



DIRO
IFT 6150

TRAITEMENT D'IMAGES FILTRAGE FRÉQUENTIEL

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FILTRAGE FRÉQUENTIEL

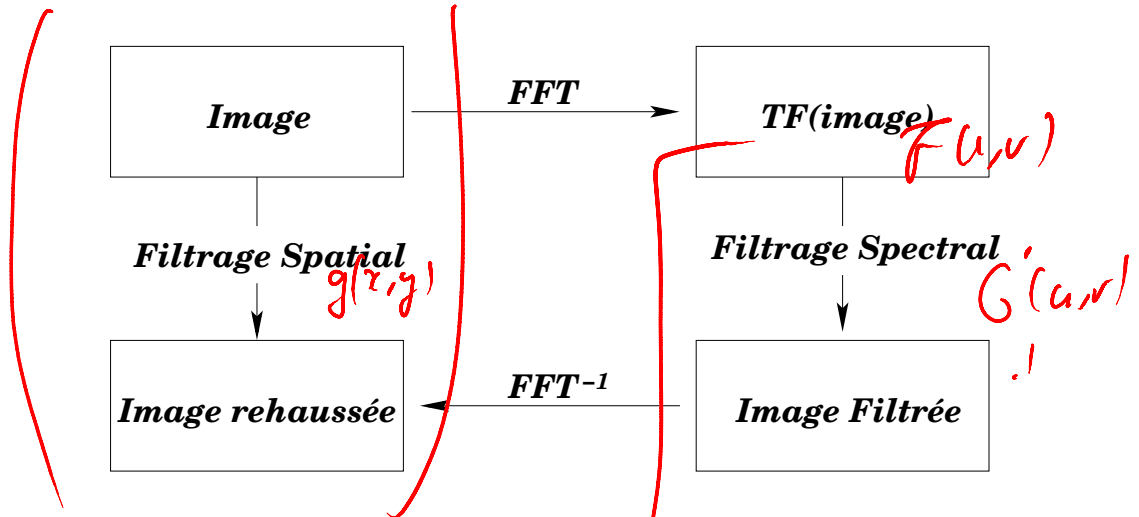
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FILTRAGE FRÉQUENTIEL

INTRODUCTION

Rehaussement d'Images par Filtrage Spatial/Fréquentiel



Théorème de Convolution -Rappel-

$$\begin{aligned} f(x, y) * g(x, y) &\leftrightarrow F(u, \nu) \cdot G(u, \nu) \\ f(x, y) \cdot g(x, y) &\leftrightarrow F(u, \nu) * G(u, \nu) \end{aligned}$$

donc, si $f(x, y)$ est l'image à filtrer et $G(u, \nu)$, le filtre fréquentiel

$$f(x, y) * g(x, y) = \mathcal{F}^{-1}\{F(u, \nu) \cdot G(u, \nu)\}$$

↑ ↑
produit de convolution

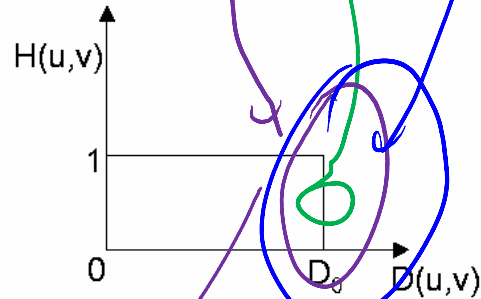
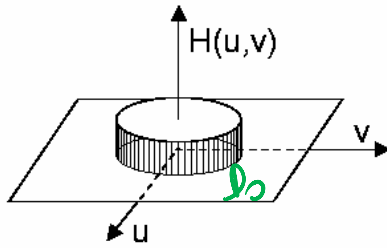
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (1)

$$H(u, \nu) = \begin{cases} 1 & D(u, \nu) \leq D_0 \\ 0 & D(u, \nu) > D_0 \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

