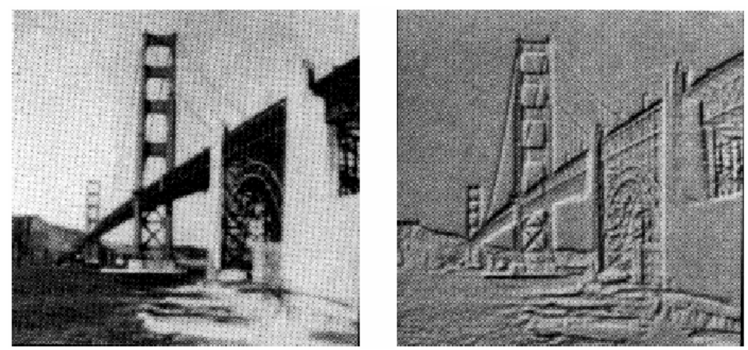
$$H(u,\nu) = \begin{cases} 1 & D(u,\nu) \geq D_0 \\ 0 & D(u,\nu) < D_0 \end{cases}$$

$$D(u,\nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure



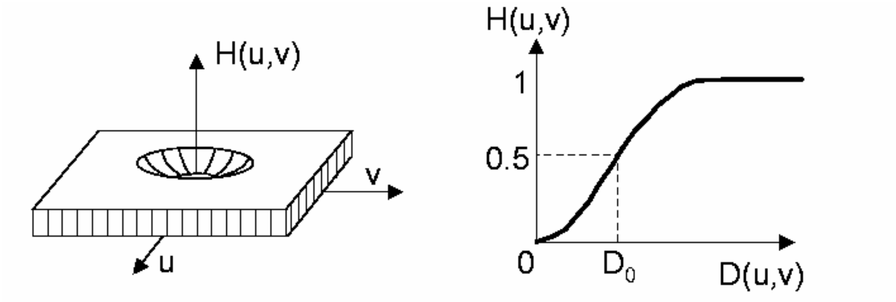
Exemple

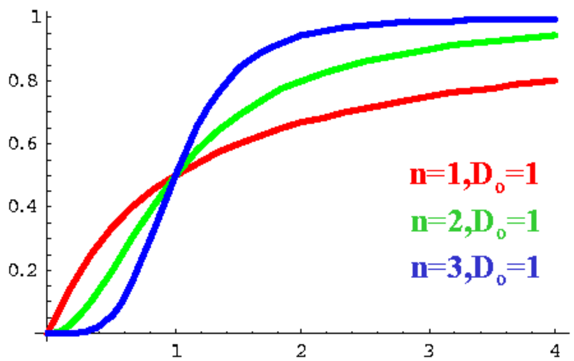


FILTRAGE FRÉQUENTIEL
FILTRE PASSE-HAUT DE BUTTERWORTH

$$H(u,\nu) = \frac{1}{1+\left(D_0/D(u,\nu)\right)^{2n}}$$
$$D(u,\nu) = \sqrt{u^2+\nu^2}$$

