



TRAITEMENT D'IMAGES

FILTRAGE FRÉQUENTIEL

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FILTRAGE FRÉQUENTIEL

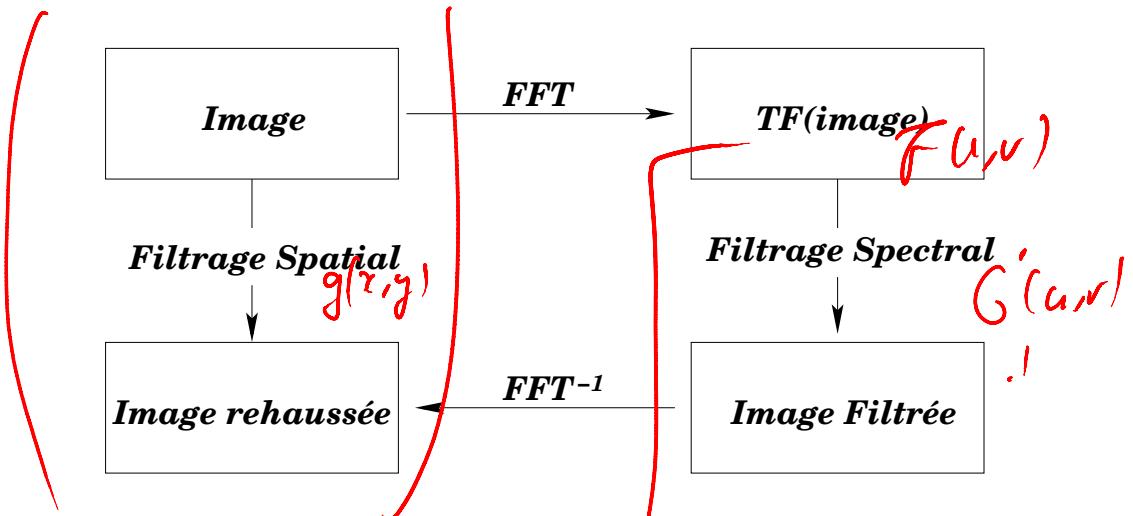
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FILTRAGE FRÉQUENTIEL

INTRODUCTION

Rehaussement d'Images par Filtrage Spatial/Fréquentiel



Théorème de Convolution -Rappel-

$$\begin{aligned} f(x, y) * g(x, y) &\Leftrightarrow F(u, v) \cdot G(u, v) \\ f(x, y) \cdot g(x, y) &\Leftrightarrow F(u, v) * G(u, v) \end{aligned}$$

donc, si $f(x, y)$ est l'image à filtrer et $G(u, v)$, le filtre fréquentiel

$$f(x, y) * g(x, y) = \mathcal{F}^{-1}\{F(u, v) \cdot G(u, v)\}$$

pour dég

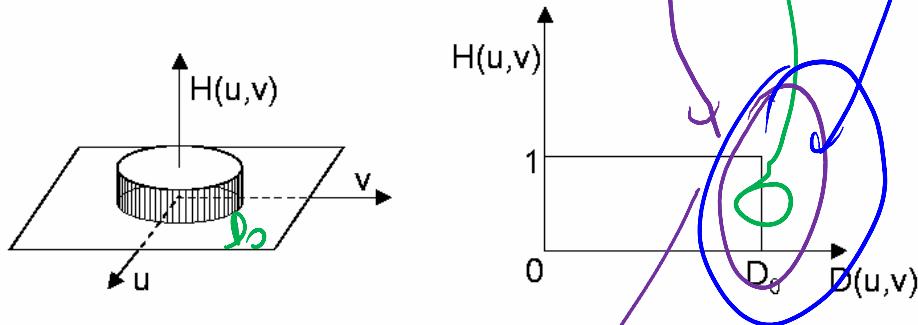
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (1)

$$H(u, v) = \begin{cases} 1 & D(u, v) \leq D_0 \\ 0 & D(u, v) > D_0 \end{cases}$$

$$D(u, v) = \sqrt{u^2 + v^2}$$

D_0 : Fréquence de Coupe



Problème

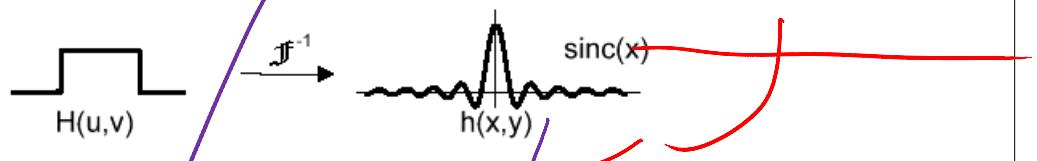
$$G(u, v) = F(u, v) \cdot H(u, v)$$

Convolution Theorem

$$g(x, y) = f(x, y) * h(x, y)$$

① discontinuité \geq Sinc freq Gras

Spatial Spatial tis gris \geq discont



$\uparrow D_0 \rightarrow$ Rayons des ondulations (-flou)