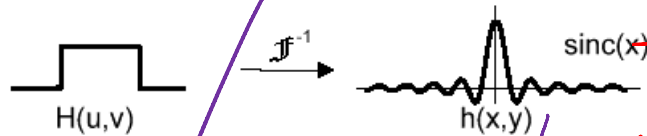


Problème

$$G(u, \nu) = F(u, \nu) \cdot H(u, \nu)$$

Convolution Theorem

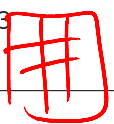
$$g(x, y) = f(x, y) * h(x, y)$$



$\uparrow D_0 \rightarrow \downarrow$ Rayons des ondulations (-flou)

① discontinue \Rightarrow saut
freq
grao

Saut
Spatial
pas gran \Rightarrow discont

② $\omega \neq 0$
 \times  \Rightarrow filtre
pas gran

coef
+ ~~×~~ filtre
pas gran

③ ondulations

Ring, y

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (2)



(a)



(b)



(c)



(d)



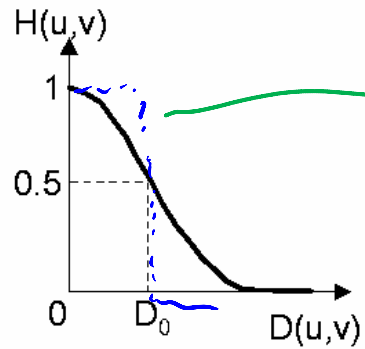
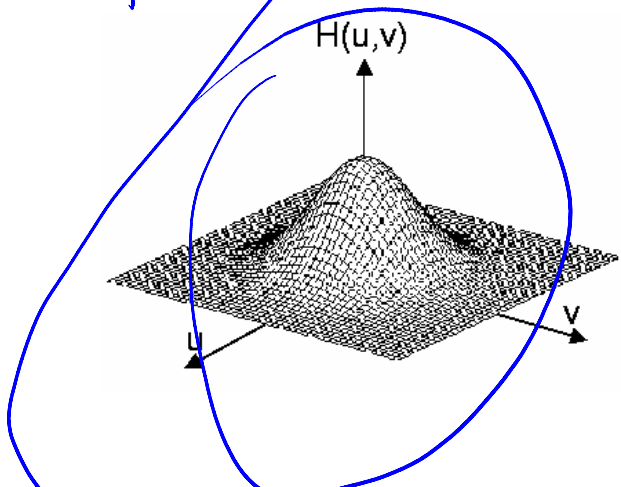
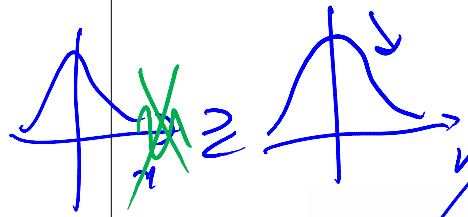
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS DE BUTTERWORTH (1)

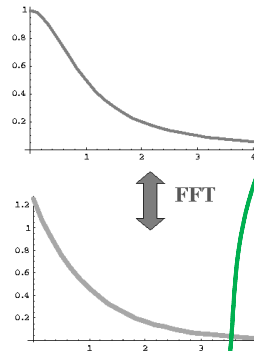
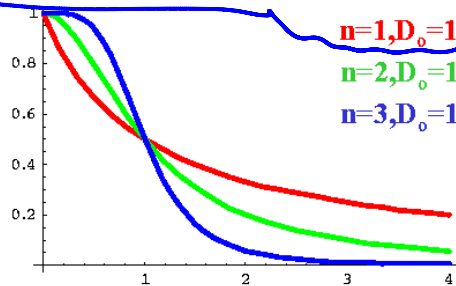
$$H(u, \nu) = \frac{1}{1 + (D(u, \nu)/D_0)^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure



