



DIRO
IFT 6150

TRAITEMENT D'IMAGES

FILTRAGE FRÉQUENTIEL

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FILTRAGE FRÉQUENTIEL

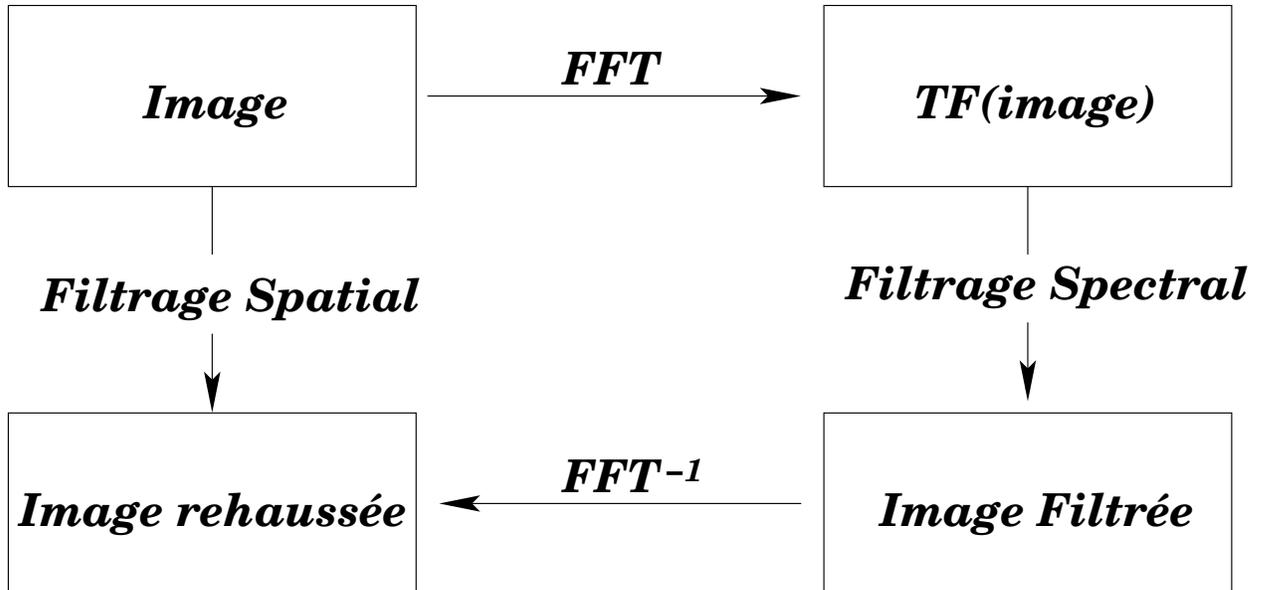
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FILTRAGE FRÉQUENTIEL

INTRODUCTION

Rehaussement d'Images par Filtrage Spatial/Fréquentiel



Théorème de Convolution -Rappel-

$$\begin{aligned} f(x, y) * g(x, y) &\leftrightarrow F(u, \nu) \cdot G(u, \nu) \\ f(x, y) \cdot g(x, y) &\leftrightarrow F(u, \nu) * G(u, \nu) \end{aligned}$$

donc, si $f(x, y)$ est l'image à filtrer et $G(u, \nu)$, le filtre fréquentiel

$$f(x, y) * g(x, y) = \mathcal{F}^{-1} \{ F(u, \nu) \cdot G(u, \nu) \}$$

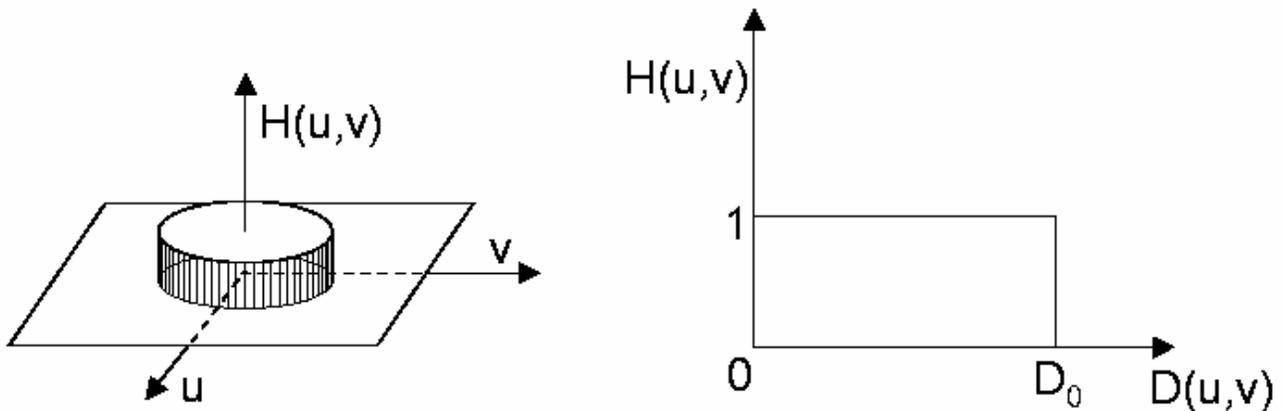
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (1)

$$H(u, \nu) = \begin{cases} 1 & D(u, \nu) \leq D_0 \\ 0 & D(u, \nu) > D_0 \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