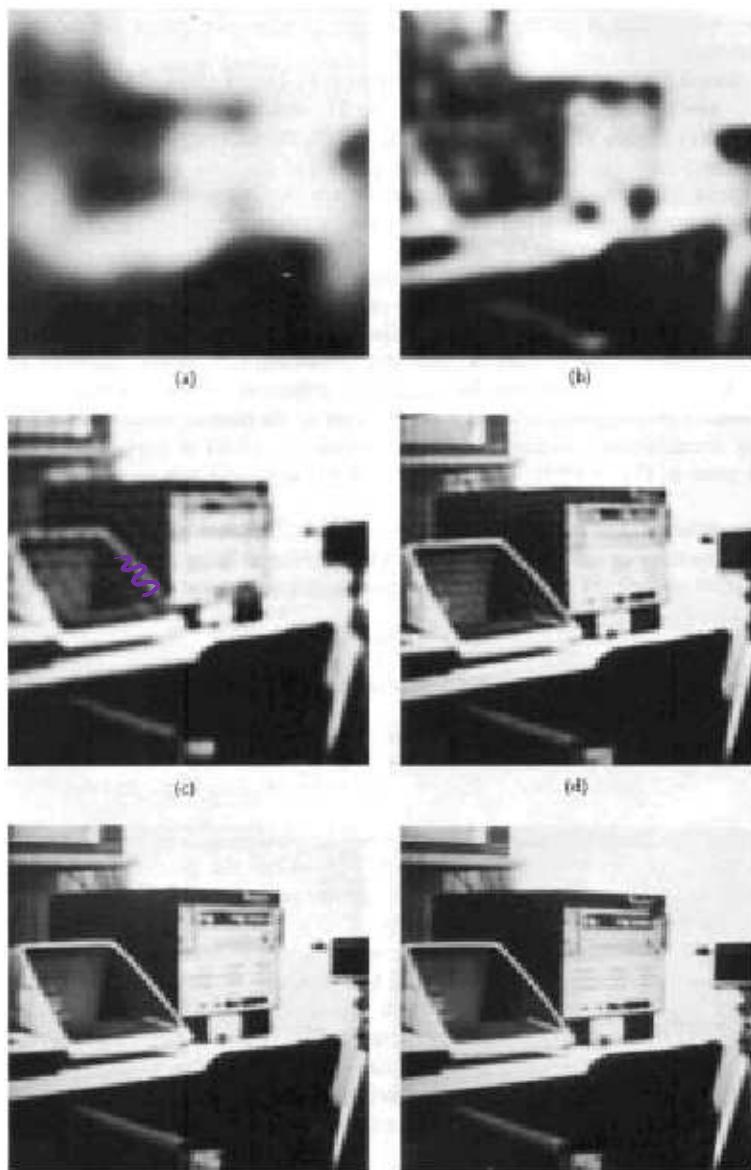
② \times \Rightarrow filtre passe bas

③ ondulations

Ring

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS IDÉAL (2)

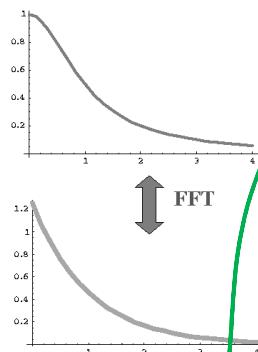
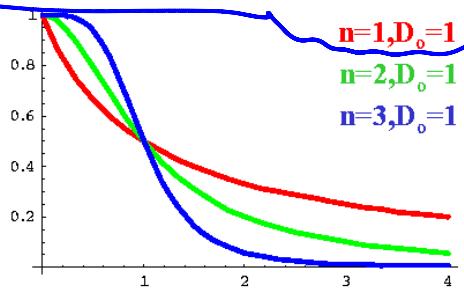
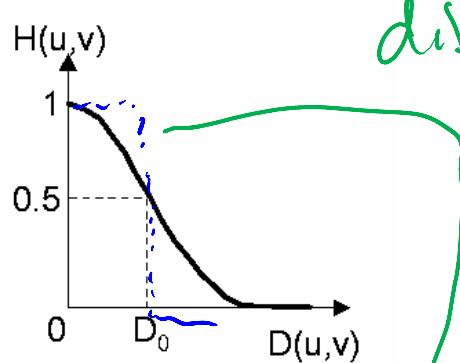
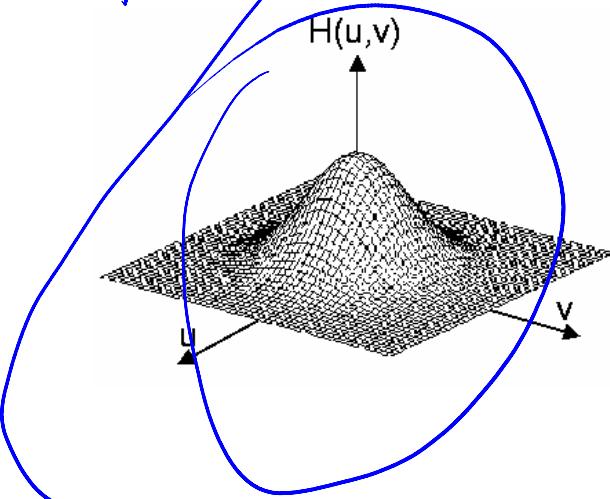
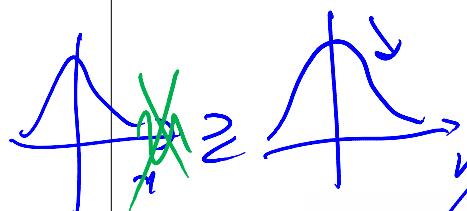