discrete



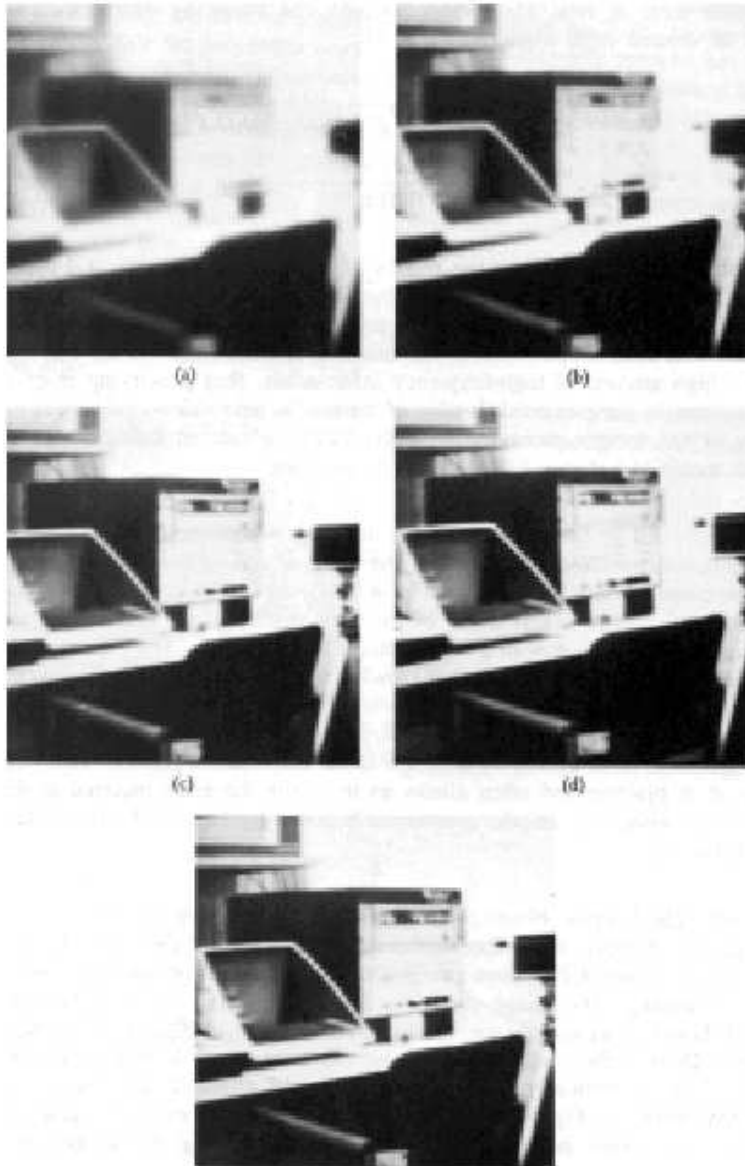
Note

- Flou moins brutal et aucune ondulation -

ringing

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS DE BUTTERWORTH (2)



6

Ungefiltert
 signal 100%
 bruit 100%
 RSB = 1

Butterworth
 haut

Signal 10%
 bruit 75%
 RSB = 0.15

FILTRAGE FRÉQUENTIEL

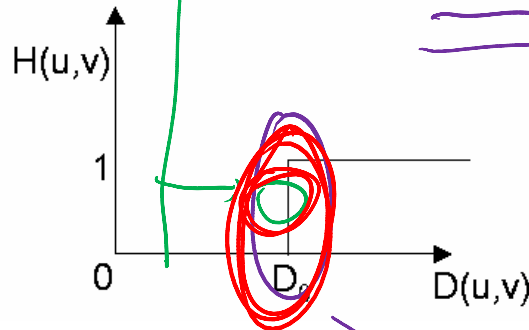
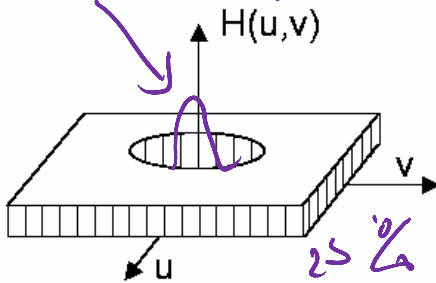
FILTRE PASSE-HAUT IDÉAL

$$H(u, \nu) = \begin{cases} 1 & D(u, \nu) \geq D_0 \\ 0 & D(u, \nu) < D_0 \end{cases}$$