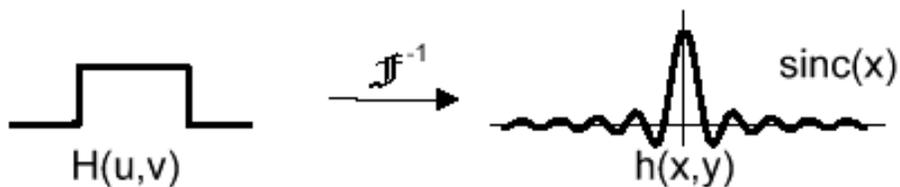


Problème

$$G(u, \nu) = F(u, \nu) \cdot H(u, \nu)$$

Convolution Theorem

$$g(x, y) = f(x, y) * h(x, y)$$



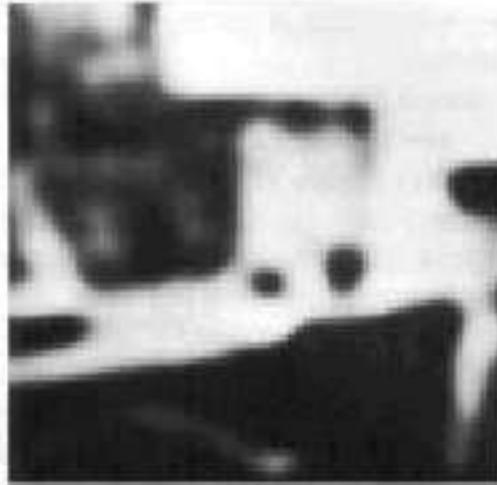
$\uparrow D_0 \rightarrow \downarrow$ Rayons des ondulations (-flou)

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FILTRE PASSE-BAS IDÉAL (2)



(a)



(b)



(c)



(d)



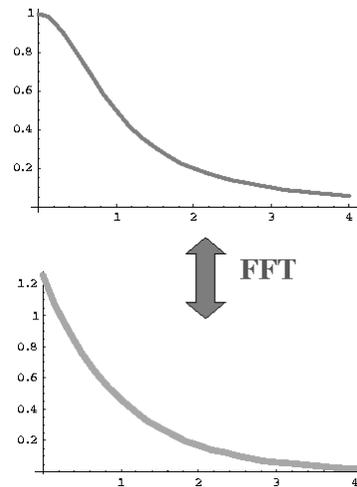
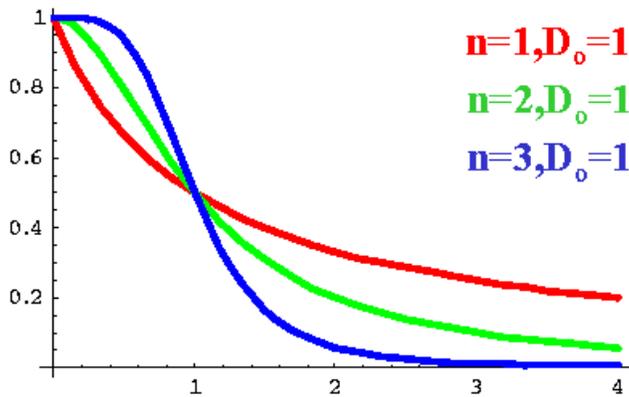
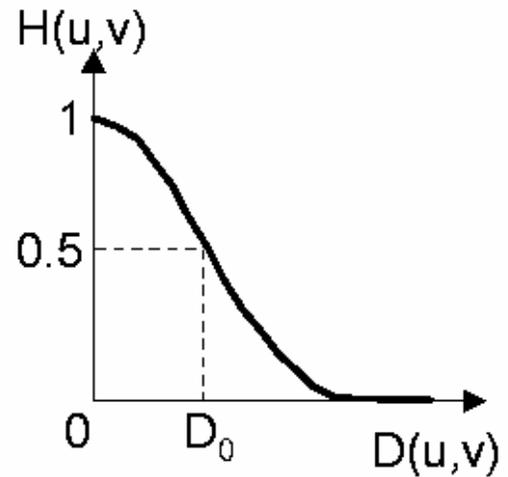
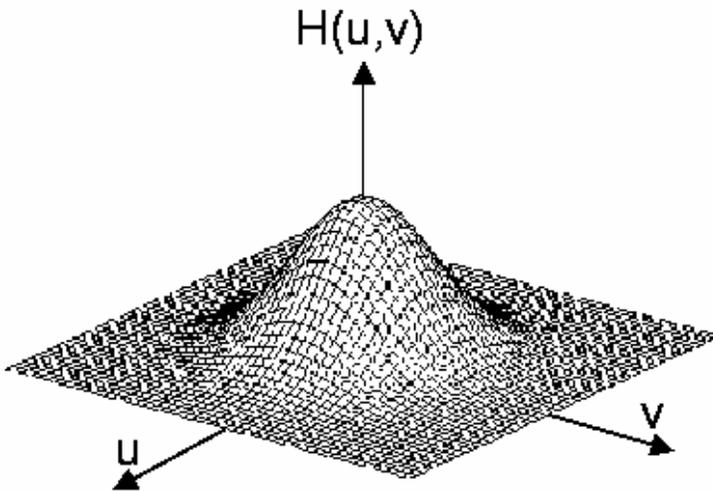
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS DE BUTTERWORTH (1)