FILTRAGE FRÉQUENTIEL FILTRE PASSE-BAS DE BUTTERWORTH (1)

$$H(u, \nu) = \frac{1}{1 + (D(u, \nu)/D_0)^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupe



Note

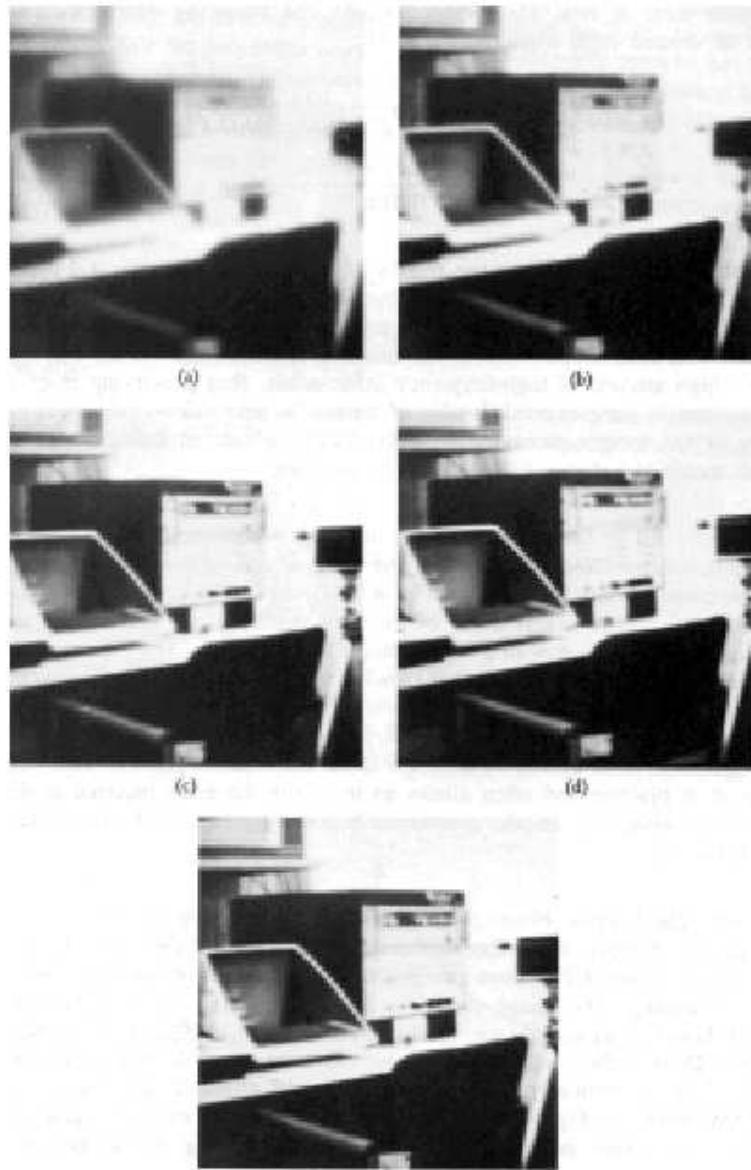
- Flou moins brutal et aucune ondulation -

Hингин

discontinu

FILTRAGE FRÉQUENTIEL

FILTRE PASSE-BAS DE BUTTERWORTH (2)



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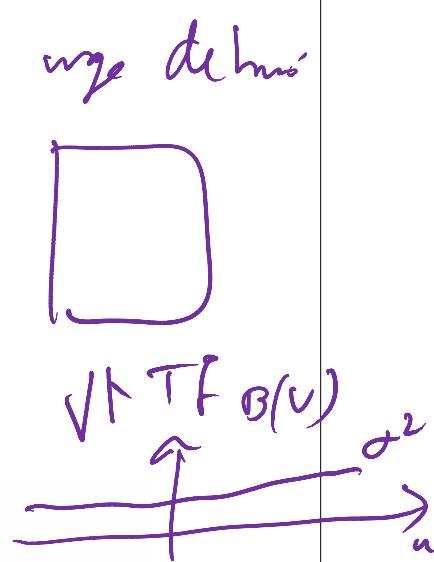
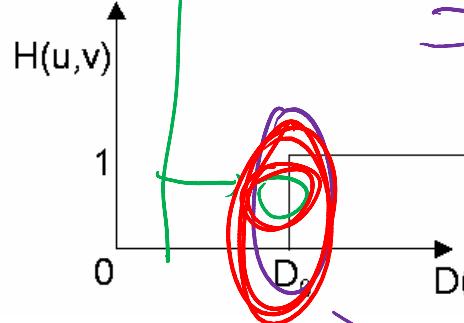
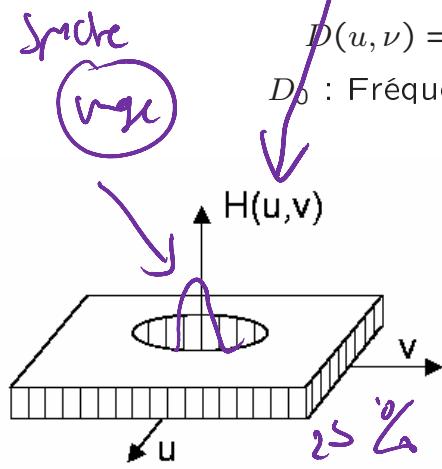