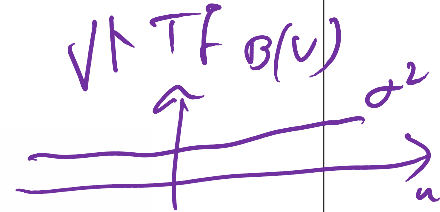
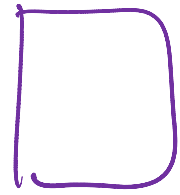
$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

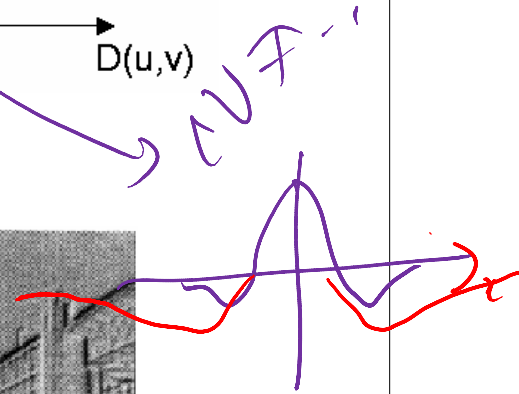
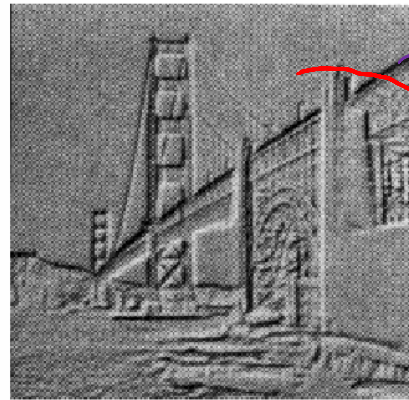
Spécifie
vague



vague de l'hor



Exemple



$$i \times \begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$l \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$\begin{array}{r} 110 \pm 2 \\ 100 \pm 2 \\ \hline 10 \pm 4 \end{array}$$



+ pole

wague = 10

pole negon

2 ss

gris neg

0

Recel (0 - 2 ss)

⇒ dérivée première (d. l'hor hve)

Amplification du bruit

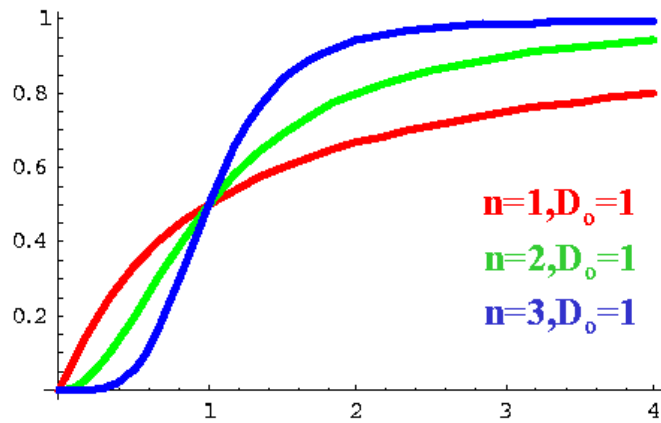
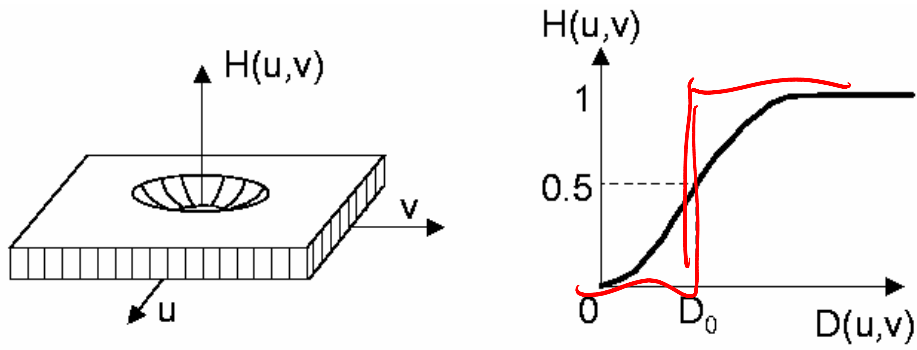
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-HAUT DE BUTTERWORTH

$$H(u, \nu) = \frac{1}{1 + (D_0/D(u, \nu))^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