D_0 : Fréquence de Coupure

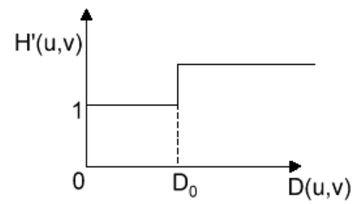




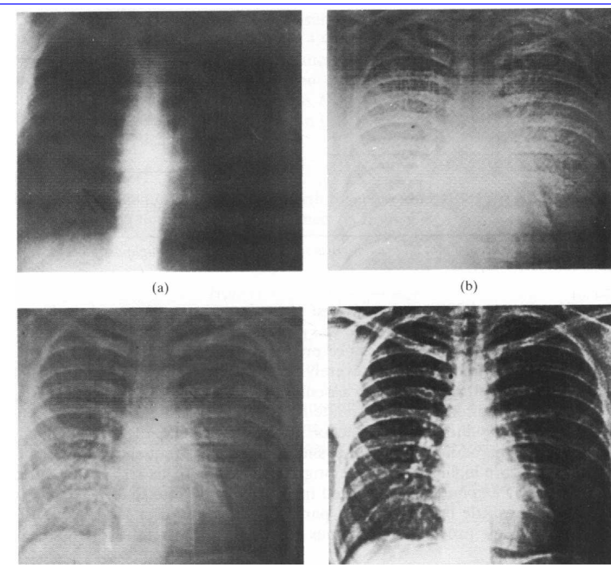
FILTRAGE FRÉQUENTIEL
REHAUSSEMENT DES HAUTES FRÉQUENCES

- Maintient la moyenne et les BF
- Amplifie les HF

$$\boxed{H'(u,\nu) = K_0 + H(u,\nu)}$$



Exemple : Filtre PH Butterworth+Rehaussement HF+Égalisation



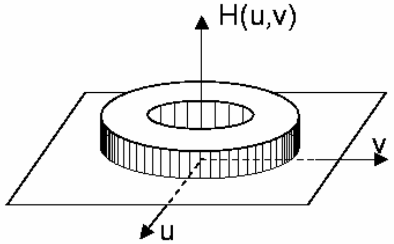
FILTRAGE FRÉQUENTIEL

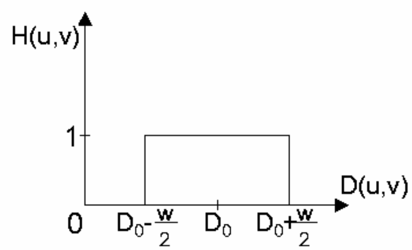
FILTRE PASSE-BANDE

$$H(u,\nu) = \begin{cases} 0 & D(u,\nu) \le D_0 - \frac{w}{2} \\ 1 & D_0 - \frac{w}{2} < D(u,\nu) < D_0 + \frac{w}{2} \\ 0 & D(u,\nu) \ge D_0 + \frac{w}{2} \end{cases}$$

$$D(u,\nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure
w : Largeur de Bande





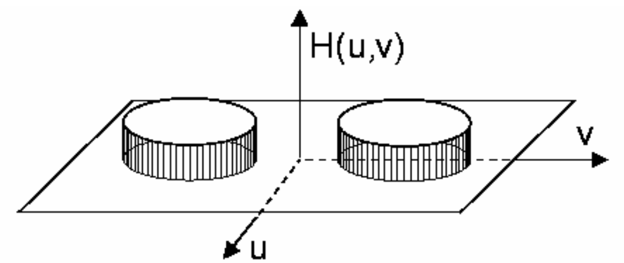
FILTRAGE FRÉQUENTIEL

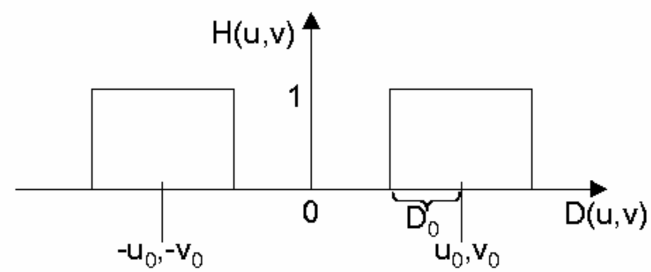
FILTRE SPECTRAL LOCAL

$$H(u,\nu) = \begin{cases} 1 & D_1(u,\nu) \le D_0 \text{ ou } D_2(u,\nu) \le D_0 \\ 0 & \text{sinon} \end{cases}$$

$$\begin{aligned} D_1(u,\nu) &= \sqrt{(u-u_0)^2 + (\nu-\nu_0)^2} \\ D_2(u,\nu) &= \sqrt{(u+u_0)^2 + (\nu+\nu_0)^2} \end{aligned}$$

D_0 : Rayon autour de la fréquence locale
u_0, ν_0 : Coordonné de la fréquence locale



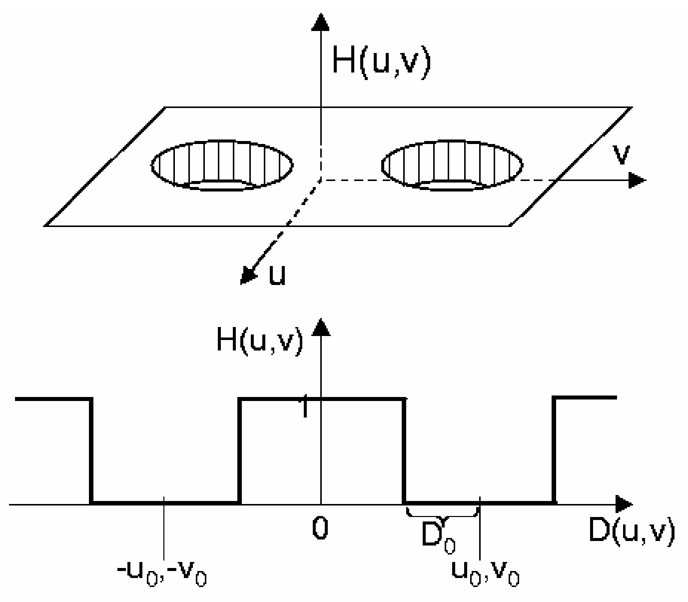


FILTRAGE FRÉQUENTIEL
FILTRE A REJECTION DE BANDE (1)