FILTRAGE FRÉQUENTIEL FILTRE PASSE-HAUT IDÉAL

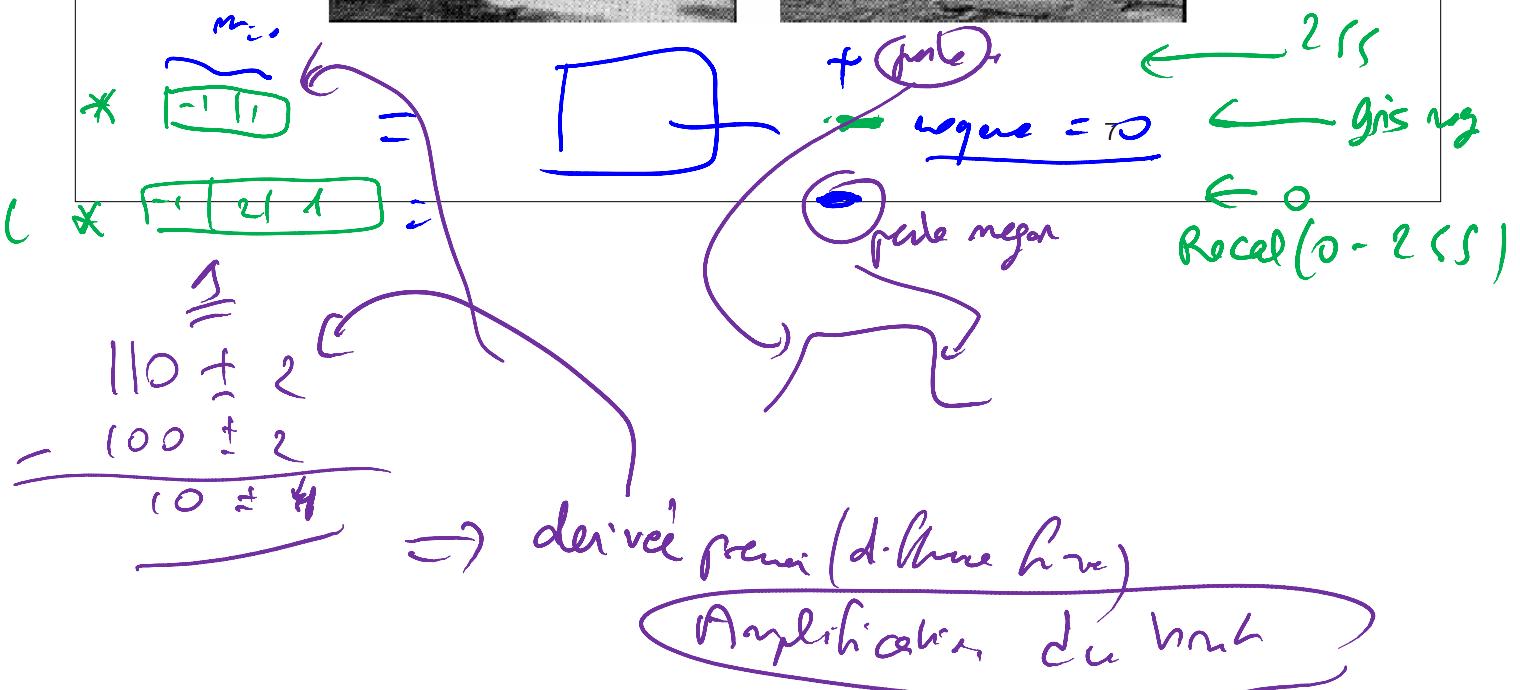
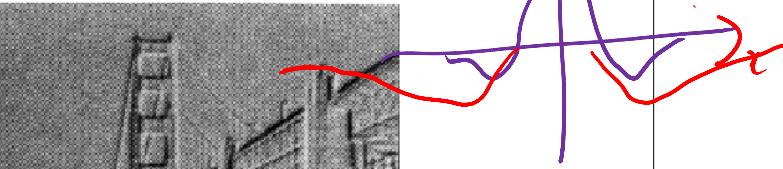
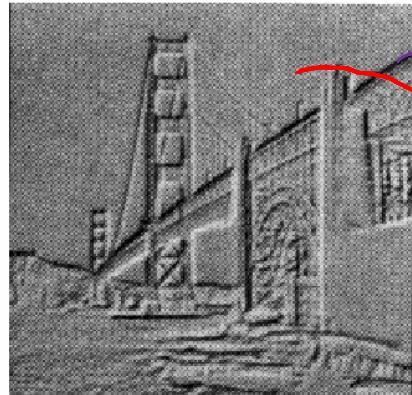
$$H(u, v) = \begin{cases} 1 & D(u, v) \geq D_0 \\ 0 & D(u, v) < D_0 \end{cases}$$