$$H(u, \nu) = \frac{1}{1 + (D(u, \nu)/D_0)^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure



Note

- Flou moins brutal et aucune ondulation -

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS DE BUTTERWORTH (2)



(a)



(b)



(c)



(d)



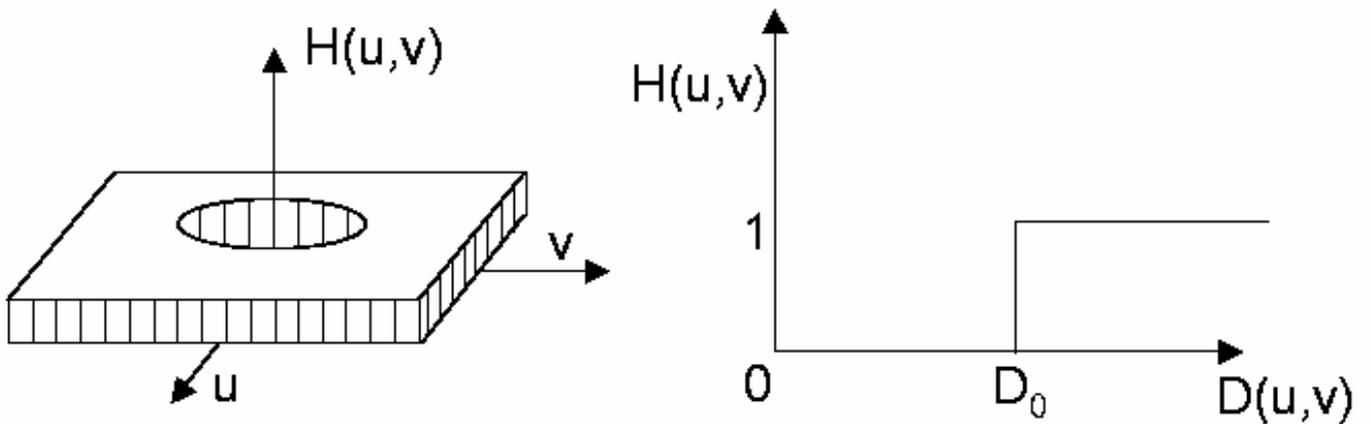
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-HAUT IDÉAL

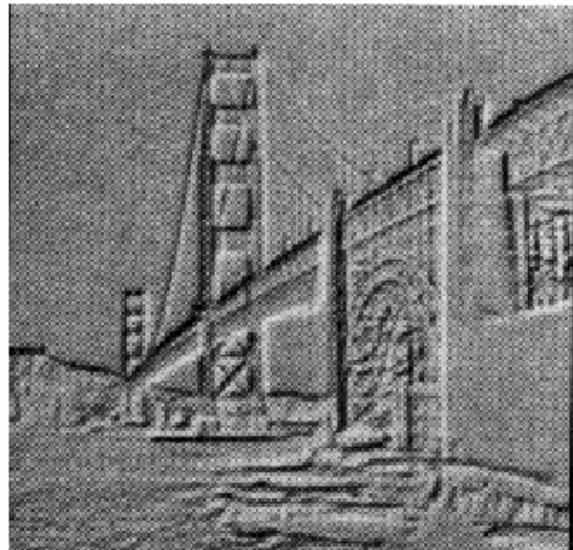
$$H(u, \nu) = \begin{cases} 1 & D(u, \nu) \geq D_0 \\ 0 & D(u, \nu) < D_0 \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure



