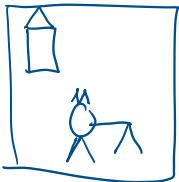


- today:
- latent variable SVMstruct - CCCP
  - deep learning - RNN

### Latent variables

motivation: semantic segmentation



Segmentation is espansive  $\rightarrow z$  "latent variable"

perhaps only have class labels  $\rightarrow y$

also: [Felzenszwalb et al TPAMI 2010  
"deformable parts models" for object recognition

$\hookrightarrow z$  there was an object part configuration

before, we had  $s(x, y; w) = \langle w, \varphi(x, y) \rangle$

now, consider  $s(x, y, z; w) = \langle w, \varphi(x, y, z) \rangle$

as before, could predict with argmax  
 $y, z \in Z$   $s(x, y, z; w)$

Learning  $\leftarrow$  CRF approach  $p(y|z)$   $\xrightarrow{\text{generalize}}$  hidden CRF  $p(y, z|x)$

similarly to latent variable model  
with graph. model  
 $\rightarrow$  marginalize  $z$  out

ML  $\rightarrow$  EM (expectation-maximization)

SVMstruct  
(surrogate loss  
minimization)

analogy for latent SVMstruct is CCCP  
(majorization minimization  
procedure)

### Latent SVMstruct

$$\ell(y, (\tilde{y}, \tilde{z}))$$

Generalized structured hinge loss

$$\ell(x, y, w) \stackrel{?}{=} \max_{\tilde{y}, \tilde{z}} \langle w, \varphi(x, \tilde{y}, \tilde{z}) \rangle + \ell(y, (\tilde{y}, \tilde{z}))$$

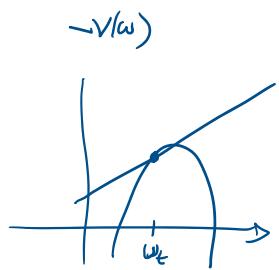
$\geq u(w)$

best score for  
ground truth  
 $\max_{z' \in Z} \langle w, \varphi(x, y, z') \rangle \geq \ell(y, \varphi(x, z'))$   
 $\geq v(w)$

here,  $\ell(x, y, w) = u(w) - v(w)$  where  $u, v$  are convex fct. of  $w$

"difference of convex functions"

$\hookrightarrow$  CCCP procedure is to approx. minimize this



### CCCP procedure:

- Linearize  $v(w)$  at  $w_t$  to get an upper bound on  $-v(w)$
- $w_{t+1}$  is obtained by minimizing the upperbound  $l(w) + \text{linearize piece}(w)$
- repeat  $\hookrightarrow$  a majorization-minimization procedure (EM is another example)

$$\left\{ \begin{array}{l} \delta_t(w) = v(w) - [v(w_t) + \langle \nabla v(w_t), w - w_t \rangle] \geq g(w) \quad \forall w \\ \text{(or subgradient)} \\ w_{t+1} = \underset{w}{\operatorname{argmin}} \delta_t(w) \end{array} \right. \quad \text{and} \quad \delta_t(w_t) = g(w_t)$$

Properties of procedure: • like EM, descent procedure i.e.  $g(w_{t+1}) \leq g(w_t)$

$$g(w_t) = \delta_t(w_t) \geq \delta_t(w_{t+1}) \geq g(w_{t+1}) \quad \text{upper bound} //$$

- local linear convergence to a stationary pt.

for latent SVMstruct

[see NeurIPS OPT workshop  
2012 paper]

\* CCCP for SVMstruct :  $v(w) = \max_{z'} \langle w, \ell(x, y, z') \rangle \Rightarrow \underset{z'}{\operatorname{argmax}} \langle w_t, \ell(x, y, z') \rangle$

 $\nabla v(w_t) = \ell(x, y, \hat{z}(x, y; w_t))$ 
 $\hat{z}(x, y, w_t)$ 
 $\Rightarrow \delta_t(w) = \max_{(\tilde{y}, \tilde{z})} \langle w, \ell(x, \tilde{y}, \tilde{z}) \rangle + l(y, \tilde{y}, \tilde{z}) - \langle w, \ell(x, y, \hat{z}_t) \rangle + \text{const.}$ 