$$D(u, v) = \sqrt{u^2 + v^2}$$

D_0 : Fréquence de Coupe



Exemple



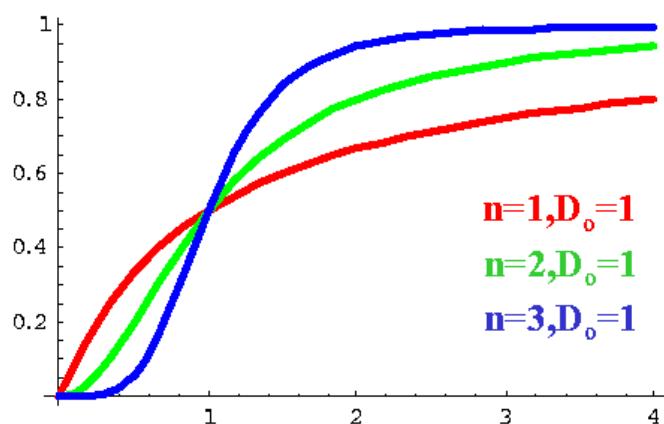
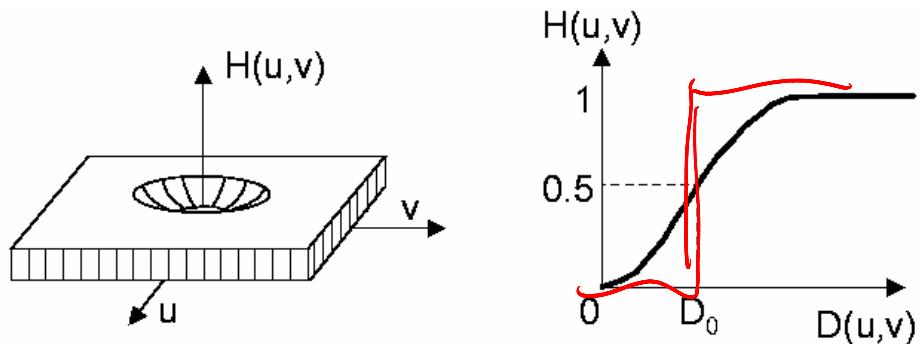
FILTRAGE FRÉQUENTIEL

FILTRE PASSE-HAUT DE BUTTERWORTH

$$H(u, \nu) = \frac{1}{1 + (D_0/D(u, \nu))^{2n}}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

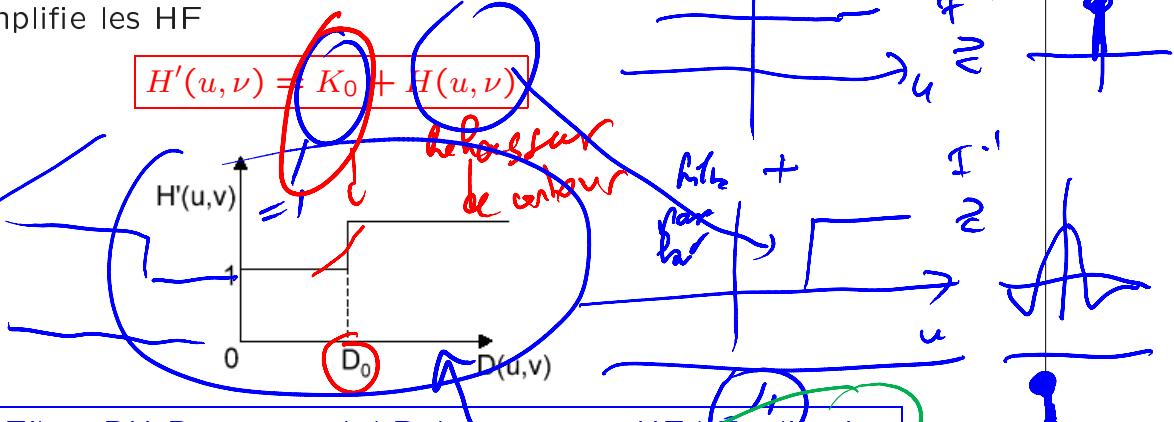
D_0 : Fréquence de Coupure



FILTRAGE FRÉQUENTIEL REHAUSSEMENT DES HAUTES FRÉQUENCES

- Maintient la moyenne et les BF
- Amplifie les HF

$$H'(u, v) = K_0 + H(u, v)$$

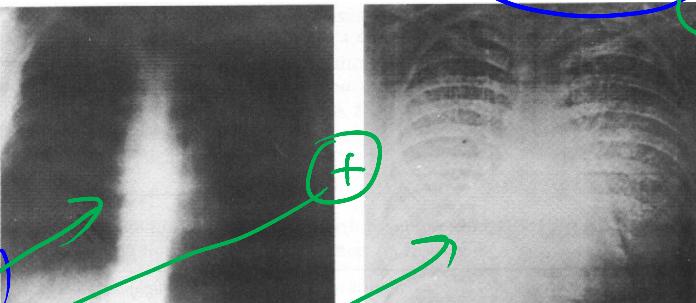


Exemple : Filtre PH Butterworth+Rehaussement HF+Egalisation

$$H'(u, v) = 1 + H(u, v)$$

$$I(u, v) \cdot (1 + H(u, v))$$

$$I(u, v) + I(u, v) \cdot H(u, v)$$

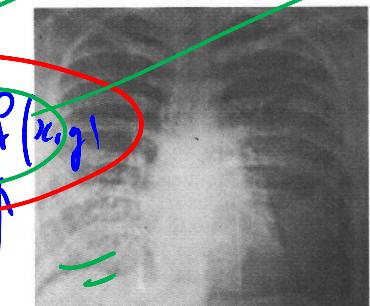


(a)