Exemple



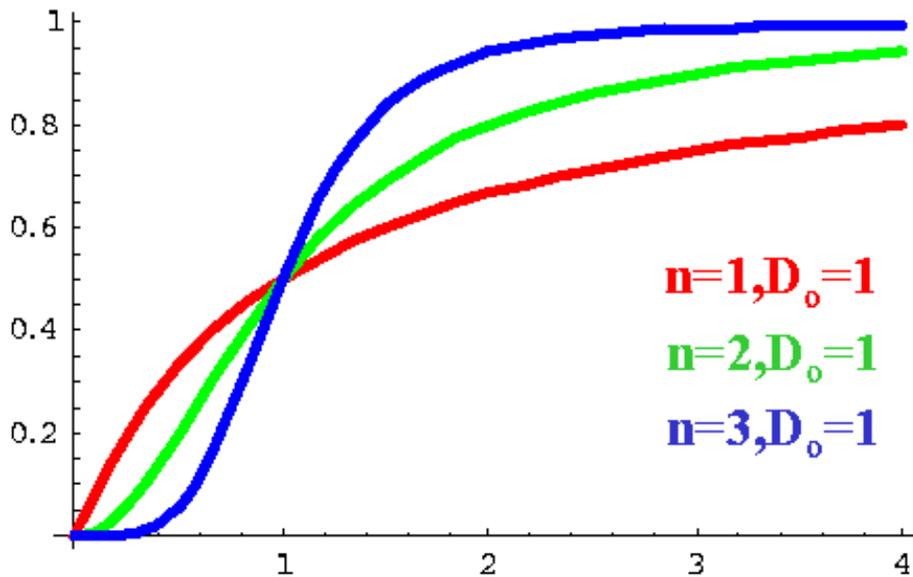
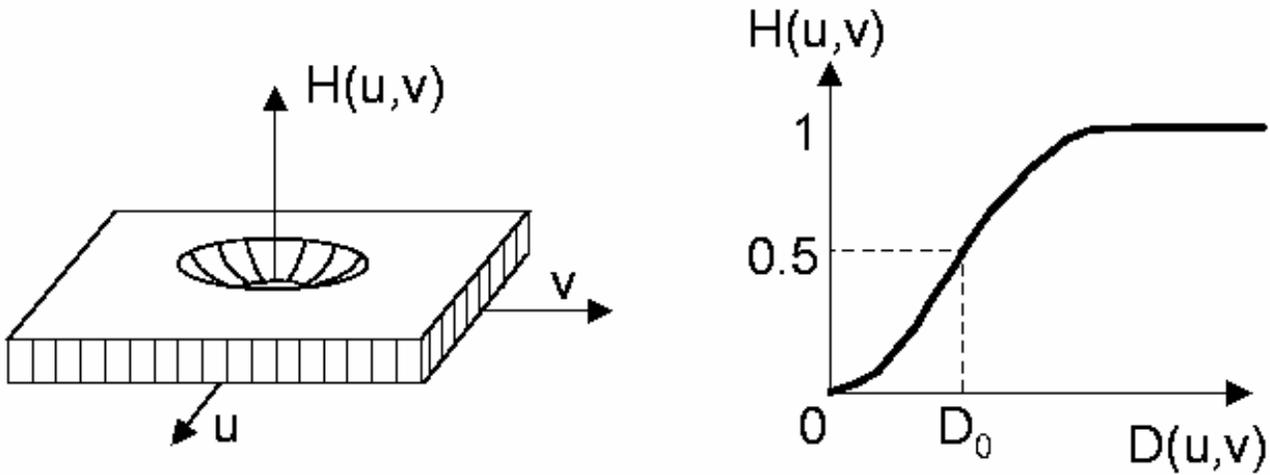
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-HAUT DE BUTTERWORTH

$$H(u, \nu) = \frac{1}{1 + (D_0/D(u, \nu))^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