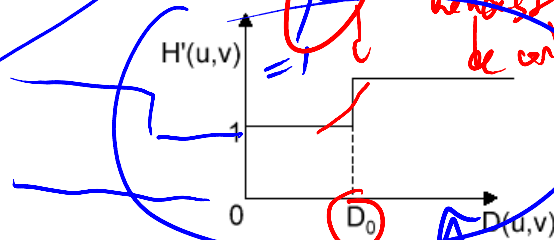


FILTRAGE FRÉQUENTIEL

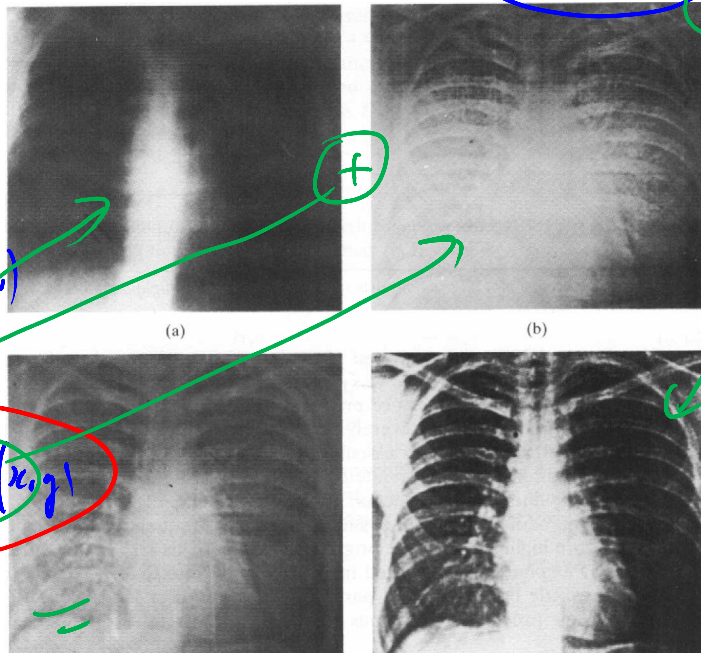
REHAUSSEMENT DES HAUTES FRÉQUENCES

- Maintient la moyenne et les BF
- Amplifie les HF

$$H'(u,v) = K_0 + H(u,v)$$



Exemple : Filtre PH Butterworth + Rehaussement HF + Egalisation



$$H'(u,v) = 1 + H(u,v)$$

$$I(u,v) \cdot (1 + H(u,v))$$

$$I(u,v) + I(u,v) \cdot H(u,v)$$

$$I(x,y) + I(x,y) \cdot \text{psf}(x,y)$$

psf assoc un filtre passe bas

FILTRAGE FRÉQUENTIEL