(b)

JP

$$K_L(x, y) + I(x, y) \otimes PSF(x, y)$$



PSF assoc
un filtre pour c

FILTRAGE FRÉQUENTIEL

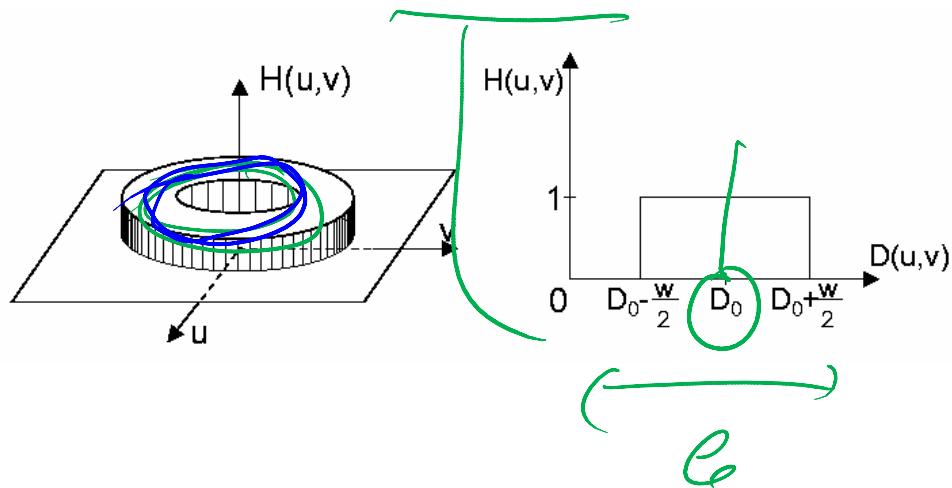
FILTRE PASSE-BANDE

$$H(u, \nu) = \begin{cases} 0 & D(u, \nu) \leq D_0 - \frac{w}{2} \\ 1 & D_0 - \frac{w}{2} < D(u, \nu) < D_0 + \frac{w}{2} \\ 0 & D(u, \nu) \geq D_0 + \frac{w}{2} \end{cases}$$

$$D(u, \nu) = \sqrt{u^2 + \nu^2}$$

D_0 : Fréquence de Coupure

w : Largeur de Bande

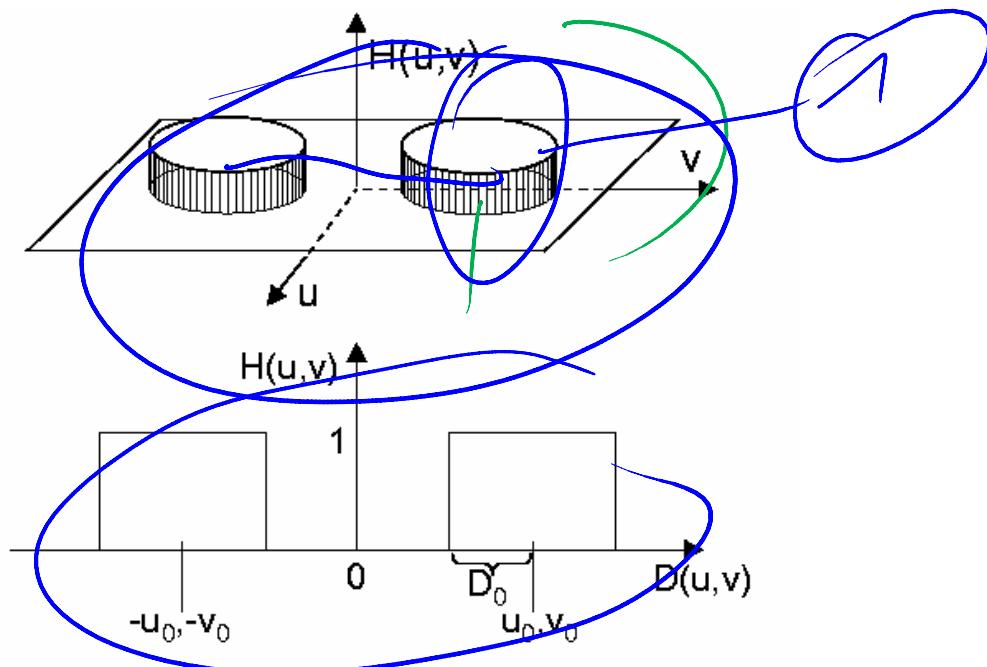


FILTRAGE FRÉQUENTIEL FILTRE SPECTRAL LOCAL

$$H(u, \nu) = \begin{cases} 1 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 0 & \text{sinon} \end{cases}$$

$$\begin{aligned} D_1(u, \nu) &= \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2} \\ D_2(u, \nu) &= \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2} \end{aligned}$$

D_0 : Rayon autour de la fréquence locale
 u_0, ν_0 : Coordonnée de la fréquence locale