$\sim$  like SVMstruct objective

CCCP algorithm for latent SVMstruct:

repeat :

- fill in  $\hat{z}_t^{(i)}$  for all ground truth  $y^{(i)}$  using  $w_t$
- solve a standard SVMstruct to get  $w_{t+1}$
- repeat

10h 16

### Deep learning

go from  $\langle w, \ell(x, y) \rangle$  to  $\langle w, \ell(x, y; \theta) \rangle$

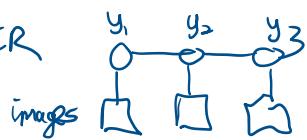
can learn

$\rightarrow$  now for "Deep learning" features in a structured model

go from  $\langle w, \varphi(x, y) \rangle$  to  $\langle w, \Psi(x, y, \underline{\Theta}) \rangle$

I) plug in "deep learning" features in a structured pred. model

example: OCR



example:  
[Vu & al. ICCV 2015]  
"context aware CNNs for person head detection"

$$\text{so far } \varphi_t(x_t, y_t) = \begin{pmatrix} 0 \\ x_t \\ 0 \end{pmatrix}_{\text{on } y_t}$$

$$\text{Instead } \varphi_t(x_t, y_t) = \begin{pmatrix} \text{NN}_t(x_t) \\ 0 \end{pmatrix}_{\text{on } y_t}$$

pre-trained on  
image net e.g.

II) "end-to-end" training structured prediction energy networks (SPENs)



$$\varphi(x, y, \underline{\Theta}) = \langle w, \Psi(x, y, \underline{\Theta}) \rangle$$

III) recurrent neural networks (RNN)

$$\text{motivation: } p(y|x) = \prod_{t \leq T} p(y_t | y_{1:t-1}, x) \xrightarrow[\text{chain rule}]{\text{graphical modeling approach}}$$

$$y_t \perp \!\!\! \perp y_{1:t-1} | y_{1:t-1}$$

$$\frac{1}{Z(x)} \prod_{t \leq T} p(y_t | y_{1:t-1}, x)$$

[Directed]

[Undirected]

RNN  $\hookrightarrow$  "structured parameterization" with no condi. independence assumptions

$$\text{of } p(y_t | y_{1:t-1}, x)$$

using a BN

$\hookrightarrow$  usually loose  
exact decoding

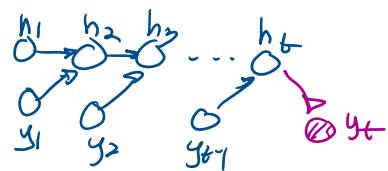
$$h_{t+1} \triangleq f(h_t, x, y_t, w)$$

$$h_t = f(f(\dots f(\underbrace{f(h_1, x, y_1, w)}_{h_2}, x, y_2), \dots), x, y_{t-1}, w)$$

define

$$p(y_t | y_{1:t-1}, x) \propto \exp(c(y_t)^T \tilde{W} h_t)$$

$\uparrow$  e.g.  
"code" for  $y_t$



$$p(y_t | y_{1:t-1}, x)$$

$\hookrightarrow$  given by a deep NN architecture

e.g. word embedding  
in NMT and can be fine tuned

standard training : using MCL

standard learning: using MCL

$$\text{f.e. } \min_{\mathbf{w}, \tilde{\mathbf{w}}} \sum_{i=1}^n \log p(y^{(i)} | x^{(i)})$$

$\sum_t \log p(\hat{y}_t | y_{1:t-1}, x)$   
output of a deep RNN

"backer forcing"  
yield  
"exposure problem"  
i.e. don't know  $p(y_t | \text{unseen}(y_{1:t-1}))$

for ML, do SGD on obj.