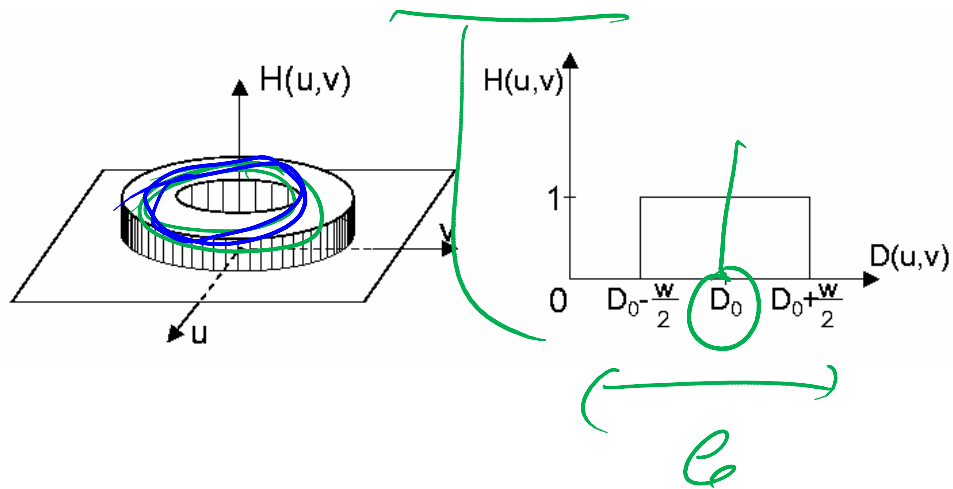
FILTRE PASSE-BANDE

$$H(u, \nu) = \begin{cases} 0 & D(u, \nu) \leq D_0 - \frac{w}{2} \\ 1 & D_0 - \frac{w}{2} < D(u, \nu) < D_0 + \frac{w}{2} \\ 0 & D(u, \nu) \geq D_0 + \frac{w}{2} \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

w : Largeur de Bande



FILTRAGE FRÉQUENTIEL FILTRE SPECTRAL LOCAL

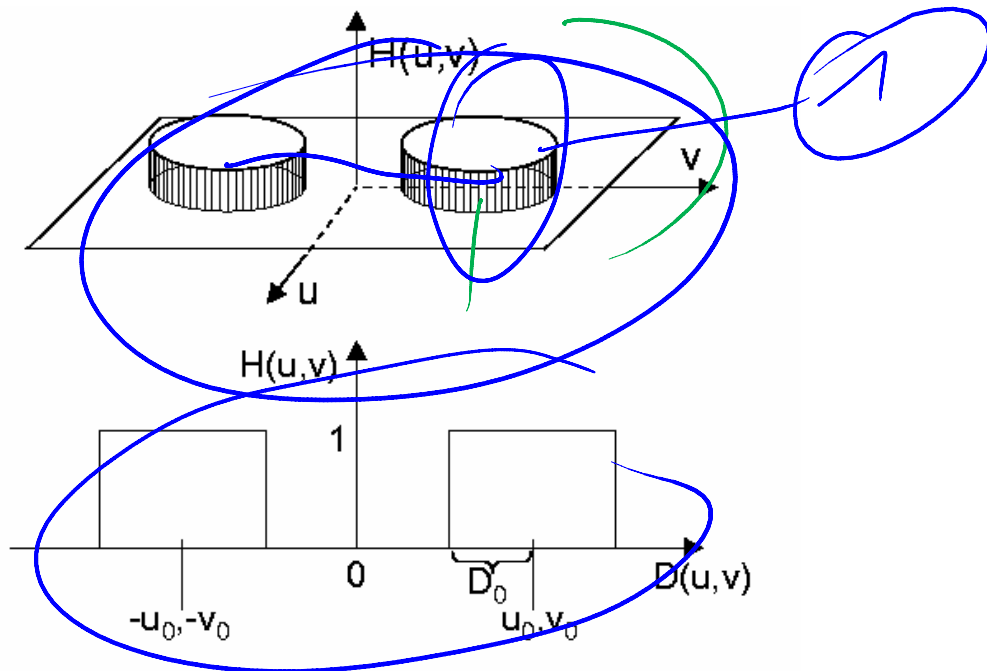
$$H(u, \nu) = \begin{cases} 1 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 0 & \text{sinon} \end{cases}$$