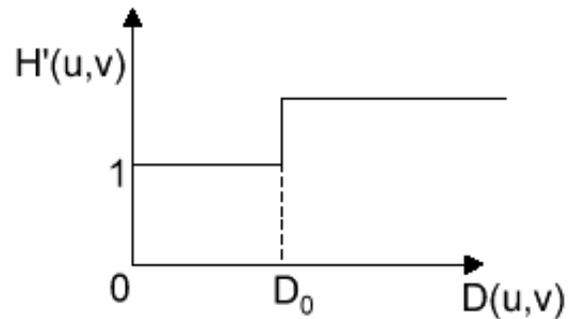


FILTRAGE FRÉQUENTIEL

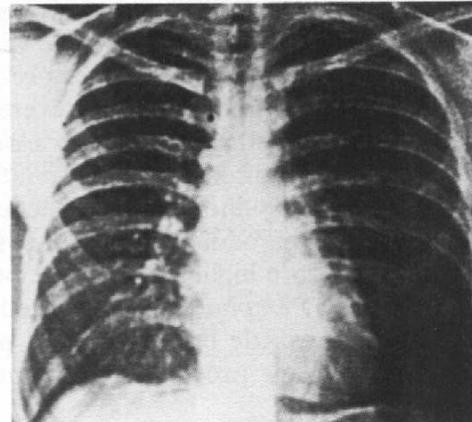
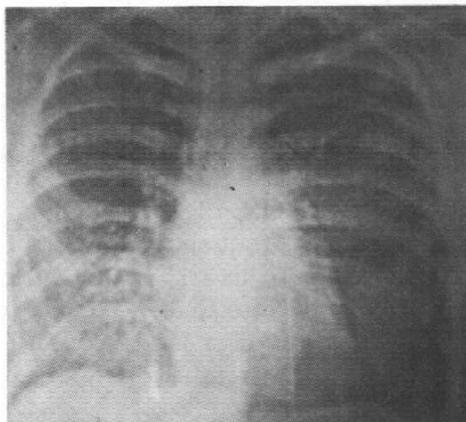
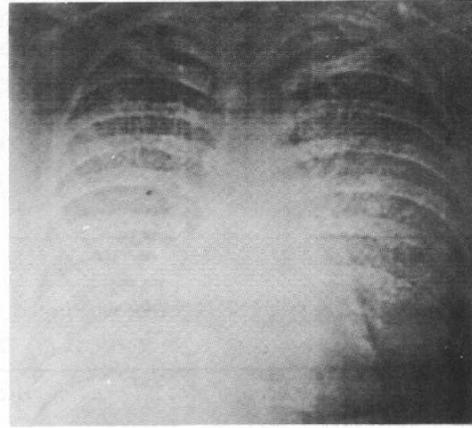
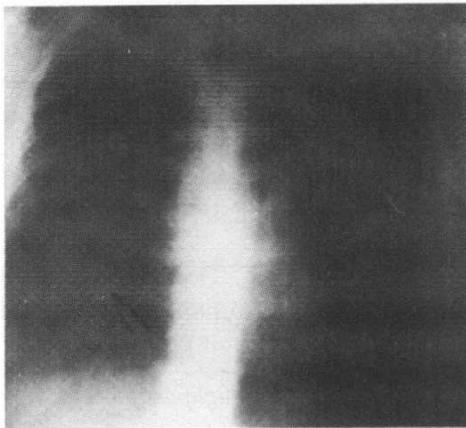
REHAUSSEMENT DES HAUTES FRÉQUENCES

- Maintient la moyenne et les BF
- Amplifie les HF

$$H'(u, \nu) = K_0 + H(u, \nu)$$



Exemple : Filtre PH Butterworth + Rehaussement HF + Égalisation



FILTRAGE FRÉQUENTIEL

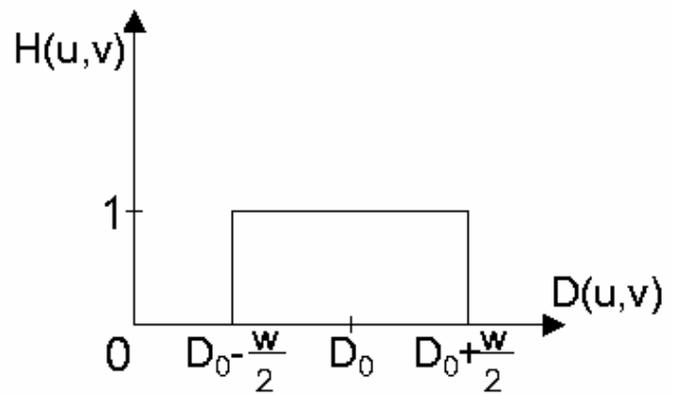
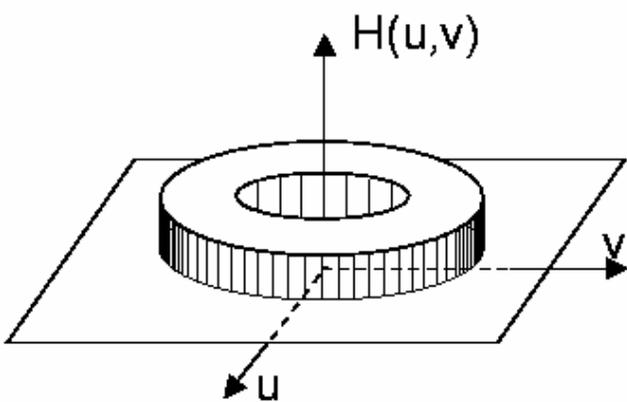
FILTRE PASSE-BANDE

$$H(u, \nu) = \begin{cases} 0 & D(u, \nu) \leq D_0 - \frac{w}{2} \\ 1 & D_0 - \frac{w}{2} < D(u, \nu) < D_0 + \frac{w}{2} \\ 0 & D(u, \nu) \geq D_0 + \frac{w}{2} \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