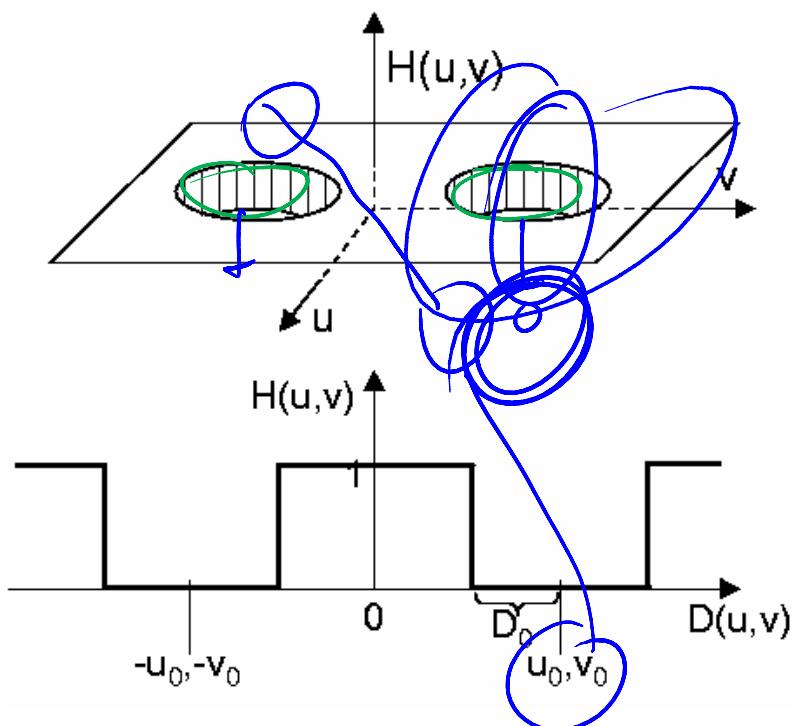
FILTRAGE FRÉQUENTIEL FILTRE A REJECTION DE BANDE (1)

$$H(u, \nu) = \begin{cases} 0 & D_1(u, \nu) \leq D_0 \text{ ou } D_2(u, \nu) \leq D_0 \\ 1 & \text{sinon} \end{cases}$$

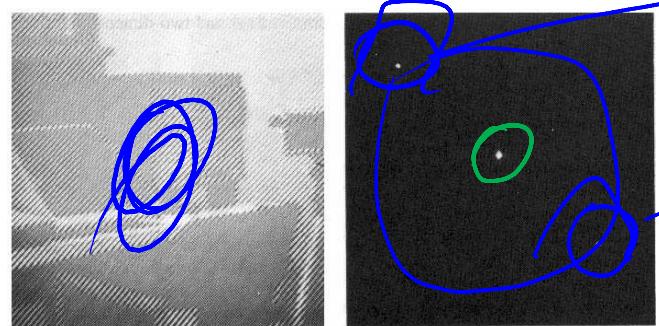
$$D_1(u, \nu) = \sqrt{(u - u_0)^2 + (\nu - \nu_0)^2}$$

$$D_2(u, \nu) = \sqrt{(u + u_0)^2 + (\nu + \nu_0)^2}$$

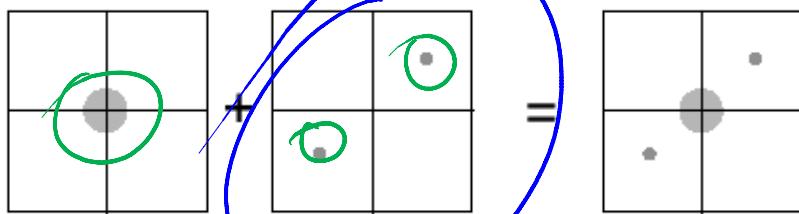
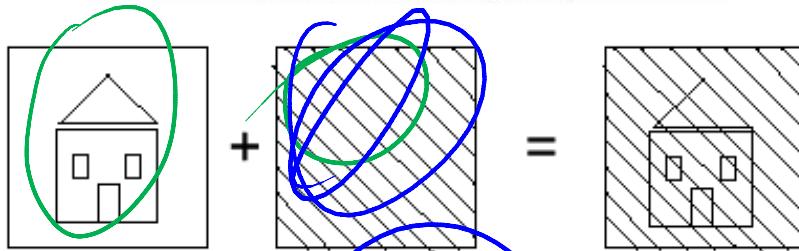
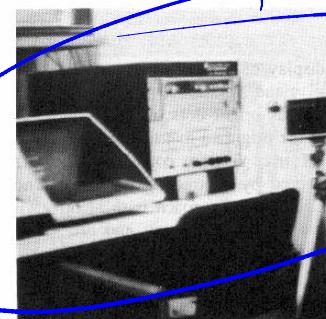
D_0 : Rayon autour de la fréquence locale
 u_0, ν_0 : Coordonnée de la fréquence locale



FILTRAGE FRÉQUENTIEL FILTRE A REJECTION DE BANDE (2)