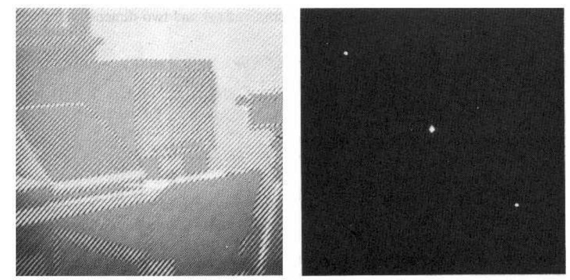
$$H(u,\nu) = \begin{cases} 0 & D_1(u,\nu) \leq D_0 \text{ ou } D_2(u,\nu) \leq D_0 \\ 1 & \text{sinon} \end{cases}$$

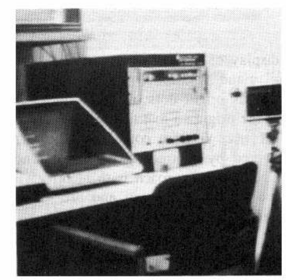
$$\begin{aligned} D_1(u,\nu) &= \sqrt{(u-u_0)^2 + (\nu-\nu_0)^2} \\ D_2(u,\nu) &= \sqrt{(u+u_0)^2 + (\nu+\nu_0)^2} \end{aligned}$$

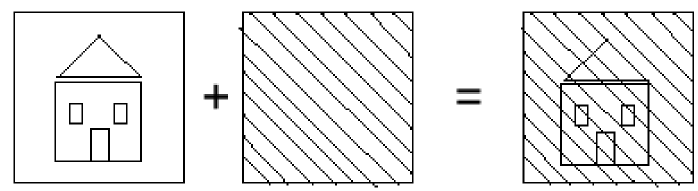
D_0 : Rayon autour de la fréquence locale
u_0, ν_0 : Coordonné de la fréquence locale

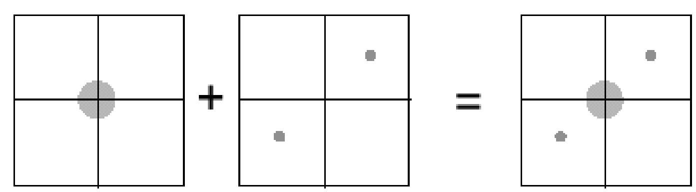


FILTRAGE FRÉQUENTIEL
FILTRE A REJECTION DE BANDE (2)







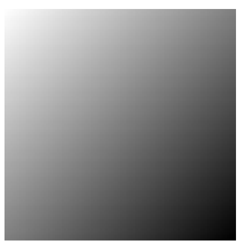
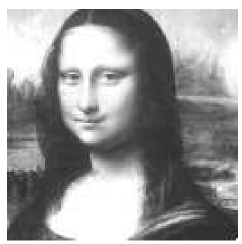


FILTRAGE FRÉQUENTIEL
FILTRAGE HOMOMORPHIQUE (1)

$$f(x,y) = i(x,y) \cdot r(x,y)$$

- $i(x,y)$: Illumination ▶ très basses fréquences
- $r(x,y)$: Réflectance ▶ plus hautes fréquences

▶ Opération Ponctuelle

 ×  = 

▶ Filtrage Homomorphique

FILTRAGE FRÉQUENTIEL
FILTRAGE HOMOMORPHIQUE (2)

$$\boxed{\begin{aligned} z(x,y) &= \mathbf{log}(f(x,y)) \\ &= \mathbf{log}(i(x,y) \cdot (r(x,y)) = \mathbf{log}(i(x,y)) + \mathbf{log}(r(x,y)) \end{aligned}}$$

$$\Big\Downarrow \mathfrak{F}$$

$$Z(u,v) = I(u,v) + R(u,v)$$

Filtre Passe-haut H(u,v):

$$S(u,v) = H(u,v) \cdot Z(u,v) = H(u,v) \cdot I(u,v) + H(u,v) \cdot R(u,v)$$

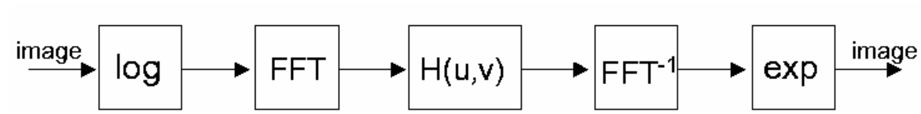
$$\Big\Downarrow \mathfrak{F}^{-1}$$

$$s(x,y) = i'(x,y) + r'(x,y)$$

$$g(x,y) = \mathbf{exp}(s(x,y)) = \mathbf{exp}(i'(x,y)) \cdot \mathbf{exp}(r'(x,y))$$

$$= i_0(x,y) \cdot r_0(x,y)$$

Diagramme Bloc



FILTRAGE FRÉQUENTIEL
FILTRAGE HOMOMORPHIQUE (3)