w : Largeur de Bande



FILTRAGE FRÉQUENTIEL

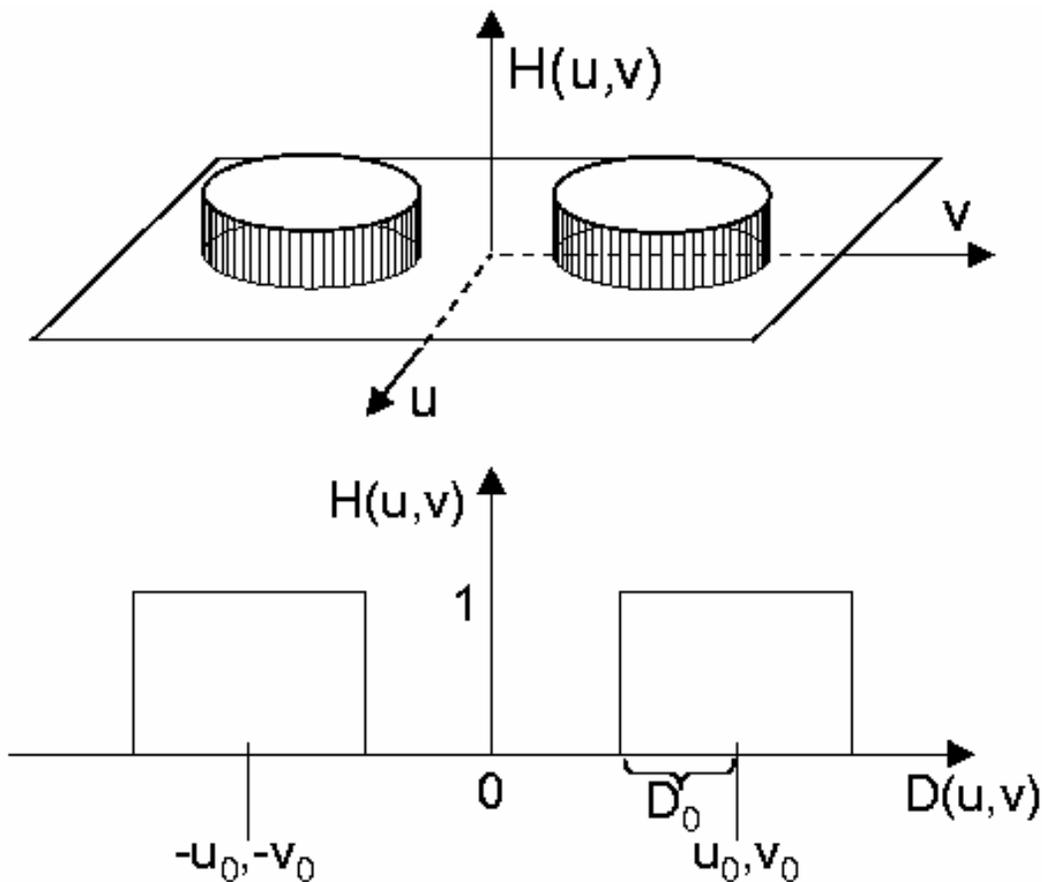
FILTRE SPECTRAL LOCAL

$$H(u, \nu) = \begin{cases} 1 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 0 & \text{sinon} \end{cases}$$

$$D_1(u, \nu) = \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2}$$

$$D_2(u, \nu) = \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2}$$

D_0 : Rayon autour de la fréquence locale
 u_0, ν_0 : Coordonné de la fréquence locale