$$D_1(u, \nu) = \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2}$$

$$D_2(u, \nu) = \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2}$$

D_0 : Rayon autour de la fréquence locale

u_0, ν_0 : Coordonnée de la fréquence locale



FILTRAGE FRÉQUENTIEL

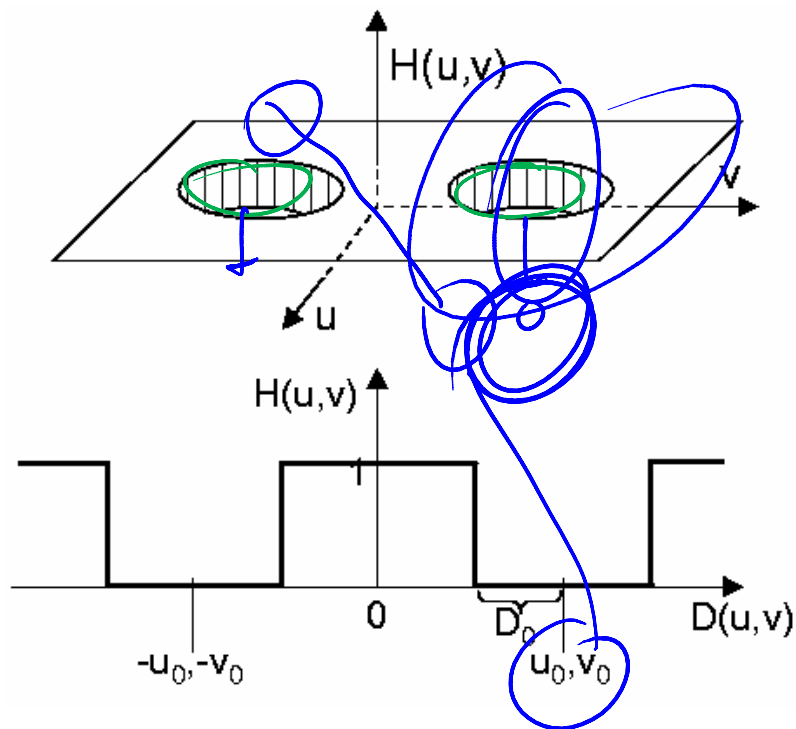
FILTRE A REJECTION DE BANDE (1)

$$H(u, \nu) = \begin{cases} 0 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 1 & \text{sinon} \end{cases}$$

$$D_1(u, \nu) = \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2}$$

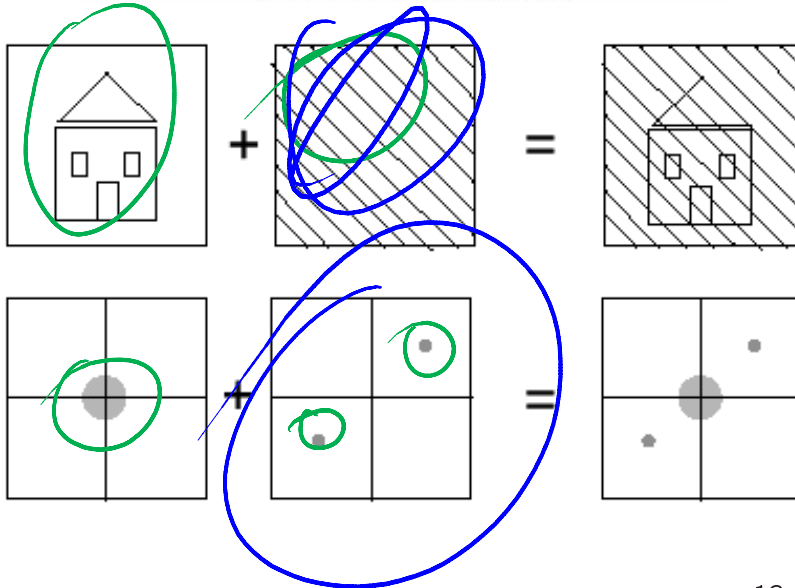
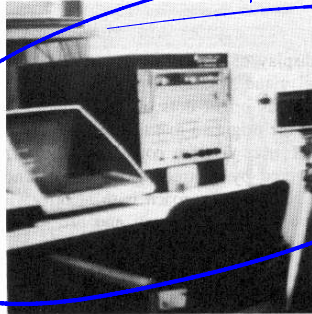
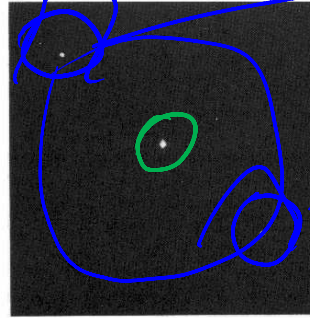
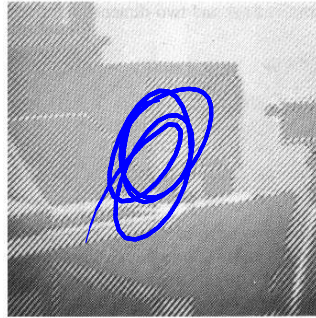
$$D_2(u, \nu) = \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2}$$