$$\text{gradient of } \log p(y^{(i)} | y_{1:t-1}, x^{(i)}; \mathbf{w}, \tilde{\mathbf{w}}) \\ \rightarrow \text{use back propagation}$$

decoding  $\Leftarrow$  argmax  $\sum_{\hat{y} \in \mathcal{Y}} \log p(\hat{y}_t | \hat{y}_{1:t-1}, x) \rightarrow \text{NP hard?}$

$\rightsquigarrow$  need approximation

greedy decoding  $\hat{y}_t = \underset{\hat{y} \in \mathcal{Y}_t}{\text{argmax}} p(y_t | \hat{y}_1, \dots, \hat{y}_{t-1}, x)$

beam search

"greedy decoding with memory of size  $k$ "  
 $\hookrightarrow$  size of beam

beam search: goal: construct  $\hat{y}_1, \dots, \hat{y}_T$

beam of size  $L$  (memory)

- at step  $t$ , you have  $L$  candidate solution prefixes  $y_{1:t}^{(1)}, y_{1:t}^{(2)}, \dots, y_{1:t}^{(L)}$

expand possible next choice  $|\mathcal{Y}_{t+1}| \cdot L$

score them (e.g.  $\log p(y_{t+1} | y_{1:t}^{(1)}, x) + \log p(y_{1:t}^{(1)}, x)$ )

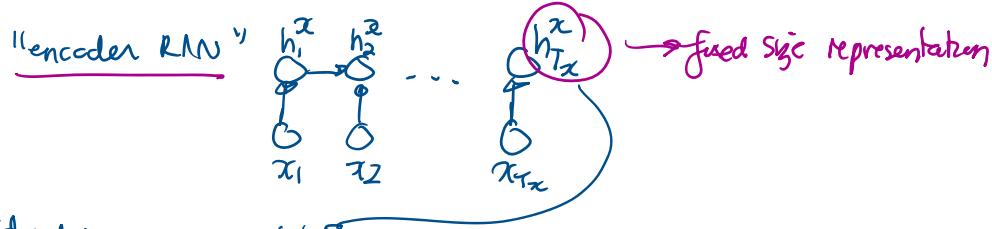
then keep top  $L$  candidates as  $\{y_{1:t+1}^{(1)}, \dots, y_{1:t+1}^{(L)}\}$

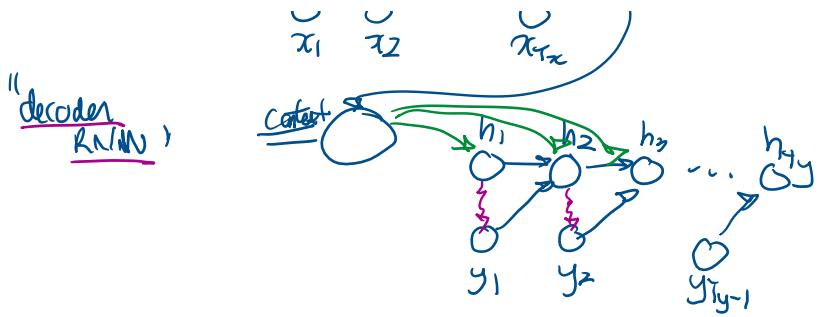
vs.

Viterbi alg. which does "backtracking" to correct past mistakes

seq2seq a.k.a. encoder / decoder architecture

$\hookrightarrow$  useful way to get  $p(y_t | y_{1:t-1}, x)$  for a RNN when  $x$  has variable length





### Issues:

a) variable length output?  $\rightarrow$  end-of-sequence symbol

b) how to handle long input sequence  $x$ ?

problem: Need to summarize input sentence in one context vector of fixed length

solution: "attention mechanism"

c) vanishing gradient?

- LSTM
- gated recurrent unit (GRU)
- etc. ..

d) sequential processing  
 $\rightarrow$  fixed by "transformer" architecture

"attention is all you need"

$\rightsquigarrow$  GPT & al.

"self-attention")

$\hookrightarrow$  quadratic cost  
 $\rightarrow$  need tricks to handle  
long context / sequence