FILTRAGE FRÉQUENTIEL

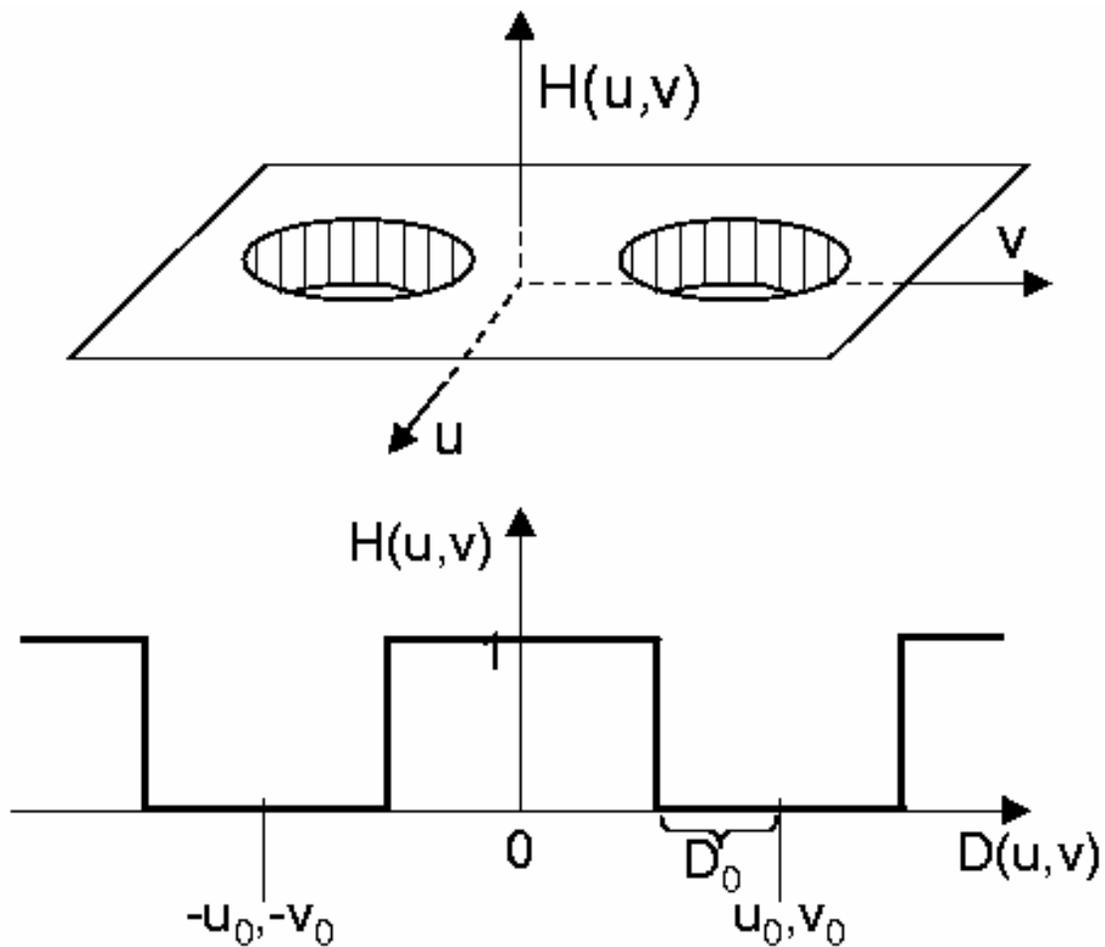
FILTRE A REJECTION DE BANDE (1)

$$H(u, \nu) = \begin{cases} 0 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 1 & \text{sinon} \end{cases}$$

$$D_1(u, \nu) = \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2}$$

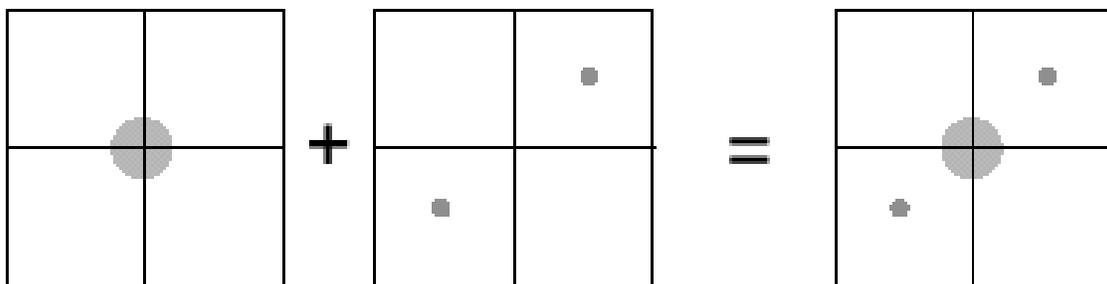
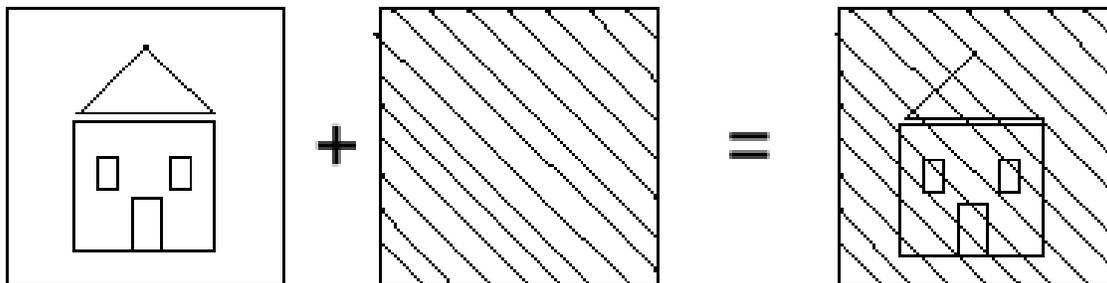
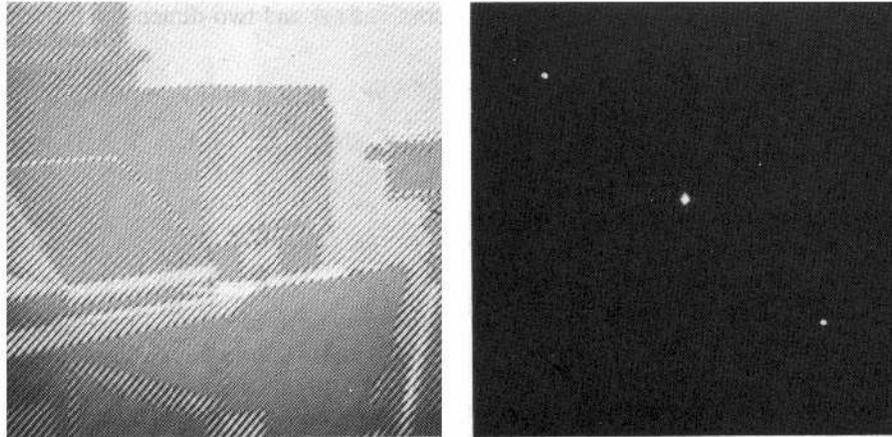
$$D_2(u, \nu) = \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2}$$