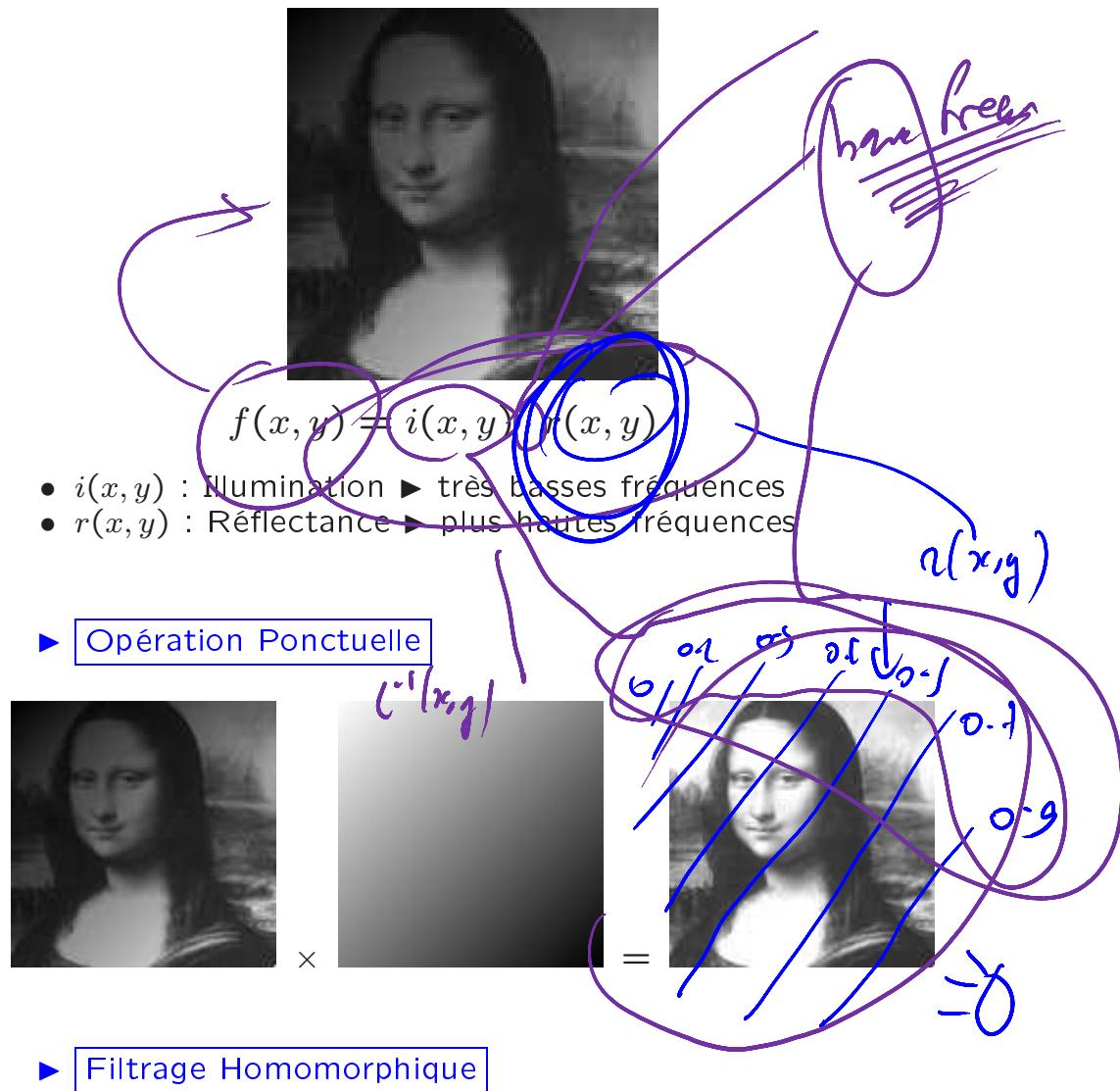


V_c



FILTRAGE FRÉQUENTIEL FILTRAGE HOMOMORPHIQUE (1)

u filtre



FILTRAGE FRÉQUENTIEL FILTRAGE HOMOMORPHIQUE (2)

$$z(x,y) = \log(f(x,y)) \\ = \log(i(x,y) \cdot r(x,y)) = \log(i(x,y)) + \log(r(x,y))$$

$$Z(u,v) = I(u,v) + R(u,v)$$

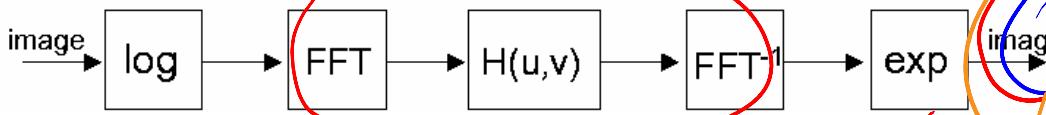
Filtre Passe-haut $H(u,v)$:

$$S(u,v) = H(u,v) \cdot Z(u,v) = H(u,v) \cdot I(u,v) + H(u,v) \cdot R(u,v)$$

$$s(x,y) = i'(x,y) + r'(x,y)$$

$$g(x,y) = \exp(s(x,y)) = \exp(i'(x,y)) \cdot \exp(r'(x,y)) \\ = i_0(x,y) \cdot r_0(x,y)$$

Diagramme Bloc



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① →
② →

$$\log(1 + \alpha g)$$

Centré sur 0

de polar
pour l'autre
très bas
freq.
Sans
compte
moyens

$\log(\text{base})$
base
et moyenne
fre

$\log(\text{depr})$
très haute
freq.

haute
et moy
freq

lente
grise

Thème
fre

FILTRAGE FRÉQUENTIEL
FILTRAGE HOMOMORPHIQUE (3)