D_0 : Rayon autour de la fréquence locale
 u_0, ν_0 : Coordonné de la fréquence locale



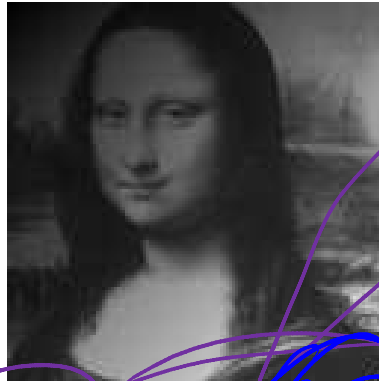
FILTRAGE FRÉQUENTIEL

FILTRE A REJECTION DE BANDE (2)



FILTRAGE FRÉQUENTIEL

FILTRAGE HOMOMORPHIQUE (1)



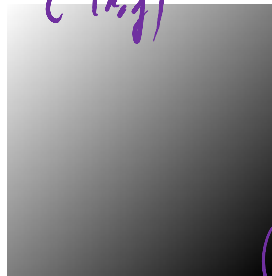
$$f(x, y) = i(x, y) r(x, y)$$

- $i(x, y)$: Illumination ► très basses fréquences
- $r(x, y)$: Réflectance ► plus hautes fréquences

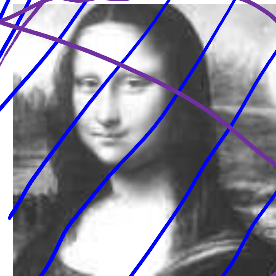
► Opération Ponctuelle



×



=



► Filtrage Homomorphique

FILTRAGE FRÉQUENTIEL FILTRAGE HOMOMORPHIQUE (2)

$$z(x,y) = \log(f(x,y)) \\ = \log(i(x,y) \cdot r(x,y)) = \log(i(x,y)) + \log(r(x,y))$$

$$Z(u,v) = I(u,v) + R(u,v)$$

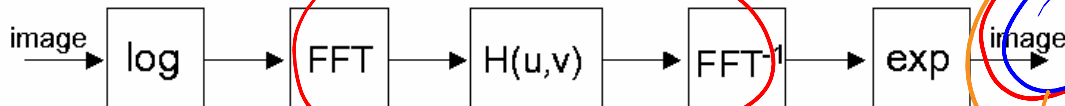
Filtre Passe-haut $H(u,v)$:

$$S(u,v) = H(u,v) \cdot Z(u,v) = H(u,v) \cdot I(u,v) + H(u,v) \cdot R(u,v)$$

$$s(x,y) = i'(x,y) + r'(x,y)$$

$$g(x,y) = \exp(s(x,y)) = \exp(i'(x,y)) \cdot \exp(r'(x,y)) \\ = i_0(x,y) \cdot r_0(x,y)$$

Diagramme Bloc



15

①

②

→

→

$\log(1+r_s)$

Centrée sur 0

$\log(1+r_s)$
have
et regere
free

$\log(\text{degré})$
have
free

have
et regere
free

Tham
be

degré
par le

have
free

Save
Cognent
no yes

Rike pen

have
free

FILTRAGE FRÉQUENTIEL

FILTRAGE HOMOMORPHIQUE (3)