D_0 : Rayon autour de la fréquence locale
 u_0, ν_0 : Coordonné de la fréquence locale



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FILTRE A REJECTION DE BANDE (2)



FILTRAGE FRÉQUENTIEL

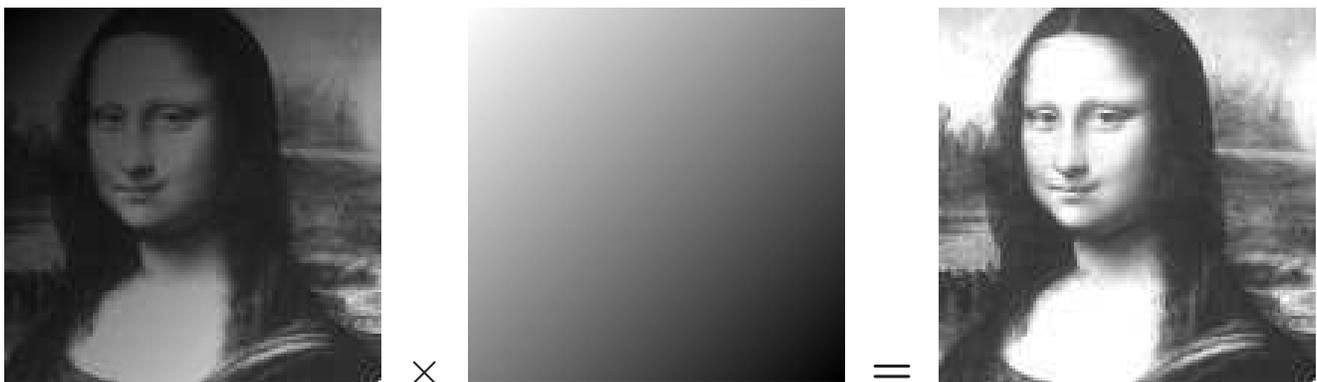
FILTRAGE HOMOMORPHIQUE (1)



$$f(x, y) = i(x, y) \cdot r(x, y)$$

- $i(x, y)$: Illumination ► très basses fréquences
- $r(x, y)$: Réflectance ► plus hautes fréquences

► Opération Ponctuelle



► Filtrage Homomorphique

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FILTRAGE HOMOMORPHIQUE (2)

$$\begin{aligned} z(x,y) &= \log(f(x,y)) \\ &= \log(i(x,y) \cdot r(x,y)) = \log(i(x,y)) + \log(r(x,y)) \end{aligned}$$



$$Z(u,v) = I(u,v) + R(u,v)$$

Filtre Passe-haut $H(u,v)$:

$$S(u,v) = H(u,v) \cdot Z(u,v) = H(u,v) \cdot I(u,v) + H(u,v) \cdot R(u,v)$$

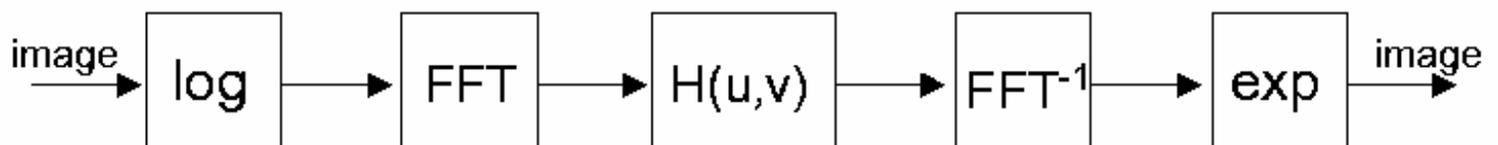


$$s(x,y) = i'(x,y) + r'(x,y)$$

$$g(x,y) = \exp(s(x,y)) = \exp(i'(x,y)) \cdot \exp(r'(x,y))$$

$$= i_0(x,y) \cdot r_0(x,y)$$

Diagramme Bloc



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FILTRAGE HOMOMORPHIQUE (3)

